A Backstepping Design of a Control System for a Magnetic Levitation System

Examensarbete utfört i Reglerteknik
vid Tekniska Högskolan i Linköping
av

Nawrous Ibrahim Mahmoud

Reg nr: LiTH-ISY-EX-3383
Linköping 2003

A Backstepping Design of a Control System for a Magnetic Levitation System

Examensarbete utfört i Reglerteknik
vid Tekniska Högskolan i Linköping
av

Nawrous Ibrahim Mahmoud

Reg nr: LiTH-ISY-EX-3383

Supervisor: **Magnus Åkerblad**

Examiner: **Torkel Glad**

Linköping 5th September 2003.

Avdelning, Institution
Division, Department

Institutionen för Systemteknik
581 83 LINKÖPING

Datum
Date
2003-09-01

Språk
Language
Svenska/Swedish
X Engelska/English

Rapporttyp
Report category
Licentiatavhandling
X Examensarbete
C-uppsats
D-uppsats
Övrig rapport

ISBN

ISRN LITH-ISY-EX-3383-2003

Serietitel och serienummer
Title of series, numbering

ISSN

URL för elektronisk version
http://www.ep.liu.se/exjobb/isy/2003/3383/

Titel
Title

En Backstepping Design av Reglersystem för Magnetsvävare

A Backstepping Design of a Control System for a Magnetic Levitation System

Författare
Author

Nawrous Ibrahim Mahmoud

Sammanfattning
Abstract

The subject of this thesis is the design of a control law for a magnetic levitation system, which in this case is the system 33-210. The method used is backstepping technique and specifically adaptive observer backstepping due to parameter uncertainties and lack of access to all the states of the system. The second state of the system, the speed of the steel ball, was estimated by a reduced order observer. The model used gave us the opportunity to estimate a parameter which in the literature is denoted virtual control coefficient. Backstepping method gives us a rather straight forward way to design the controlling unit for a system with these properties. Stabilization of the closed-loop system is achieved by incorporating a Lypapunov function, which were chose a quadratic one in this thesis. If the derivative of this function is rendered negative definite by the control law, then we achieve stability. The results of the design were evaluated in simulations and real-time measurements by testing the tracking performance of the system. The simulation results were very promising and the validations in real-time were satisfying. Note that this has been done in previous studies; the new aspect here is the limitation of the voltage input. The real-time results showed that the parameter estimation converges only locally.

Nyckelord
Keyword

Backstepping, unknown virtual control coefficients, magnetic levitation system, clf

Abstract

The subject of this thesis is the design of a control law for a magnetic levitation system, which in this case is the system 33-210. The method used is backstepping technique and specifically adaptive observer backstepping due to parameter uncertainties and lack of access to all the states of the system. The second state of the system, the speed of the steel ball, was estimated by a reduced order observer. The model used gave us the opportunity to estimate a parameter which in the literature is denoted virtual control coefficient. Backstepping method gives us a rather straight forward way to design the controlling unit for a system with these properties. Stabilization of the closed-loop system is achieved by incorporating a Lypapunov function, which were chose a quadratic one in this thesis. If the derivative of this function is rendered negative definite by the control law, then we achieve stability. The results of the design were evaluated in simulations and real-time measurements by testing the tracking performance of the system. The simulation results were very promising and the validations in real-time were satisfying. Note that this has been done in previous studies; the new aspect here is the limitation of the voltage input. The real-time results showed that the parameter estimation converges only locally.

Keywords: Backstepping,unknown virtual control coefficients,magnetic levitation system, clf

Sammanfattning

Syftet med detta examensarbete är att utforma ett reglersystem for en magnetsvävare, som här är processen 33-210. Metoden jag har använt är adaptiv observatörs Backstepping, ty alla tillstånd är inte mätbara och det finns osäkerhet i modellens parametrar. Här har jag utnyttjat en reducerad observatör för att skatta det andra tillståndet i systemet, som är kulans hastighet. Den modell som används i detta arbete möjliggör att skatta en parameter som i litteraturen kallas "virtual control coefficient". Backstepping metoden tillhandahåller en ganska enkel tillvägagångs sätt för att ta fram en regulator för ett system med dessa egenskaper. Det slutna systemets stabilitet är garanterad med hjälp av en kvadratisk Lyapunov funktion. Detta genom att tvinga dess tidsderivata att vara negativ med hjälp av en regulator. Jag har utvärderat resultaten i simuleringar och senare i realtidsmätningar mot processen 33-210 genom att testa regulatorn förmåga att reglera utefter en fyrkantsvåg. Resultaten från simuleringarna var mycket lovande och mätningarna i realtid har varit tillfredsställande. Notera att detta har utförts i tidigare arbeten; den nya aspekten vi tar hänsyn till här är begränsning i insignalen. Mätningar i realtid på processen 33-210 visade att parameterskattningen är endast lokalt konvergent.

Acknowledgment

Presenting this thesis to you, I complete my Master of Science Degree in Applied Physics and Electrical Engineering at Linköping University. Here I want to thank some of those people whom been helping me through out the last 5 years, without whom I would not have been here and writing this thesis.

First of all I would like to thank my family, specially my father and mother for supporting me, regardless of what I have been doing. Their struggle against tyranny and oppression have always been an inspiration source for me. They inspired me to surpass myself every time I faced a challenge. Thank you for learning me that hard work always pay off.

Working on this thesis, Lic. Magnus Åkerblad was the one whom provided me with guidance and help almost on a daily bases. Thanks for having time for my questions although you had a very busy schedule. I also would like to thank Professor Torkel Glad for showing interest in my thesis.

Other people I need to mention here are Fredrik, Hjalmar, Conny, Magnus and Johan, good people whom I have the privilege of having them as friends. Talking to Fredrik after a long day at school or going out on a rock climbing tour with Hjalmar was my way to calm down and stay focused after a hard day. Thank all of you guys, without you I would have not been here today. I also would like to thank Naz for her advises on the linguistic correctness of this work. Your inputs and comments made the writing of this thesis much easier.

Last but not the leas: Thanks Namam! The way you handel people and life is so inspiring. Thanks for been there for me!

September 2003
Nawrous Ibrahim Mahmoud

Contents

Chapter 1

Introduction

1.1 Background

Nonlinear control theory has been the subject of very strong devolvement during the last two decades. The tools developed in this area suddenly made the design and implementation of controlling units in nonlinear systems more structured and rather straight forward. One of the concepts which are well known today is Backstepping theory. This method gives us a tool for recursive design of the control law based on the Lyapunov theory.

The magnetic levitation system is one of those nonlinear systems which have been subject to intensive studies in order to find a fully stabilizing control unit. The most known implementation of this system is in the transportation field and the manufacturing of trains suspending on magnetic railways. Transrapid in Germany is one of these projects. The system has been inherently unstable, it made a perfect test platform to implement the backstepping theory and trying to see and analyze its properties.

1.2 Objective

The objective of this thesis is to implement a control law for the magnetic levitation system according to backstepping technique. In the cases where it is possible we test the controller in realtime on the MagLev system 33-210 and compare it with the simulation results. This kind of controllers have been implemented in different previous studies. The new aspect of our thesis is the limitation of the input voltage, which we have to account for.

Chapter 2

Backstepping

Control systems have one main goal to achieve, and that is the stability of the controlled system. There are different kinds of stability problems which occur when studying dynamical systems. Here we are concerned with stability of equilibrium points. Let us first briefly review Lyapunov stability and formalize this requirement.(For more details se [3] and [2]).

2.1 Lyapunov stability

Definition 2.1 (Lyapunov stability) *Consider the system*

$$\dot{x} = f(x(t)) \tag{2.1}$$

with the initial condition $x(0)$. Let $x^*(t)$ be the solution to the differential equation (2.1) with the corresponding initial condition $x^*(0)$. The solution is then labelled

- *stable, if for each $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that*

  $$\| x^*(0) - x(0) \| < \delta \Longrightarrow \| x^*(t) - x(t) \| < \epsilon \quad for\ all \quad t \geq 0$$

  ($x(t)$ is the solution corresponding to the initial condition $x(0)$.)

- *unstable, if it is not stable*

- *asymptotically stable, if it is stable and in addition there exists δ such that*

  $$\| x^*(0) - x(0) \| < \delta \Longrightarrow \| x^*(t) - x(t) \| \rightarrow 0 \qquad as \qquad t \rightarrow \infty$$

Figure 2.1 illustrates this definition. The distance δ from $x^*(0)$ marks the area in which the trajectory must start in order to stay within the ϵ-distance from $x^*(t)$.

The solutions of a given system may be stable or unstable. For instance, (2.1) may have stable and unstable *equilibria*, that is, constant solutions $x(t; x_e) \equiv x_e$ satisfying $f(x_e) \equiv 0$. If an equilibrium x_e is asymptotically stable, then it has a

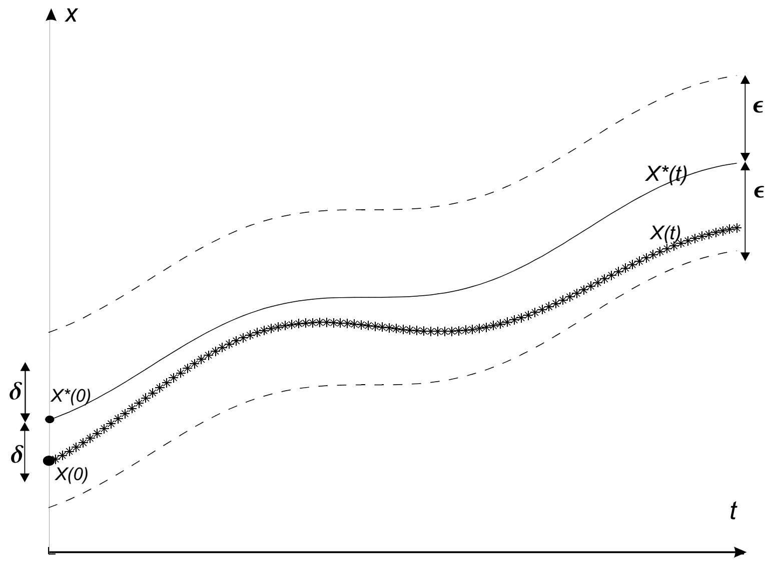

Figure 2.1. Definition of stability

region of attraction - a set Ω of initial states x_0 such that $x(t; x_0) \to x_e$ as $t \to \infty$ for all $x_0 \in \Omega$. When the region of attraction is the whole space $\mathbb{R}^n$, then the stability properties are *global*, otherwise they are called *local*. This insight in the close relationship between the solution and the equilibrium of a system gives us the idea of extending the definition to also include the later. This way we can analyze the behavior of the solution by the properties of the equilibrium. Let us define this more stringent.

Definition 2.2 *Let the system (2.1) have the equilibrium x_e and let $x^*(t)$ be the solution of (2.1) with the initial condition $x^*(0) = x_e$. This implies $x^*(t) = x_e$ for all t. The equilibrium is then stable, unstable or asymptotically stable iff the solution $x^*(t)$ have the same property.*

An equilibrium point is thereby asymptotically stable if all solutions which start nearby stay nearby and tend to the point as time approaches infinity. This is a very desirable property of a control system. Even more favorable would it be if the state tended to the equilibrium from an arbitrary initial condition, which leads us to the following definition.(For more details se [1]).

Definition 2.3 *An equilibrium point x_e of the system (2.1) is globally asymptotically stable (GAS), if it is stable and $x(t) \to x_e$ as $t \to \infty$*

Now, as seen from the definitions above, to show a certain type of stability, we have to determine $x(t)$, the explicit solution of (2.1). This solution generally cannot not be found analytically. Fortunately there are other ways of proving stability. A. M. Lyapunov, a Russian mathematician and engineer, came up with the idea of using the state vector $x(t)$ for constructing a scalar function $V(x)$. This function would measure how far the system is from the equilibrium. $V(x)$ is energy-like, radially unbounded and positive definite function. If $V(x)$ can be shown to continuously decrease, then the system itself must be moving towards the equilibrium.

This approach of showing stability is called Lyapunov's direct method and can be found in [2] and [3]. Before we go further, let us clarify concepts that we will use throughout this thesis.

Definition 2.4 *A scalar function $V(x)$ is said to be*

- *positive definite if $V(0) = 0$ and*

$$V(x) > 0, x \neq 0$$

- *positive semidefinite if $V(0) = 0$ and*

$$V(x) \geq 0, x \neq 0$$

- *negative(semi-)definite if $-V(x)$ is positive (semi-)definite*
- *radially unbounded if*

$$V(x) \rightarrow \infty \; as \parallel x \parallel \rightarrow \infty$$

Now we can state the main theorem for proving stability.

Theorem 2.1 (LaSalle-Yoshizawa) *Let $x = 0$ be an equilibrium point for (2.1). Let $V(x)$ be a scalar, continuously differentiable function of the state x such that*

- *$V(x)$ is positive definite*
- *$V(x)$ is radially unbounded*
- *$\dot{V}(x) = V_x(x)f(x) \leq -W(x)$ where $W(x)$ is positive semidefinite*

Then, all solutions of (2.1) satisfy

$$\lim_{t\rightarrow\infty} W(x(t)) = 0$$

In addition, if $W(x)$ is positive definite, then the equilibrium $x = 0$ is GAS.

Proof. See [3] or [2]. □

Note that any equilibrium under investigation can be mapped to the origin by substituting x with $z = x - x_e$. Therefore, there is no loss of generality in standardizing results for the zero solution $z \equiv 0$. But demanding $\dot{V}$ to be negative definite, in order to claim stability, may cause problems. The following example, ([1], ex. 12.4), shows this problem.

Example 1
Let the system be

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_2 - x_1^3\end{aligned}$$

choosing $V = \alpha x_1^4 + x_2^2$ we get

$$V_x f(x) = (4\alpha - 2)x_1^3 x_2 - 2x_2^2$$

The choice $\alpha = 1/2$ results in $V_x f(x) = -2x_2^2 \leq 0$. Obviously we cannot use theorem 2.1 as it is because $\dot{V}$ is negative semidefinite.

The previous example motivate us to define the following corollary.

Corollary 2.1 *Let $x = 0$ be the only equilibrium point of (2.1). Let $V(x)$ be a scalar, continuously differentiable function of the state x which is positive definite and radially unbounded. Let $E = \{x : \dot{V}(x) = 0\}$ and suppose that no other solution than $x(t) \equiv 0$ can stay forever in E. Then $x = 0$ is GAS.*

Proof. See [3] or [2]. □

Example 2
From the example above we found $V_x f = -2x_2^2 \leq 0$. Now we rely on the previous lemma to show stability of the origin. In order for the solution to stay for ever in the region where $V_x f = 0$, x_2 must be 0 and x_1 can have an arbitrary value. But as we se from the equations of the system, $x_2 \equiv 0 \Longrightarrow x_1 \equiv 0$. We can now conclude that the origin is GAS.

Now that we laid the foundation of Lyapunov stability the main question appearing is how to find these functions. The theorems above do not offer any systematic method of finding these functions. In the case of electrical or mechanical systems there are natural Lyapunov function candidates like total energy functions. In other cases, it is basically a matter of trial and error.

The backstepping approach is so far the only systematic and recursive method for constructing a Lyapunov function, along the design of the stabilizing control law. Yet, the system must have a lower triangular structure in order to apply the method, as we will see later. Before we can explore this state-of-the-art technique in adaptive control of nonlinear systems, we have to extend the systems handled so far to those including a control input.

2.2 Control Lyapunov functions (clf)

Let us now add a control input and consider the system

$$\dot{x} = f(x, u) \tag{2.2}$$

Our main objective of this thesis is the design of a closed-loop system with desirable stability properties, rather than to analyze the properties of the system itself. Therefore we are interested in an extension of the Lyapunov function concept. This concept is called *control Lyapunov function* and labelled *clf* for convenience. Given the stability results from the previous section, we want to find a control law

$$u = \alpha(x)$$

such that the desired state of the closed-loop system

$$\dot{x} = f(x, \alpha(x)) \tag{2.3}$$

is a globally asymptotically stable equilibrium point. Once again we consider the origin to be the goal state for simplicity. We can choose a function $V(x)$ as a Lyapunov candidate, and require that its derivative along the solutions of (2.3) satisfy $\dot{V}(x) \leq -W(x)$, where $W(x)$ is positive definite function. Then closed loop stability follows from Theorem (2.1). We therefore need to find $\alpha(x)$ to guarantee that for all $x \in \mathbb{R}^n$

$$\dot{V}(x) = \frac{dV}{dx}(x) f(x, \alpha(x)) \leq -W(x) \tag{2.4}$$

The pair V and W must be chosen carefully otherwise (2.4) will not be solvable. This motivate the following definition, which can be found in [3].

Definition 2.5 (Control Lyapunov function (clf)) *A smooth positive definite and radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+$ is called a control Lyapunov function (clf) for (2.2) if for every $x \neq 0$*

$$\dot{V}(x) = V_x(x) f(x, u) < 0 \quad for\ some\ u \tag{2.5}$$

The significance of this definition is in establishing the fact that, the existence of a globally stabilizing control law is equivalent to the existence of a clf. If we have a clf for the system then we can certainly find a globally stabilizing control law. The reverse is also true. This is known as Artestin's theorem and can be found in [6]. Now that we defined the concept clf, we can move on and explore the backstepping theory, which is the main tool have been utilized in this thesis.

2.3 Backstepping

The main deficiency of the clf concept as a design tool is that for most nonlinear systems a clf is not known. The task of finding an appropriate clf my be as complex

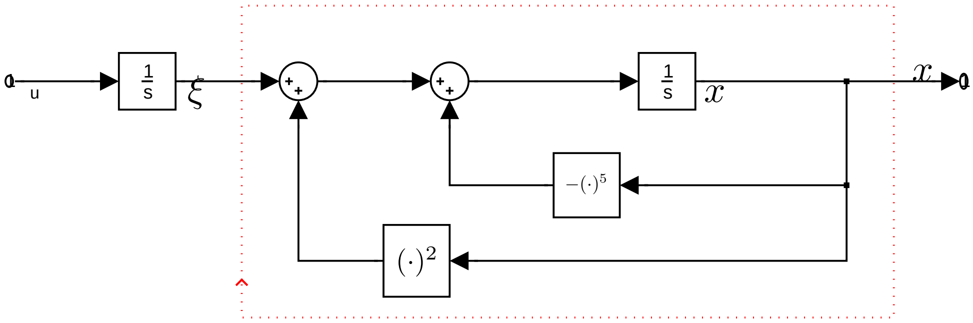

Figure 2.2. Block diagram of the system (2.6)

as that of designing a stabilizing feedback law. The backstepping procedure solve these two problems for us simultaneously. The following is a standard result and can be found in [3] and [2]. To spare the reader from the labor of understanding the main ideas of backstepping by a theorem, we will start this section with an example, and hope it will clarify the concepts before we state the theorem. Inspired by ([3], sec. 2.2.1) and ([2], ex. 13.6) we construct the following example.

Example 3
Consider the second-order system

$$\dot{x} = x^2 - x^5 + \xi \tag{2.6a}$$
$$\dot{\xi} = u. \tag{2.6b}$$

We want to design a feedback control law for regulation of $x(t)$ towards its equilibrium, which we choose to be $x = 0$, for all $x(0)$,$\xi(0)$. We remind the reader that by regulation we mean $x(t) \to 0$ as $t \to \infty$. The only equilibrium with $x = 0$ for (2.6a) is at $(x, \xi) = (0, 0)$.The design goal is fulfilled by rendering this equilibrium GAS. In the block diagram in Figure 2.2 the scalar system (2.6a) appears in the dotted box. In this step of the design, let us forget the equation (2.6b) for a moment and think of ξ as the control input of (2.6a). In that case, we choose the clf

$$V(x) = \frac{1}{2}x^2 \tag{2.7}$$

with the time-derivative

$$\dot{V} = V_x f(x) = x(\xi - x^5 + x^2) \tag{2.8}$$

The question arising now is how can we choose the control law in order to render the derivative of $V(x)$ negative definite? Here we have room for variations and can rely on different concepts for this choice. This degree of freedom in the choice of

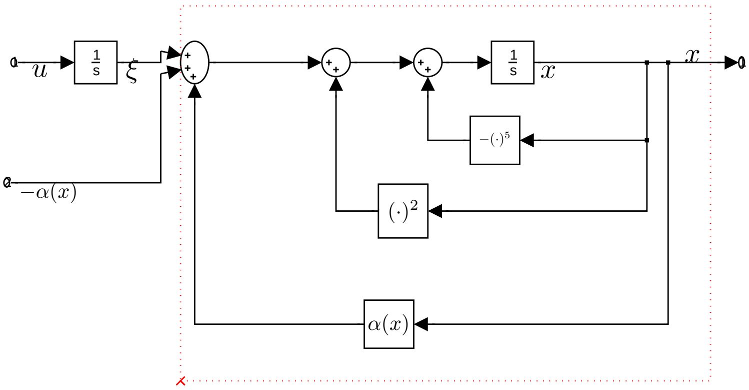

Figure 2.3. Introducing $\alpha(x)$as the desired value of ξ

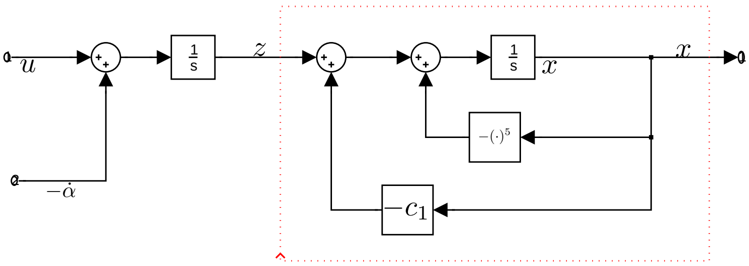

Figure 2.4. Closing the feedback loop in the dotted box with $+\alpha$ and "backstepping" through the integrato

the controller, is one of the trademarks of backstepping. Remembering that this example is just for the demonstration of backstepping, we just choose a controller. For example, the controller is

$$\xi = -c_1 x - x^2 \tag{2.9}$$

and with $V(x)$ as above we get $W(x) = x^2$, and we fulfill the condition (2.4). Now we are finished stabilizing (2.6a). Of course ξ is just a state variable and not the control. So we define its "desired value" as

$$\xi^{des} = -c_1 x - x^2 \triangleq \alpha(x) \qquad c_1 > 0. \tag{2.10}$$

Let z be the deviation of ξ from its desired value:

$$z = \xi - \xi^{des} = \xi - \alpha = \xi + c_1 x + x^2. \tag{2.11}$$

We call ξ a *virtual control*, and its desired value $\alpha(x)$ a *stabilizing function*. Rewriting the system (2.6) in the (x, z)-coordinates result in a more convenient form, which is illustrated in Figures 2.3 and 2.4. Starting from (2.6) and Figure 2.2, we add and subtract the stabilizing function $\alpha(x)$ to the $\dot{x}$-equation as shown in Figure 2.3. Then we use $\alpha(x)$ as the feedback control inside the dotted box and "backstep"-$\alpha(x)$ through the integrator, as in Figure 2.4. In the new coordinates (x, z) the system is expressed as

$$\dot{x} = -c_1 x - x^5 + z \tag{2.12a}$$

$$\dot{z} = u + (c_1 + 2x)(-c_1 x - x^5 + z) \tag{2.12b}$$

We now need to construct a clf V_a for the system (2.6). The most simple choice is to augment $V(x)$ with a quadratic term in the error variable z:

$$V_a(x, \xi) = V(x) + \frac{1}{2}z^2 = \frac{1}{2}x^2 + \frac{1}{2}(\xi + c_1 x + x^2)^2 \tag{2.13}$$

and calculate its time-derivative as below:

$$\dot{V}_a = x[-c_1 x - x^5 + z] + z[u + (c_1 + 2x)(-c_1 x - x^5 + z)] \tag{2.14}$$

$$= -c_1 x^2 - x^6 + z[x + u + (c_1 + 2x)(-c_1 x - x^5 + z)]. \tag{2.15}$$

Now we can design u to render $\dot{V}_a$ negative definite. For this reason the cross-term xz is grouped together with u. This maneuver is possible because u is also multiplied by z due to the chosen form of V_a. The simplest way to achieve this is to make the bracketed term in the last equation equal to $-c_2 z$, where $c_2 > 0$:

$$u = -c_2 z - x - (c_1 + 2x)(-c_1 x - x^5 + z) \tag{2.16}$$

With this control law, the clf derivative is

$$\dot{V}_a = -c_1 x^2 - c_2 z^2 - x^6, \tag{2.17}$$

which proves that in the (x, z)-coordinates the equilibrium $(0, 0)$ is GAS, which imposes the same property on the equilibrium $(0, 0)$ in the (x, ξ)-coordinates, and we reach our goal.

This example showed how we can design a stabilizing controller for a system in which the actual input is in a range of one integration from the system itself. We will formalize this result in the following lemma. The extension to the case of whole chain of integrators and even more complex subsystems than just an integration is straightforward. For details see [3] and [2].

Assumption 2.1 *Consider the system*

$$\dot{x} = f(x) + g(x)u, \qquad f(0) = 0, \tag{2.18}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}$ is the control input. There exist a continuously differentiable feedback control law

$$u = \alpha(x), \qquad \alpha(0) = 0, \tag{2.19}$$

and a smooth, positive definite, radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}$ such that

$$\frac{\partial V}{\partial x}[f(x) + g(x)\alpha(x)] \leq -W(x) \leq 0, \qquad \forall x \in \mathbb{R}^n, \tag{2.20}$$

where $W : \mathbb{R}^n \to \mathbb{R}$ is positive semidefinite.

Lemma 2.1 *Let the system* (2.18) *be augmented by an integrator:*

$$\dot{x} = f(x) + g(x)\xi \tag{2.21a}$$
$$\dot{\xi} = u, \tag{2.21b}$$

and suppose that (2.21a) *satisfies Assumption 2.1 with $\xi \in \mathbb{R}$ as its control. Then, if $W(x)$ is positive definite,*

$$V_a(x, \xi) = V(x) + \frac{1}{2}[\xi - \alpha(x)]^2 \tag{2.22}$$

is a clf for the full system (2.21)*, that is, there exists a feedback control $u = \alpha_a(x, \xi)$ which renders $(x, \xi) = (0, 0)$ the GAS equilibrium of* (2.21)*. One such control is*

$$u = -c(\xi - \alpha(x)) + \frac{\partial \alpha}{\partial x}[f(x) + g(x)\xi] - \frac{\partial V}{\partial x}g(x), \qquad c > 0. \tag{2.23}$$

Proof. See [3]. □

Once again, the choice of the control (2.23) is neither the only nor necessarily the best globally stabilizing control law. The main result of backstepping is not the specific form of the control law, but rather the construction of a Lyapunov function whose derivative can be made negative by a wide variety of control laws. This fact has been stressed in both [3] and [2].

Just for the demonstration of this fact, we go back to the first step in the last example and will se how we can choose this controller differently and how the choice affects the closed loop system properties.

Example 4
(Cont. example 2.6) Here we will compare two different designs to calculate the control law (2.9)

In feedback linearization design, the control law

$$\xi = -x^2 + x^5 - x \tag{2.24}$$

cancels both nonlinearities (x^2 and $-x^5$) and replace them by $-x$ so that the resulting feedback system is linear: $\dot{x} = -c_1 x$. Taking

$$V(x) = \frac{1}{2}x^2 \tag{2.25}$$

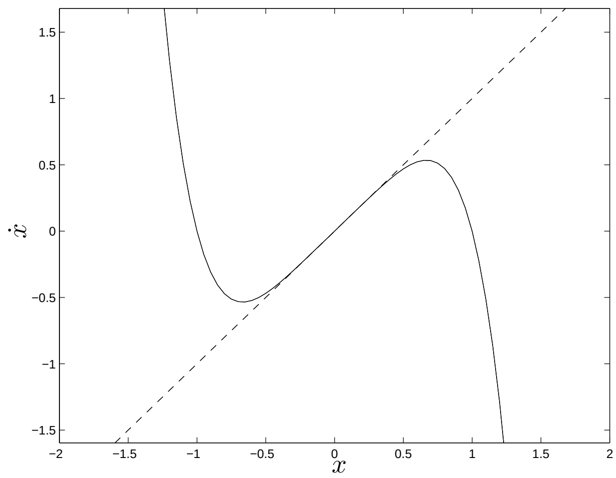

Figure 2.5. The dynamic of the system $\dot{x} = -x^5 + x$ near the origin. The dotted line is $\dot{x} = x$ and the solid line is $\dot{x} = -x^5 + x$

as a clf for (2.6a) we see that this control law satisfies the requirement (2.4) with $W(x) = x^2$, that is, $\dot{V}(x) \leq -x^2$. But this controller is irrational because it cancels also the useful nonlinearity $-x^5$. It is useful in the sense that it helps the system to reach its equilibrium faster and with less control effort, as can be seen from Figure 2.5. This is specially true for large values of x, where the x^5-term dominates the dynamics and push the system towards the origin(the equilibrium state). But near the origin the linear term x dominates the dynamics and acts destabilizing by pushing the dynamics from the origin. Thus, a more reasonable choice is not to cancel $-x^5$. Therefore we picked, without further explanation,

$$u = -x^2 - x \tag{2.26}$$

which with $V(x) = \frac{1}{2}x^2$ as before we get $W(x) = x^2$, and we fulfill (2.4).

2.4 Structural constraints

The systems handled in [3] are:

- *pure-feedback systems*, a class of lower triangular systems,:

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\xi \\
\dot{\xi}_1 &= f_1(x, \xi_1, \xi_2) \\
\dot{\xi}_2 &= f_2(x, \xi_1, \xi_2, \xi_3) \\
&\vdots \\
\dot{\xi}_{k-1} &= f_{k-1}(x, \xi_1, \ldots, \xi_k) \\
\dot{\xi}_k &= f_k(x, \xi_1, \ldots, \xi_k, u)
\end{aligned}
$$

  where $\xi \in \mathbb{R}$. The x-subsystem must satisfy Assumption 2.1 in order for the design to succeed. In addition, f_i, $i = 1 \ldots k-1$ must be invertible w.r.t. ξ_{k+1} and f_k must be invertible w.r.t u.

- *strict-feedback systems*, systems where the new variable enter in an affine way:

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\xi_1 \\
\dot{\xi}_1 &= f_1(x, \xi_1) + g_1(x, \xi_1)\xi_2 \\
\dot{\xi}_2 &= f_2(x, \xi_1, \xi_2) + g_2(x, \xi_1, \xi_2)\xi_3 \\
&\vdots \\
\dot{\xi}_{k-1} &= f_{k-1}(x, \xi_1, \ldots, \xi_{k-1}) + g_{k-1}(x, \xi_1, \ldots, \xi_{k-1})\xi_k \\
\dot{\xi}_k &= f_k(x, \xi_1, \ldots, \xi_k) + g_k(x, \xi_1, \ldots, \xi_k)u
\end{aligned}
$$

  the reason for referring to the ξ-subsystem as "strict-feedback" is that the nonlinearities f_i, g_i in the $\dot{\xi}_i$-equation $(i = 1, \ldots, k)$ depend only on $x, \xi_1, \ldots, \xi_i$, which are the states "fed back". Strict-feedback systems are nice to deal with and often used for deriving results related to backstepping.

Now we have the tools to design a control law and determine its stability properties. We also know what kind of systems we can handle. Next step would be to apply this method to a specific system, which in this case is the magnetic levitation system. But we have to first account for two very important features that often are present in realistic systems. Those are *parametric uncertainties* and systems were not all states are measurable. These issues are subject for an extended investigation in [3].

2.5 Adaptive backstepping

For systems with parametric uncertainties, a *parameter update law* is designed such that the closed loop stability is guaranteed when the estimator is used by the controller. This is achieved by extending the Lyapunov function $V(x)$ with a term penalizing the estimation error. The idea is to employ backstepping to design a control law for the system as if all the parameters were known and then replace the unknown parameters by their estimates, a "certainty equivalence" way of thinking. Let us illustrate this in the following example, which can be found in [3]

Example 5
Consider the plant

$$\dot{x} = u + \theta x \tag{2.27}$$

where u is the control and θ is the unknown constant parameter. The ambition is to achieve regulation of the state $x(t)$: $x(t) \to 0, t \to \infty$. Here we seek a parameter update law for the estimate $\hat{\theta}(t)$,

$$\dot{\hat{\theta}} = \tau(x, \hat{\theta}) \tag{2.28}$$

which, along with a control law $u = \alpha(x, \hat{\theta})$, will make the derivative of the clf $V(x, \hat{\theta})$ negative. As we mentioned in the preceding of this section, on of the terms in the clf is to penalize the estimation error, $\tilde{\theta}$. A simple choice is the quadratic term, $\frac{1}{2}\tilde{\theta}^2$. This result in the clf

$$V(x, \hat{\theta}) = \frac{1}{2}x^2 + \frac{1}{2}(\hat{\theta} - \theta)^2 \tag{2.29}$$

which is a radially unbounded fuction of time. We express the derivative of V as a function of u and $\dot{\hat{\theta}}$ and seek $\alpha(x, \hat{\theta})$ and $\tau(x, \hat{\theta})$ to guarantee that $\dot{V} \leq -px^2$ with $p > 0$.

$$\dot{V} = x(u + \theta x) + (\hat{\theta} - \theta)\dot{\hat{\theta}} \leq -px^2.$$

Rearranging the terms we get

$$xu + \hat{\theta}\dot{\hat{\theta}} + \theta(x^2 - \dot{\hat{\theta}}) \leq -px^2.$$

Since neither α nor τ is allowed to depend on the unknown θ, we must take $\tau = x^2$,

$$\dot{\hat{\theta}} = x^2.$$

The remaining condition

$$xu + \theta\hat{x}^2 \leq -px^2$$

allows us to select $\alpha(x, \hat{\theta})$in various ways. For instance we can choose

$$\alpha = -(p + \hat{\theta})x.$$

This controller renders, along with the update law, the derivative of clf negative and the closed loop system is guaranteed stable.

2.5.1 Unknown virtual control coefficients

Here we have to highlight an extension to adaptive backstepping tool due to unknown parameters called *high gain* constants. In [3] the problem is addressed under the name *unknown virtual control coefficients*, where finding an update law for this parameter is solved by requiring the knowledge of the sign of the parameter. We consider the system

$$\dot{x} = f(x) + g(x)u \tag{2.30}$$

where with the control law $u = \alpha(x)$ and the clf $V(x)$ we get

$$\dot{V} = V_x(f(x) + g(x)\alpha(x)) = -q(x)$$

where q is positive definite. Instead, if the system is

$$\dot{x} = f(x) + bg(x)u$$

where $b > 0$ is constant and unknown, [3] suggests the controller

$$u = \hat{\varrho}\alpha(x).$$

Here $\hat{\varrho}$ is interpreted as an estimate for $1/b$. We augment the clf with a quadratic function to penalize the deviation of the estimate from the true value of the parameter. This result in the clf

$$V_1 = V(x) + \frac{b}{2\gamma}\tilde{\varrho}^2 \tag{2.31}$$

where

$$\tilde{\varrho} = \frac{1}{b} - \hat{\varrho}.$$

The time derivative of the clf is then

$$\begin{aligned}
\dot{V}_1 &= V_x(f + g\alpha + bg\hat{\varrho}\alpha - g\alpha) - \frac{b}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} \\
&= -q(x) + (V_x g)(b\hat{\varrho} - 1)\alpha - \frac{b}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} \\
&= -q(x) - b\tilde{\varrho}(V_x g\alpha + \frac{1}{\gamma}\dot{\hat{\varrho}}) \\
&= -q(x)
\end{aligned}$$

if we choose

$$\dot{\hat{\varrho}} = -sgn(b)\gamma(V_x g)\alpha \tag{2.32}$$

and we fulfill (2.4).

Note that [3] do not point out the convergency properties of the update law. In fact the parameter update law does not have to converge to the true parameter value, but just to a value which is bounded. In search of an *optimal control law* where the optimality means that the controller and the parameter update law

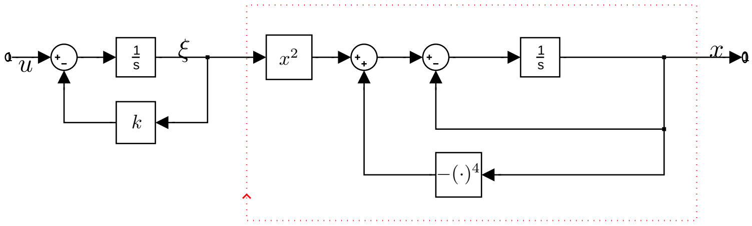

Figure 2.6. The block diagram of the system (2.33a),(2.33b)

fulfill a meaningful cost functional one can further study [4] This functional incorporates integral penalty on the control effort, the tracking error and the parameter estimation error. I choose to, because of lack of time, terminate this investigation here and go further on with the second issue of backstepping, which is observer backstepping.

2.6 Observer Backstepping

In a more realistic case, all states are not accessible for measurement. That is why we need an estimation of these states through time by the knowledge of the systems input and output. For linear systems this problem can be decomposed into two subproblems which can be solved separately: the design of a state observer, and the design of a state-feedback controller. But the separation principle does not apply for nonlinear systems. [3] presents a recursive design procedure to solve this issue, by using an estimate of the state in the plant system and considering the estimation error as disturbance. The effect of the disturbance is counteracted by adding *nonlinear damping terms.* The following example will illustrate the basics that is needed in order to understand the implementation which will we present later in this thesis. For further explanations see [3].

Example 6
Let us consider the plant

$$\dot{x} = -x + x^4 + x^2\xi \quad (2.33a)$$
$$\dot{\xi} = -k\xi + u \quad (2.33b)$$

where $k > 0$, and the equilibrium $(x, \xi) = (0, 0)$. The block diagram is given in Figure 2.6. When both x and ξ are measured, this system can be stabilized using backstepping. Using ξ as virtual control in (2.33a), an obvious choice of stabilizing control is $\alpha_1(x) = -x$, which reduce (2.33a) to $\dot{x} = -x^2$. Introducing the first error variable $z = \xi - \alpha_1(x)$ and rewriting the equations, we get

$$\begin{aligned}\dot{x} &= -x + x^2 z \\ \dot{z} &= -k\xi + u + 2x(-x + x^2 z)\end{aligned}$$

We choose the clf as $V(x,\xi) = \frac{1}{2}(x^2 + z^2)$ which have the time derivative

$$\dot{V} = -x^2 + z[x^3 - k\xi + u + 2x(-x + x^2 z)].$$

Hence the choice of control law

$$u = -cz - x^3 + k\xi - 2x(-x + x^2 z) \tag{2.34}$$

with $c > 0$ as a design constant, yields $\dot{V} = -x^2 - cz^2$ and renders $(0,0)$ the GAS equilibrium of the closed-loop system.

Now, suppose ξ is not measurable. Hence the first virtual control cannot be ξ and the error variable $z = \xi + x^2$ is not implementable. Led by the ideas of observer construction from linear control, we chose

$$\dot{\hat{\xi}} = -k\hat{\xi} + u \tag{2.35}$$

Subtracting (2.35) from (2.33b) shows that the state estimation error converges exponentially to zero:

$$\dot{\tilde{\xi}} = -k\tilde{\xi} \longrightarrow \tilde{\xi}(t) = \tilde{\xi}(0)e^{-kt}.$$

Led by the certainty equivalence idea, we replace ξ with $\hat{\xi} + \tilde{\xi}$ in (2.33a):

$$\dot{x} = -x + x^4 + x^2\hat{\xi} + x^2\tilde{\xi}.$$

Introducing the observer (2.35) we manipulate the system (2.33a), as seen from Figure (2.7) and (2.8), and (2.33b) into the following form:

$$\dot{x} = -x + x^4 + x^2\hat{\xi} + x^2\tilde{\xi} \tag{2.36a}$$

$$\dot{\hat{\xi}} = -k\hat{\xi} + u \tag{2.36b}$$

$$\dot{\tilde{\xi}} = -k\tilde{\xi} \tag{2.36c}$$

The next step is to design a stabilizing control law for (2.36). We know that $z = \xi - \alpha_1(x) = \tilde{\xi} + \hat{\xi} - \alpha_1(x)$. We also know that $\tilde{\xi}$ is exponentially decaying. This fact might tempt us to ignore its effect on the closed-loop system and instead use $z = \hat{\xi} - \alpha_1(x)$. This way we have the following closed loop-system

$$\begin{aligned}\dot{x} &= -x + x^2 z + x^2\tilde{\xi} \\ \dot{z} &= -cz - x^3 + 2x^3\tilde{\xi} \\ \dot{\tilde{\xi}} &= -k\tilde{\xi}\end{aligned}$$

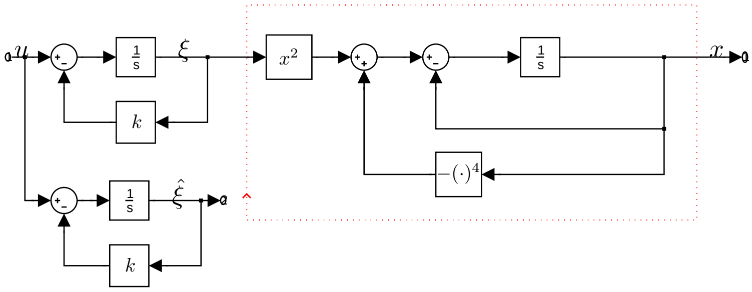

Figure 2.7. Adding the observer to the system (2.33a) and (2.33b)

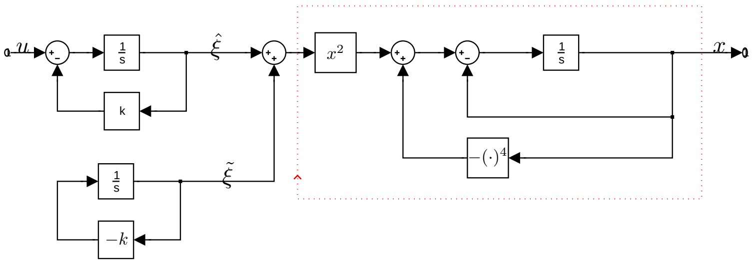

Figure 2.8. Replace ξ by $\hat{\xi}$ as the virtual control

Consider the case of $z \equiv 0$, then we have

$$\dot{x} = -x + x^2\tilde{\xi}, \qquad \widetilde{\xi(t)} = \tilde{\xi}(0)e^{-kt} \tag{2.37}$$

with the solution

$$x(t) = \frac{x(0)(1+k)}{[1+k-\tilde{\xi}(0)]e^t + \tilde{\xi}(0)x(0)e^{-kt}} \tag{2.38}$$

This solution escapes to infinity in finite time for all conditions $\tilde{\xi}(0)x(0) > 1 + k$. To overcome this obstacle [3] incorporates nonlinear damping terms.

The nonlinear damping term is affected by the way the disturbance enters the equations. The main idea is to have a term in the controller which allows the completion of squares with the disturbance term. If $\tilde{\xi}$ entered the plant equation multiplied by a function, which is bounded by a constant or a linear function, the former choice of controller might have been satisfying. Obviously it is inadequate in this case. To this end, we choose the first stabilizing function to be

$$\alpha_1 = -x^2 - d_1(x^2)^2, \quad d_1 > 0. \tag{2.39}$$

The clf for (2.36a) is $V(x) = \frac{1}{2}x^2$ and we will, in the same manner as in adaptive backstepping section above, augment it with a new term penalizing the estimation error:

$$V_1(x,\tilde{x}) = V(x) + \frac{1}{2d_1k}\tilde{\xi}^2 = \frac{1}{2}x^2 + \frac{1}{2d_1k}\tilde{\xi}^2 \tag{2.40}$$

and the derivative of the clf is, using the stabilizing function above

$$\dot{V}_1 = \dot{V} + \frac{1}{d_1}\tilde{\xi} - x^2 \leq -x^2 + x^3z - \frac{3}{4d_1}\tilde{\xi}^2. \tag{2.41}$$

Hence, if $z \equiv 0$ the stabilizing function will render $(0,0)$ the GAS equilibrium of the $(x,\tilde{\xi})$ system.

The derivative of z is now expressed as

$$\dot{z} = -k\hat{\xi} + u - \frac{\partial\alpha_1}{\partial x}\big(-x + x^4 + x^2\hat{\xi}\big) - \frac{\partial\alpha_1}{\partial x}x^2\tilde{\xi}$$

The estimation error appears again, so its effect must be accounted for by adding another nonlinear damping term. To this end, we augment the clf with a new quadratic term in $\tilde{\xi}$, beside the z^2-term:

$$V_2 = V_1 + \frac{1}{2}z^2 + \frac{1}{2d_2k}\tilde{\xi}^2$$

Its time derivative is then

$$\begin{aligned}
\dot{V}_2 &\leq -x^2 + x^3z - \frac{3}{4d_1}\tilde{\xi}^2 + z\big[-k\hat{\xi} + u - \frac{\partial\alpha_1}{\partial x}(-x + x^4 + x^2\hat{\xi})\big] \\
&= -x^2 - \frac{3}{4d_1}\tilde{\xi}^2 - \frac{1}{d_2}\tilde{\xi}^2 + z\big[x^3 - k\hat{\xi} + u - \frac{\partial\alpha_1}{\partial x}(-x + x^4 + x^2\hat{\xi})\big] - z\frac{\partial\alpha_1}{\partial x}x^2\tilde{\xi}
\end{aligned}$$

If we choose controller

$$u = -cz - d_2z\Big(\frac{\partial\alpha_1}{\partial x}x^2\Big)^2 - x^3 + k\hat{\xi} + \frac{\partial\alpha_1}{\partial x}\big(-x + x^4 + x^2\hat{\xi}\big) \tag{2.42}$$

yields

$$\begin{aligned}
\dot{V}_2 &\leq -x^2 - cz^2 - \frac{3}{4d_1}\tilde{\xi}^2 - d_2z^2\Big(\frac{\partial\alpha_1}{\partial x}x^2\Big)^2 - z\frac{\partial\alpha_1}{\partial x}x^2\tilde{\xi} - \frac{1}{d_2}\tilde{\xi}^2 \\
&= -x^2 - cz^2 - \frac{3}{4d_1}\tilde{\xi}^2 - d_2\Big(z\frac{\partial\alpha_1}{\partial x}x^2 + \frac{1}{2d_2}\tilde{\xi}\Big)^2 - \frac{3}{4d_2}\tilde{\xi}^2 \\
&\leq -x^2 - cz^2 - \frac{3}{4}\Big(\frac{1}{d_1} + \frac{1}{d_2}\Big)\tilde{\xi}^2
\end{aligned}$$

and we fulfill the condition (2.4) and the origin is the GAS equilibrium of the closed-loop system.

Now that we have all the tools needed for the controller design in hand we will in the next chapter proceed with the building of a model for the magnetic levitation system. This model will be the benchmark at which we will test the design method presented here.

Chapter 3

Modelling of a Magnetic Levitation system

As the heading of this chapter indicates, in this part of the thesis we will build a model for the magnetic levitation system. The methods and theories we stated in the previous chapter need a test plant in order to be tested. We will present a mathematical model, in the form of differential equations, which will describe, as close as possible the dynamics of the system. We will base the model on known laws of physics and specially electromagnetics in order to derive these equations. Much work on modelling magnetic levitation system dynamics have been done.

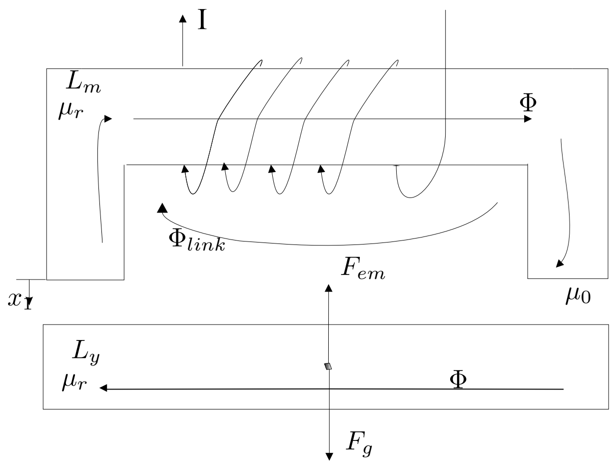

Figure 3.1. Electromagnet configuration.

The most fundamental and important ideas are outlined in [5] and [7]. Previous works on this subject with the propose of designing a control law, adaptive or only robust, incorporating backstepping technique are done. For instance, in [8] and [9], the models are rather complex and the tracking performance are excellent. Most of the examples studied in [3] shows that the control law gets rather complex when the model does. This was even found in [8] and [9]. The question is whether we have to consider this complexity as an obstacle when it comes to realtime implementation. The MagLev system 33-210 has a limitation on the control input, $\mid u \mid < -10[V]$. The controllers designed in [8] and [9] fluctuate rapidly in an area where $\mid u \mid \leq -60[V]$. These controllers are not implementable in our case. But note that this is duality between the level of complexity and the magnitude is not complete. Even one simple integrator can cause large control inputs due to windup-phenomenon. Avoiding a situation where we can not explain why the control input is too large, we decide to derive a new model. A model which is simpler and easy to understand but yet describe the essence of the system dynamics.

Let us start with the well known problem of an electromagnet shown in Figure 3.1. This have been used in most undergraduate courses when studying flux linkage and electromagnetic force magnitude w.r.t. the yoke displacement from the magnet. If the air gap between the magnet and the yoke is small, then we can consider the whole system as a closed electrical system with a corresponding magnetic flux across the yoke and the magnet. The magnetic co-energy can be derived using the magnetic flux. The derivative of this energy w.r.t. to the yokes distance from the magnet is the electromagnetic force acted on the yoke. Let us derive these ideas mathematically.

The magnetic field intensity H relate to the magnetic flux Φ through

$$H = \frac{\Phi}{\mu A}$$

where A is the cross section of the yoke and μ is permeability of the medium. Apmères' law in electrostatics gives us

$$\oint_c \overline{H} * \overline{dL} = I_{enclosed}$$

where $I_{enclosed}$ is the current going through the area which inclosed by c. Applying this formula to the two parts of this joint circuit we get

$$H_m L_m + H_y L_y + H_a L_a = I_{enclosed} = NI$$

where L_m, L_y, L_a are the length of the magnet, the yoke and the air gap respectively. H_m, H_y and H_a are the corresponding magnetic field intensities. Assuming the air gap is little, we can neglect the flux linkage Φ_{link} and the magnetic flux is the same everywhere in the circuit. This means

$$H_m = H_y = \frac{\Phi}{A\mu_r\mu_0}$$
$$H_a = \frac{\Phi}{A\mu_0}$$

where μ_r is the relative permeability and μ_0 is for vacuum. These two equations result in the expression for the magnetic flux

$$\Phi = \frac{NIA\mu_0}{2z + \frac{L_m + L_y}{\mu_r}}$$

and in turn the magnetic field intensity and magnetic flux density resp.

$$\begin{aligned} H &= \frac{NI}{2z + \frac{L_m+L_y}{\mu_r}} \\ B &= \frac{NI\mu_0}{2z + \frac{L_m+L_y}{\mu_r}}. \end{aligned}$$

The magnetic co-energy is

$$W = 0.5 * \int_\tau \overline{BH} d\tau$$

where τ is the whole space. In our case the whole space is the volume of the system, which is symmetric and simple to compute. Therefore we get

$$W = \frac{(NI)^2 A\mu_0\mu_r}{4(\frac{L_m+L_y}{\mu_r} + z)}$$

and the electromagnetic force can be computed as

$$F_{em} = - \bigtriangledown W.$$

Therefore

$$F_{em} = \frac{(NI)^2 A\mu_0}{4(\frac{L_m+L_y}{2\mu_r} + x_1)^2}$$

where N is the number of turns, A is the cross section where the yoke and the magnet have contact, L_y is the length of the yoke, L_m is the length of the magnet, μ is the permeability of air, x_1 is the air gap length and I is the current through the wire winding. The constants are known and we can simplify the form to

$$F_{em} = \frac{aI^2}{(b + x_1)^2} \tag{3.1}$$

where

$$a = \frac{AN^2\mu_0}{4} \qquad b = \frac{L_1 + L_2}{2\mu_r} \tag{3.2}$$

On the other hand, the force equation for the yoke is

$$m\ddot{x}_1 = mg - F_{em} \tag{3.3}$$

so denoting the distance and velocity of the yoke as x_1 respective x_2, we get the following differential equations

$$\dot{x}_1 = x_2 \tag{3.4}$$

$$\dot{x}_2 = g - \frac{1}{m}F_{em} \tag{3.5}$$

So far there is no sign of the controlling voltage that we have at hand as input to the system. If we consider the link from the voltage signal to the current as a RL-link, we get the following equation

$$Ri(t) + \frac{\partial}{\partial t}(L(t)i(t)) = v(t)$$

where $L(t)$ is the inductance of the coil, $v(t)$ is the applied voltage to the circuit and $i(t)$ is the current throw the circuit. The dependence of L on time comes from the fact that the coil inductance will be affected by the mutual inductance between the magnet and the yoke. Considering the case where the yoke is replaced by a steel ball, we assume that this dependence is very weak, and choose $L(t) = L$.

This assumption, and remembering that the input voltage is our control signal, gives us our final equation for the system.

$$\frac{\partial i}{\partial t} = \frac{-R}{L}i + \frac{1}{L}u$$

Denoting our third variable as $x_3 = i^2$ we get the complete differential equation

$$\dot{x}_1 = x_2 \tag{3.6a}$$

$$\dot{x}_2 = g - \theta_1\lambda(x_1)x_3 \tag{3.6b}$$

$$\dot{x}_3 = -2\theta_2 x_3 + 2\theta_3\sqrt{x_3}u \tag{3.6c}$$

where $\lambda(x_1) = \frac{1}{(x_1+b)^2}$ and

$$\theta_1 = \frac{a}{m} \qquad \theta_2 = \frac{R}{L} \qquad \theta_3 = \frac{1}{L}$$

The interesting question now is whether (3.6) incorporates any of the required structures as in 2.4. After closer examination we see that (3.6) is in strict-feedback form. This can be realized when denoting the following.

$$f(x) = 0 \qquad g(x) = 1$$
$$f_1(x,\xi_1) = g \quad g_1(x,\xi_1) = \frac{1}{(x_1+b)^2}$$
$$f_k(x,\xi_1,\ldots,\xi_k) = x_3 \quad g_k(x,\xi_1,\ldots,\xi_k) = \sqrt{x_3}$$

3.1 Test Equipment

The process been used in this thesis is MLS $33 - 210$ and the manufacturer of this machine suggests a different relationship between the current and the voltage. Here

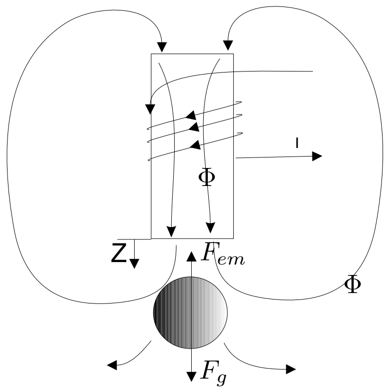

Figure 3.2. The Magnetic Levitation (MagLev) system $33-210$

we have $I = cU + d$ where c and d are constants. This new information changes the above differential equations into

$$\dot{x}_1 = x_2 \tag{3.7a}$$

$$\dot{x}_2 = g - \theta_1 \lambda(x_1)(u + u_0)^2 \tag{3.7b}$$

which have a rather simpler structure. But we have to warn for this generalization. This model is supposed to describe a system which is more like the Figure 3.2 rather than Figure 3.1. Here the flux linkage is greater than before and the cross section is not really comparable with the same in the previous section. We also have a limited voltage magnitude to use on this test equipment. The controller must be in the range

$$\parallel u \parallel \leq 10 \quad [V]. \tag{3.8}$$

One more reason to be cautious is the lack of affinity in the way the control voltage enters the last equation. Comparing this with the section 2.4 we se that this model is mixture of the pure- and strict-feedback form. In (3.7b) we have to at least fulfill the condition

$$(u + u_0)^2 \geq 0.$$

Otherwise the design will not succeed. The voltage control can not have complex values. We realize here that the set of initial conditions ,from which we can emerge and reach stability, is bounded.

We will use these two models in the next chapter and we will analyze the results.

Chapter 4

Implementation and Experimental Results

In this chapter we will present the results achieved in this thesis. The objective is to try different cases for the model and analyze how the backstepping recursive procedure will work in that particular case and what properties the closed-loop system will have. We will start with a simple model and will add further features as we go further until we arrive at the more realistic case where we have a state observer and a parameter estimator in the system. Before we begin the design let us state the following assumption, which will make our calculations simpler

Assumption 4.1 *If r is the reference signal to be tracked, then we assume*

$$\dot{r} = 0 \tag{4.1}$$
$$\ddot{r} = 0 \tag{4.2}$$
$$\dddot{r} = 0. \tag{4.3}$$

This assumption means that we probably suffer from lack of tracking during transitions, places where the reference signal change values very fast.

We need even a model for measurement noise $v(t)$. Here we assume $v(t)$ to be a normal distribution as below.

$$v(t) \sim N(0, R)$$

4.1 Two State Model With No uncertainties

As we mentioned in modelling Chapter 3, we will use 3.7 the two state model for testing and simulation. Here we will present the implementation and results.

4.1.1 Full State Feedback

Consider the model *two-state-model*

$$\begin{aligned} \dot{x}_1 &= x_2 & \text{(4.4a)} \\ \dot{x}_2 &= g - \theta\lambda(x_1)(u + u_0)^2 & \text{(4.4b)} \\ \lambda(x_1) &= \frac{1}{(x_1 + b)^2} \end{aligned}$$

where θ is a constant known parameter. Let us apply backstepping to this model. The objective of the design is, for the first state x_1, which is the distance of the steel ball from the electromagnet to track a smooth reference signal r.

Step 1

Let the first error variable be

$$z_1 = x_1 - r$$

and rewrite (4.4a) as below:

$$\dot{z}_1 = x_2.$$

A very simple clf for this equation is $V(z_1) = \frac{1}{2}z_1^2$ with the time derivative

$$\dot{V} = z_1\dot{z}_1 = z_1 x_2$$

and if we choose the first stabilizing function as

$$x_2^{des} = -c_1 z_1 \triangleq \alpha \qquad c_1 > 1. \tag{4.5}$$

then we get

$$\dot{V} = -c_1 z_1^2 \leq -z_1^2.$$

It is obvious that this controller stabilizes the first equation because

$$\begin{aligned} \dot{z}_1 + c_1 z_1 &= 0 \qquad \Rightarrow \\ z_1(t) &= constant * e^{-c_1 t} \end{aligned}$$

and with large c_1, we reach the equilibrium sufficiently fast.

Step 2

The second error variable is then

$$z_2 = x_2 - \alpha \Rightarrow$$

$$\begin{aligned} \dot{z}_1 &= z_2 + \alpha \\ \dot{z}_2 &= \dot{x}_2 - \dot{\alpha} = g - \theta_1\lambda(z_1)(u + u_0)^2 + c1(z_2 + \alpha). \end{aligned}$$

where $\lambda(z_1) = \frac{1}{(z_1+r+b)^2}$. Remembering the way we augmented the clf in different steps in chapter 1, we will do the same here. The new clf is augmented with a new term penalizing the deviation of the variable x_2 from its desired value:

$$V_1(z_1, z_2) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{4.6}$$

with the time derivative

$$\begin{aligned}
\dot{V}_1 &= z_1\dot{z}_1 + z_2\dot{z}_2 \\
&= z_1(z_2+\alpha) + z_2[g - \theta_1\lambda(z_1)(u+u_0)^2 + c_1(z_2+\alpha)] \\
&= z_1\alpha + z_2[z_1 + g - \theta_1\lambda(z_1)(u+u_0)^2 + c_1(z_2+\alpha)] \\
&\leq -z_1^2 + z_2[z_1 + g - \theta_1\lambda(z_1)(u+u_0)^2 + c_1(z_2+\alpha)] \\
&\leq -z_1^2 - z_2^2
\end{aligned}$$

if we choose

$$(u+u_0)^{2_{des}} = \frac{c_2z_2 + z_1 + g + c_1(z_2+\alpha)}{\theta_1\lambda(z_1)} \qquad c_2 > 0. \tag{4.7}$$

This choice result in the closed-loop system

$$\begin{aligned}
\dot{z}_1 &= z_2 - c_1z_1 \\
\dot{z}_2 &= -c_2z_2 - z_1
\end{aligned}$$

which simplified is

$$\dot{\mathbf{z}} = \begin{pmatrix} -c_1 & 1 \\ -1 & -c_2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

The matrix above should be Hurwitz, which means that the eigenvalues have negative real part, for stability to be fulfilled. Analytically, the eigenvalues are

$$s_{1,2} = -\frac{c_1+c_2}{2} \pm \sqrt{\frac{(c_1-c_2)^2-4}{4}}$$

and therefore, by choosing c_1 and c_2 properly, the closed-loop system is stabile. Simulations showed the good performance of the tracking course. We saw also that the control input is bounded and it is implementable if we want to test it on the magnetic levitation system 33-210, which is our intention. Even though we add noise in form of measurement noise, the performance of the controller still is the same. These simulation results can be seen in Figure 4.1. Note that this good performance of the controller is dependent on the full knowledge of the parameters in the model. Changing these parameters in the model will cost us non-vanishing tracking error, see Figure 4.2 where we changed the value of the gravitation constant g.

Hence having some kind of adaption in the controller is desirable and this will be subject of further investigation in the coming sections. The conclusion to be drawn

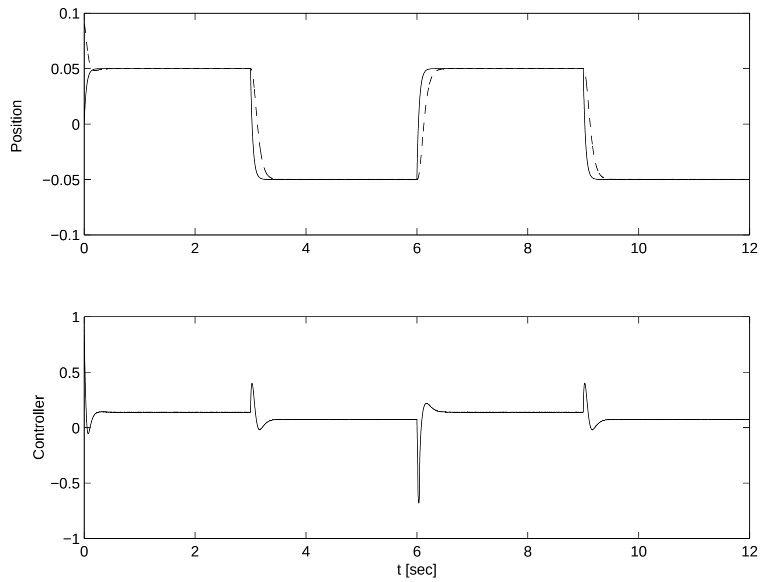

Figure 4.1. Simulation using the controller (4.7) with $(c_1, c_2) = (20, 20)$. In the first figure, the solid line is the reference signal. The dashed line is the steel ball position. Adding measurement noise, $R = 10^{-9}$

here is that in the presence of no uncertainties and when the full state is available for feedback, the backstepping technique provided a control law which stabilizes the closed-loop system. Note that we only are considering local stabilization because x_1 is limited to a bounded length in which it is allowed to vary. Even the control input is bounded as in (3.8). Note even that the only effect we see from Assumption 4.1 is the lack of tracking in the transitions, where r changes level very abruptly.

4.1.2 Output Feedback - Pseudo differentiation

Having all the states available for measurement is not realistic. In our case, the velocity is not available for measurement. Here we have to employ some kind of estimation for the velocity of the steel ball. The choices are pseudo-differentiation of the position, which is the only state available for measurement, or observer of full/reduced order. Because we already measure the position, we will implement a reduced order observer in the last case.

The pseudo-differentiation has been carried out by the time-derivative block in Simulink. This block approximate the derivative of the input by computing

$$\frac{\triangle u}{\triangle t}$$

where $\triangle u$ is the change in input value and $\triangle t$ is the change in time since the last simulation step. But in this case the sensitivity for measurement noise is very high,

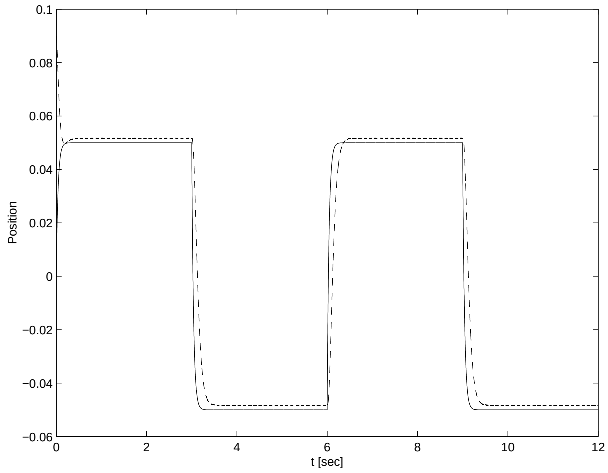

Figure 4.2. Simulation of position tracking when the model is not fully known. Here in terms of the value of the gravitation constant

which can be seen from Figure 4.3, where we added measurement noise. This fact made the effort of designing an observer, which is not that sensitive to measurement noise, worthwhile. We wanted to test the controller (4.7) on the MagLev system 33-210 and used the time-derivative block mentioned above. The results were not promising. The levitation of the steel ball was achieved only when we decrease c_1 and c_2 to ca. 2. The controller was very slow and could not achieve satisfying height control. The test result can be seen in Figure 4.5. We even see from Figure 4.4 that the velocity estimate is very noisy. This could be one of the reasons why the controller behave so poorly. (Several other probable reasons will be presented in the next section.) So, a better way of estimating the second state is needed in order for the backstepping control law to succeed.

4.1.3 Output Feedback - Reduced Order Velocity Observer

Following the reduced-observer idea from *Reglerteknik*, the observer for the system (4.4) is

$$\begin{aligned} \chi &= \hat{x}_2 - ky \\ \Rightarrow \hat{x}_2 &= \chi + ky \qquad (4.8) \\ \dot{\chi} &= g - \theta_1 \lambda(x_1)(u + u_0)^2 - k\dot{y} \\ &= g - \theta_1 \lambda(x_1)(u + u_0)^2 - k(\chi + ky) \qquad (4.9) \end{aligned}$$

where $\dot{y} = \dot{x}_1 = x_2$. Using the estimate $\hat{x}_2$ instead of x_2, the certainty equivalence approach, we get

$$\dot{y} = \chi + ky$$

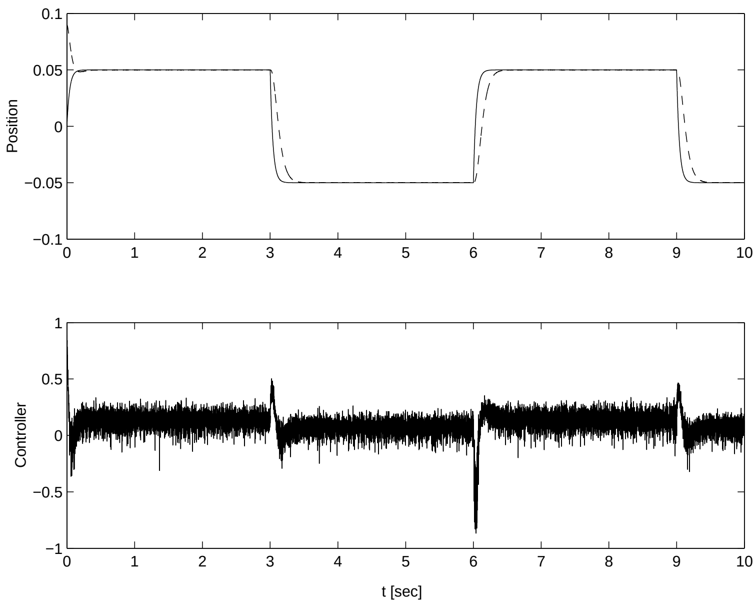

Figure 4.3. Simulation of tracking control when adding measurement noise, $R = 10^{-9}$ and $(c_1, c_2) = (20, 20)$. On the top, the dashed line is steel ball position and solid line is reference signal. On the bottom, we see measurement noise affect the input signal significantly.

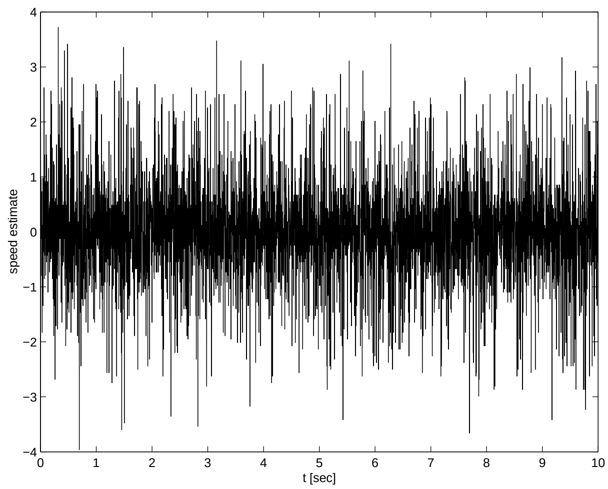

Figure 4.4. The velocity of the steel ball computed by time-derivative of the position. $(c_1, c_2) \approx (2, 2)$

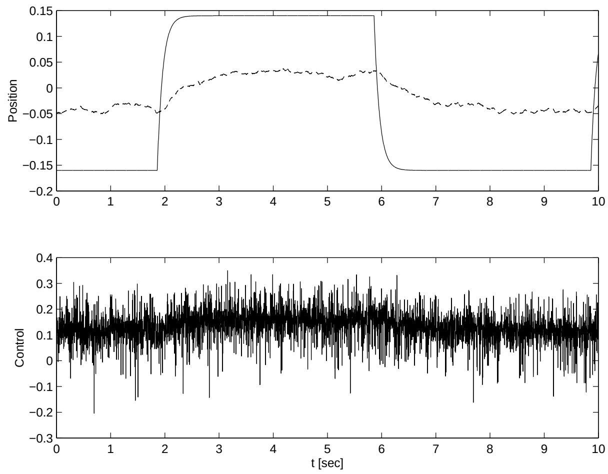

Figure 4.5. Tracking control when time-derivation is incorporated for estimating the steel ball speed. $(c_1, c_2) \approx (2, 2)$

which we used in the derivations above. The observation error is $\varepsilon = x_2 - \hat{x}_2$ with the dynamics

$$\begin{aligned} \dot{\varepsilon} &= g - \theta_1 \lambda(x_1)(u + u_0)^2 - \dot{\chi} - k\dot{y} \\ &= g - \theta_1 \lambda(x_1)(u + u_0)^2 - g + \theta_1 \lambda(x_1)(u + u_0)^2 + k(\chi + ky) - kx_2 \\ &= k\hat{x}_2 - kx_2 \\ &= -k\varepsilon. \end{aligned}$$

Therefore the error will vanish following

$$\varepsilon(t) = \varepsilon(0)e^{-kt}$$

and the equations system (4.4) will be transformed to

$$\dot{x}_1 = \chi + kx_1 + \varepsilon \quad (4.10a)$$

$$\dot{\chi} = g - \theta_1 \lambda(x_1)(u + u_0)^2 - k(\chi + kx_1) \quad (4.10b)$$

$$\dot{\varepsilon} = -k\varepsilon \quad (4.10c)$$

Remembering the approach presented in the observer backstepping section, we will apply backstepping to (4.10). The error-term will be considered as disturbance and will be accounted for, if needed, in the backstepping steps. As we saw in the section nonlinear damping terms, even harmless disturbances as this one might cause instability when certain conditions are present.

Step 1

$$z_1 = x_1 - r \Longrightarrow$$

$$\begin{aligned}
\dot{z}_1 &= \{\chi + kx_1 + \varepsilon\} \\
&= \chi + k(z_1 + r) + \varepsilon \\
\dot{\chi} &= \{g - \theta_1\lambda(z_1)(u + u_0)^2 - k(\chi + kx_1)\} \\
&= g - \theta_1\lambda(z_1)(u + u_0)^2 - k(\chi + k(z_1 + r))
\end{aligned}$$

A clf for the first equation is $V(z_1)$ where

$$\begin{aligned}
V(z_1) &= \frac{1}{2}z_1^2 \\
\dot{V} &= z_1\dot{z}_1 = z_1(\chi + k(z_1 + r) + \varepsilon)
\end{aligned}$$

The only choice we have in picking a virtual control variable is χ

$$\chi^{des} = -c_1 z_1 - k(z_1 + r) \triangleq \alpha \qquad c_1 > 0, \tag{4.11}$$

which will result in the derivative of the clf

$$\dot{V} = -c_1 z_1^2 + z_1\varepsilon.$$

Here we se the observer error occur in the clf derivative expression. We remember from the section about *nonlinear damping* that these terms were counted for by introducing a damping term. This was justified if the *disturbance* entered the equation multiplied by a function without upper bound, such in the case of $\phi(x) = x^2$. The case here is not the same, therefore the controller is satisfactory. Yet the clf is not, in the current form. The last term in $\dot{V}$ has indefinite sign. Therefore we will augment our clf with a quadratic term in the disturbance variable as following

$$\begin{aligned}
V_1 &= V + \frac{1}{2kd_1}\varepsilon^2 \quad \Longrightarrow \\
\dot{V}_1 &= z_1(\chi + k(z_1 + r) + \varepsilon) - \frac{1}{d_1}\varepsilon^2 \\
&= -c_1 z_1^2 + z_1\varepsilon - \frac{1}{d_1}\varepsilon^2
\end{aligned}$$

The completion of squares are now possible, and we get

$$\begin{aligned}
\dot{V}_1 &= -c_{1p}\big[z_1^2 - \frac{1}{c_{1p}}z_1\varepsilon + \frac{1}{d_1 c_{1p}}\varepsilon^2\big] \\
&= -c_{1p}\Big[(z_1 - \frac{1}{2c_{1p}}\varepsilon)^2 + \frac{4c_{1p} - d_1}{4d_1 c_{1p}^2}\varepsilon^2)\Big] \\
&= -c_{1p}\Big(z_1 - \frac{1}{2c_{1p}}\varepsilon\Big)^2 - \Big(\frac{4c_{1p} - d_1}{4d_1 c_{1p}}\Big)\varepsilon^2
\end{aligned}$$

and the $\dot{V}$ is thereby negative definite. This choice of the first virtual control transform the z_1-equations into

$$\dot{z}_1 = -c_1 z_1 + \varepsilon$$

Laplace transforming this equations gives

$$sZ_1(s) - z_1(0) + c_1 Z_1(s) = \frac{\varepsilon(0)}{s+k}$$

$$Z_1(s) = \frac{\varepsilon(0)}{k-c_1}\frac{k-c_1}{(s+c_1)(s+k)} + \frac{z_1(0)}{(s+c_1)}$$

and the inverse Laplace transform result in the solution

$$z_1(t) = \frac{\varepsilon(0)}{k-c_{1p}}(e^{-c_1 t} - e^{-kt}) + z_1(0)e^{-c_1 t}$$

Step 2

We define the new error variable, as the deviation of the first virtual control from its desired value,

$$z_2 = \chi - \alpha \Rightarrow$$

$$\begin{aligned}
\dot{z}_1 &= z_2 + \alpha + k(z_1 + r) + \varepsilon \\
\dot{z}_2 &= g - \theta_1\lambda(u+u_0)^2 - k(\chi + k(z_1+r)) - \dot{\alpha} \\
&= g - \theta_1\lambda(u+u_0)^2 - k(z_2 + \alpha + k(z_1+r)) \\
&+ (k+c_1)(z_2 + \alpha + k(z_1+r) + \varepsilon) \\
&= g - \theta_1\lambda(u+u_0)^2 + c_1(z_2 + \alpha + k(z_1+r)) + (k+c_1)\varepsilon
\end{aligned}$$

where

$$\dot{\alpha} = -c_1\dot{z}_1 - k\dot{z}_1.$$

The new clf will be again augmented by a quadratic term penalizing the new error variable.

$$V_2 = V_1 + \frac{1}{2}z_2^2$$

and its time derivative is

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 \\
&= z_1(z_2 + \alpha + k(z_1+r) + \varepsilon) - \frac{1}{d_1}\varepsilon^2 \\
&+ z_2\big[g - \theta_1\lambda(u+u_0)^2 + c_1(z_2 + \alpha + k(z_1+r)) + (k+c_1)\varepsilon\big] \\
&= z_1[\alpha + k(z_1+r) + \varepsilon] - \frac{1}{d_1}\varepsilon^2 \\
&+ z_2\big[z_1 + g - \theta_1\lambda(u+u_0)^2 + c_1(z_2 + \alpha + k(z_1+r)) + (k+c_1)\varepsilon\big].
\end{aligned}$$

Now its time to choose the controller u. Here we choose a control law, which equals the last bracketed term to $-c_2 z_2$, $c_2 > 0$, as the following

$$z_1 + g - \theta_1\lambda(u+u_0)^2 + c_1(z_2 + \alpha + k(z_1+r)) = -c_2 z_2 \Longrightarrow$$

$$(u + u_0)^2 = \frac{z_1 + g + c_1(z_2 + \alpha + k(z_1 + r)) + c_2 z_2}{\theta_1 \lambda(z_1)} \tag{4.12}$$

and result in the time derivative function

$$\begin{aligned}
\dot{V}_2 &= z_1(-c_1 z_1 + \varepsilon) - \frac{1}{d_1}\varepsilon^2 + z_2[-c_2 z_2 + (k + c_1)\varepsilon] \\
&= -c_1(z_1 - \frac{1}{2c_1}\varepsilon)^2 - \frac{4c_1 - d_1}{4c_1}\varepsilon^2 - c_2 z_2^2 + (k + c_1) z_2 \varepsilon.
\end{aligned}$$

Once again we se that there is no need for adding a further term in form of nonlinear damping. But we need a new term in the expression of the clf. This in order to be sure that the last sign-indefinite term in the last expression will no cause instability. Hence

$$\begin{aligned}
V_3 &= V_2 + \frac{1}{2d_2 k}\varepsilon^2 \\
&= \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2kd_1}\varepsilon^2 + \frac{1}{2kd_2}\varepsilon^2
\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}
\dot{V}_3 &= -c_1(z_1 - \frac{1}{2c_1}\varepsilon)^2 - \frac{4c_1 - d_1}{4c_1}\varepsilon^2 - c_2 z_2^2 + (k + c_1) z_2 \varepsilon - \frac{1}{d_2}\varepsilon^2 \\
&= -c_1(z_1 - \frac{1}{2c_1}\varepsilon)^2 - \frac{4c_1 - d_1}{4c_1}\varepsilon^2 - c_2\big[z_2^2 - \frac{k + c_1}{c_2} z_2 \varepsilon + \frac{1}{d_2 c_2}\varepsilon^2\big] \\
&= -c_1(z_1 - \frac{1}{2c_1}\varepsilon)^2 - \frac{4c_1 - d_1}{4c_1}\varepsilon^2 - c_2\Big[(z_2 - \frac{k + c_1}{2c_2}\varepsilon)^2 + \frac{4c_2 - d_2(k + c_1)^2}{4d_2 c_2^2}\varepsilon^2\Big] \\
&= -c_1(z_1 - \frac{1}{2c_1}\varepsilon)^2 - \frac{4c_1 - d_1}{4c_1}\varepsilon^2 - c_2\big(z_2 - \frac{k + c_1}{2c_2}\varepsilon\big)^2 - \Big(\frac{4c_2 - d_2(k + c_1)^2}{4d_2 c_2}\Big)\varepsilon^2 \\
&= -q(z, \varepsilon)
\end{aligned}$$

is negative definite, and thereby the overall controller u is stabilizing.

We tested this controller in a simulation were we initiate the observer with a different point to test its ability to converge and set $k = 10$. This should push the observer error to vanish in at time range of a $0.1s$. We even add measurement noise just to see how robust is the closed loop system. The results were very satisfactory and can be seen in Figure 4.6 and 4.7. This means that the observer backstepping technique is capable, in the presence of no other uncertainties, to stabilize the closed loop system. Once again we see the effect of Assumption 4.1 as poor tracking in the transitions in Figure 4.6.

In an effort to test consistency of this design we tested (4.12) on the Maglev system 33-210. The results differ from the simulations, which can be seen in Figure 4.9 and 4.8. There might be many explanation for this behavior but we could not for certain point out the most crucial one. For example, we noticed that the sensor, measuring the ball distance from the electromagnet, returns a value which is not the distance itself but proportional to it. Not having the full knowledge of

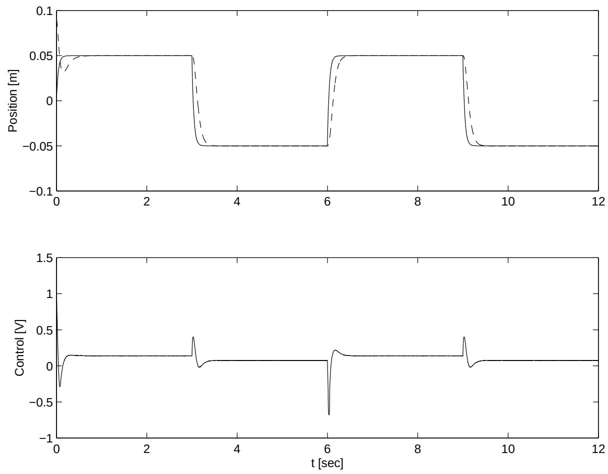

Figure 4.6. Simulation of tracking control with (4.12) when $R = 10^{-9}$,$k = 10$ and $(c_1, c_2) = (20, 20)$. The control input is in the limited range (3.8) during the simulation and the tracking control is very good.

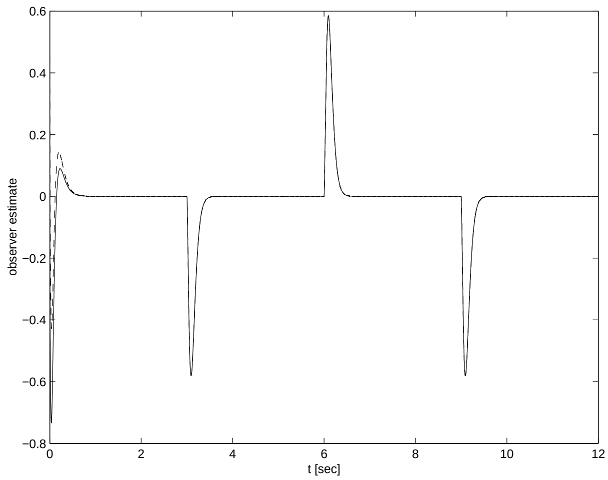

Figure 4.7. Simulation of observer response when implemented with (4.12) ,introducing measurement noise $R = 10^{-9}$, $k = 10$ and $(c_1, c_2) = (20, 20)$. The dashed line is $\hat{x}_2$ while the solid line is x_2. Except for the beginning of the simulation, the observer mange to predict the velocity very well.

this proportionality, we had to carry on experiments to compute this relationship. Levitating the ball at different heights, we measured the distance with a ruler. The sensor signal was simply computed as the mean value of the signal over a time interval. Tests from these experiments did not result in a reliable expression, and the simulations, using these expressions, failed in controlling the steel ball height. One possible explanation is the lack of accuracy of this manual measuring. The sensor is supposed to measure height differences in the range of mm accuracy, which the height-measuring procedure above could not offer. We even noticed that the system dynamics changes over time depending how long the machine were switched on. The models we used in this work are constant in time and therefore may have problems predicting the behavior of the system. This problem complicated the task of gathering data for comparison with simulation results. Even the parameter values we used in the simulation phase suffers form inaccuracy. They are results from experiments using the built-in hardware controller. The observer design is based on the knowledge we have about the system, which in this case is the used model. So, inaccuracy in the model will cause a poorly performing observer. Changing the gravitation constant in the model and setting the reference signal to a constant, we found an offset error in the observer signal. Although not much but yet have to be considered. We have to suppress that those high c_1 and c_2 values we used before in the simulations gave us numerical problems which led to the failure of the tests. Extensive examinations of this problem made clear that we have to decrease these to values to about 15 in order to carry out simulations.

Result from experiments implementing (4.12) and the observer can be seen from Figure 4.9 and 4.8. Meeting these problems, we see that there is a need for

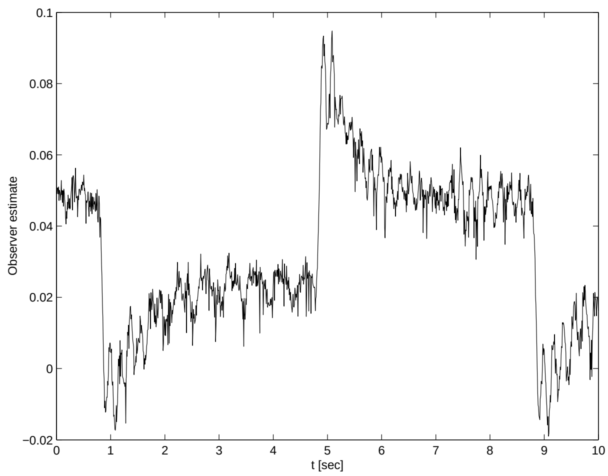

Figure 4.8. Estimated value of x_2 by the observer from test on MagLev 33-210 when $k = 10$ and $(c_1, c_2) = (15, 15)$. The offset error is clearly visible, even when the steel ball is, in mean, still.

adaption. A static controller do not have a chance in this case, at least when we seek a satisfactory performance. Therefore we hope to get better performance when we implement a parameter estimator along the controller. This is done in the next section. Our conclusion is that in order for the observer backstepping technique to succeed, we have to adapt the system, online. The control law provided by

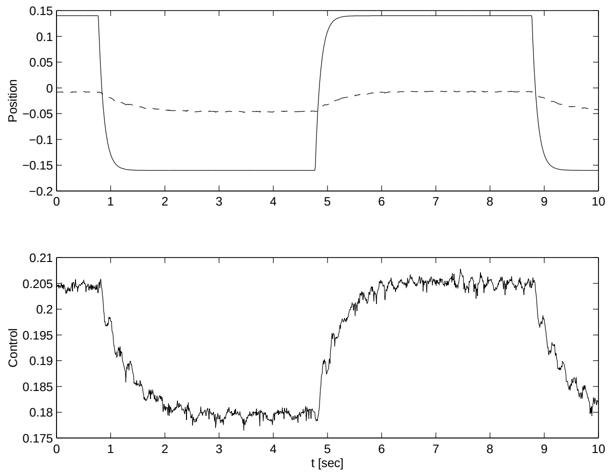

Figure 4.9. Tracking response when observer implemented with (4.12). The dashed line in the first subfigure is x_1 while the solid is the reference signal. The controller loses its ability to regulate the steel ball height in order to levitate the ball at all.

observer backstepping hade excellent performance in the simulations we conducted before. The problem is most certainly in the model and not in the method. This new degree of freedom might solve this problem without the need to change the model.

4.1.4 Adaptive observer backstepping

Following the basic ideas in section 2.5.1 we will try to construct a controller which will be an adaptive one. The only parameter perturbation we can handle, due to the structure of the model, is θ. We still require that θ is constant. Hence the control law (4.12) will change into

$$(u+u_0)^2 = \hat{\varrho}\frac{z_1 + g + c_1(z_2 + \alpha + k(z_1 + r)) + c_2 z_2}{\lambda(z_1)} \tag{4.14}$$

where

$$\hat{\varrho} = \frac{1}{\hat{\theta}}.$$

Using this estimated value instead of the unknown parameter will again lead us to the idea of penalizing the deviation of the estimation from the real value. Therefore we construct

$$V_4 = V_3 + \frac{\theta}{2\gamma}\tilde{\varrho}^2 \tag{4.15}$$

and the time derivative will be

$$\begin{aligned}\dot{V}_4 &= z_1[\alpha + k(z_1+r) + \varepsilon] - (\frac{1}{d_1} + \frac{1}{d_2})\varepsilon^2 - \frac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} \\ &+ z_2\big[z_1 + g - \theta\lambda(u+u_0)^2 + c_1(z_2 + \alpha + k(z_1+r)) + (k+c_1)\varepsilon\big].\end{aligned}$$

Using the controller (4.14) in this expression, we get

$$\dot{V}_4 = z_1(-c_1z_1 + \varepsilon) + z_2\Big[z_1 + g - \theta\lambda\hat{\varrho}\frac{z_1+g+c_1(z_2+\alpha+k(z_1+r))+c_2z_2}{\lambda(z_1)} + \\ c_1(z_2+\alpha+k(z_1+r)) + (k+c_1)\varepsilon\Big] - (\tfrac{1}{d_1} + \tfrac{1}{d_2})\varepsilon^2 - \tfrac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}}.$$

The estimation error is

$$\tilde{\varrho} = \frac{1}{\theta} - \hat{\varrho}$$

and this implies

$$\begin{aligned}\dot{V}_4 &= z_1(-c_1z_1 + \varepsilon) - \frac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} \\ &+ z_2\Big[z_1 + g - \underbrace{\theta(\frac{1}{\theta} - \tilde{\varrho})}_{(1-\theta\tilde{\varrho})}[z_1 + g + c_1(z_2+\alpha+k(z_1+r)) + c_2z_2] \\ &+ c_1(z_2+\alpha+k(z_1+r)) + (k+c_1)\varepsilon\Big] - (\frac{1}{d_1} + \frac{1}{d_2})\varepsilon^2.\end{aligned}$$

As can be seen, many terms will cancels and the remaining terms are

$$\begin{aligned}\dot{V}_4 &= z_1(-c_1z_1 + \varepsilon) - (\frac{1}{d_1} + \frac{1}{d_2})\varepsilon^2 + z_2\Big[-c_2z_2 + (k+c_1)\varepsilon \\ &+ \theta\tilde{\varrho}[z_1 + g + c_1(z_2+\alpha+k(z_1+r)) + c_2z_2]\Big] - \frac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}} \\ &= z_1(-c_1z_1 + \varepsilon) + z_2(-c_2z_2 + (k+c_1)\varepsilon) - (\frac{1}{d_1} + \frac{1}{d_2})\varepsilon^2 \\ &+ z_2\theta\tilde{\varrho}\Big[z_1 + g + c_1(z_2+\alpha+k(z_1+r)) + c_2z_2\Big] - \frac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}}.\end{aligned}$$

The reader recognize the first row of expressions after last equal sign and it is $-q(z)$ as before. For us to decide now is how to choose the update law $\hat{\varrho}$ in order for the derivative of the clf to remain negative. Here we choose to cancel this term,

$$\theta\tilde{\varrho}z_2[z_1 + g + c_1(z_2+\alpha+k(z_1r)) + c_2z_2] = \frac{\theta}{\gamma}\tilde{\varrho}\dot{\hat{\varrho}}$$

and we get the parameter update law

$$\dot{\hat{\varrho}} = \gamma z_2[z_1 + g + c_1(z_2+\alpha+k(z_1+r)) + c_2z_2]. \tag{4.16}$$

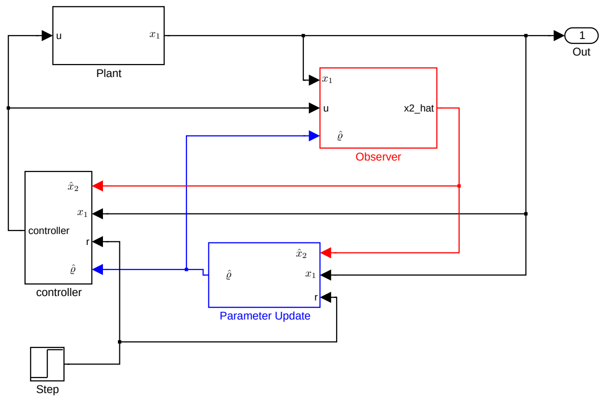

Figure 4.10. Block representation of the closed-loop system when using a parameter update for θ and an reduced order observer for estimating x_2.

The first test we conducted was a step response, which we included in a simulation as shown in Figure 4.10. We wanted to test whether the estimation converge or not, an if so, to what values. We implemented a step signal and chose $(c_1, c_2) = (7, 7)$ just to be as near the realtime tests as possible but with satisfying transient performance. (This choice will be more motivated at the end of this chapter.) We even chose $\gamma = 0.001$. The results which can be seen in Figure 4.11, shows that the parameter estimate converge to the real value of θ. "Real value" of the parameter meaning the value θ have in the model we are using. The convergence of the parameter update law to this value means it will hopefully converge to whatever values θ may take. (Still, this remains to see when we do tests on the MagLev test system 33-210.) We even carried out a simulation of the whole system. We used the same values for c_1, c_2 and γ. The reference tracking, as can be seen from Figure 4.12, were satisfying although we added measurement disturbance with $R = 10^{-9}$. From Figure 4.13 we can see that the estimate converges to the value of $1/\theta$. The reason why it does not reach the final value is because the reference signal is too fast or that we have chosen the values c_1,c_2 and γ poorly. Conclusion: the adaptive observer backstepping is able to overcome the problem we had due to uncertainties in the model. Comparing the results here with Figure 4.6 and 4.7 we see this fact clearly.

Implementing this controller in Simulink and measuring the response of the Maglev system 33-210, we got results that we found rather consistent with the simulations. First, we made a test measuring the steel ball level and $\hat{\varrho}$ when we hold the reference signal constant and then add another ball to the system. The results can be seen in Figure 4.14, where the converging property of the parameter update law towards a bounded value is obvious. The total mass has increased and therefore $\varrho = 1/\theta = m/a$ must increase too. Tests using the above chosen values of $(c_1, c_2) = (7, 7)$ were not successful and we hade to change them to $(c_1, c_2) = (3, 3)$. Testing different values of γ, we decided on 0.005. This way the parameter estimator

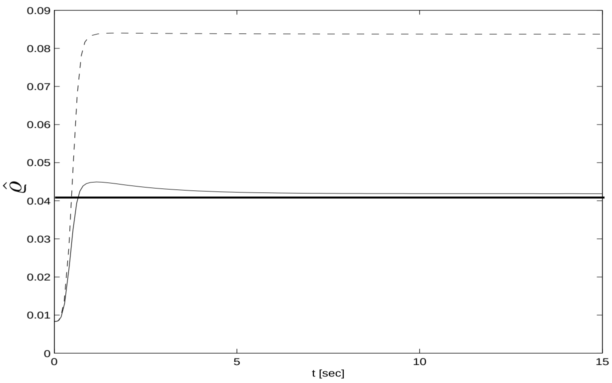

Figure 4.11. The parameter update law estimate a value which is consistent with the value used in the model. Here the solid line is the estimate when the model is including one steel ball, while the dashed line is for two. The thick solid line is $1/\theta$. Used parameters are $(c_1, c_2) = (7, 7)$, $\gamma = 0.001$ and $k = 10$.

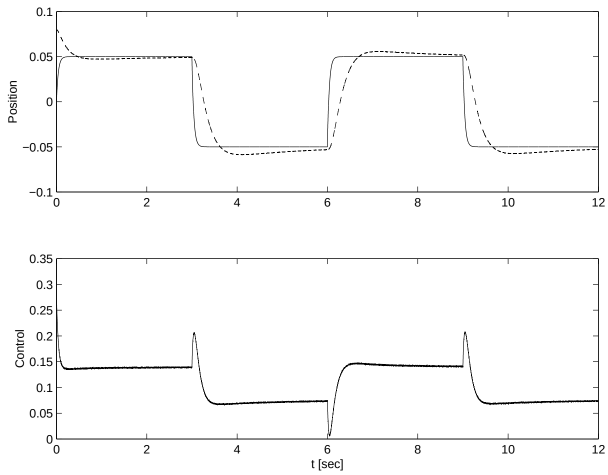

Figure 4.12. Simulation of tracking of a square signal when using controller (4.14), the observer (4.8) and the parameter update (4.16). Here, the solid line is reference signal and dashed line is x_1.

would converge faster than in the simulations made before.

Although the parameter update law estimates a higher value for the unknown parameter θ, this new value actually does improve the tracking. We carry on a test, where we let the parameter update law converge online to its value. Then we put the gain of this update law to zero. This way we were able to switch between these two values for θ, the one used in simulations and the one obtained from the update law. We see this approvement achieved in the reference tracking problem in

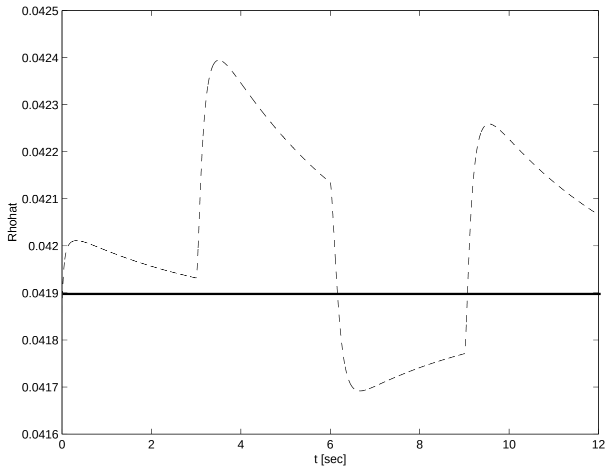

Figure 4.13. Simulation of tracking of a square signal when using controller (4.14), the observer (4.8) and the parameter update (4.16). Here, the solid line is $1/\theta$ and dashed line is$\hat{\varrho}$. The parameter estimate tend to $1/\theta$ but do not reach it before the reference signal switch to another value.

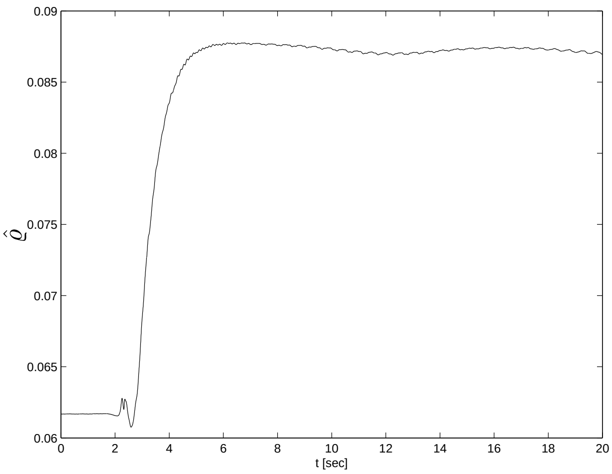

Figure 4.14. Realtime measurement using a constant reference signal. The controller is (4.14), the observer (4.8) and the parameter update (4.16). $(c_1, c_2) = (3, 3)$ and $\gamma = 0.005$. Adding a second steel ball after 2 s, we see the parameter estimate tend to another value. Using this value we were able to levitate both steel balls. The oscillation in the beginning, right after we add the second ball, is probably caused by the low reference height chosen.

the sense of less tracking error in Figure 4.15. The tracking error is not completely counteracted but yet less than before. Using a square-wave-like reference signal as

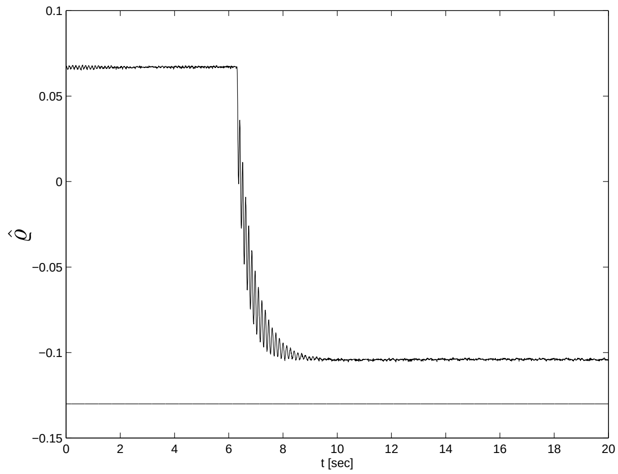

Figure 4.15. Realtime measurement showing the change in the tracking error when we toggle between the values for θ. These values are the one estimated by (4.16) and the constant value used in the simulations. $(c_1, c_2) = (3, 3)$ and $\gamma = 0.005$.

in the simulations and we can se the results in Figure 4.16. In the figure showing the level control, the only unexpected phenomenon is the oscillation of the steel ball, as can be seen from the figure. One reason could be the flux linkage which, as we mentioned earlier in the modelling chapter, increases in magnitude as we tend from the magnet. Otherwise we were satisfied with the result.

Note that the convergence property of the update law showed to be bounded to a neighborhood of its steady state value. Tests pushing the update law to start further from this value failed, even when increasing the value of γ significantly. Although we tried, we were not able to determine the size of this set due to different problems. The major one was the time variety of the system. The steady state value changed depending on how long the system were switched on. In spite of this oscillation we even got a better estimation from the observer. This can be seen from Figure 4.17, where the oscillation are prominent but yet in mean the estimated value of x_2 is better than in the non-adaption case above. Obviously, although the estimator is not that fast and the steel ball tends to oscillate when it is further from the magnet, we got much better results by using the adaptive controller than before. If we had chosen c_1, c_2 and γ better, we might have got better results. This is a matter of testing and time and we just could not find a better choice. Never the less, we can improve this controller in one more way, and that is by changing the stabilizing controller (4.11). This in the next section.

The conclusion we have to draw here, for this section, is that the adaptation approach of observer backstepping did improve the behavior of the closed-loop system. This happens despite of all the uncertainties we have. Those were the possible time-dependency of θ due to terms we have not included in the model. The possible increasing flux linkage magnitude due to the generalization we made

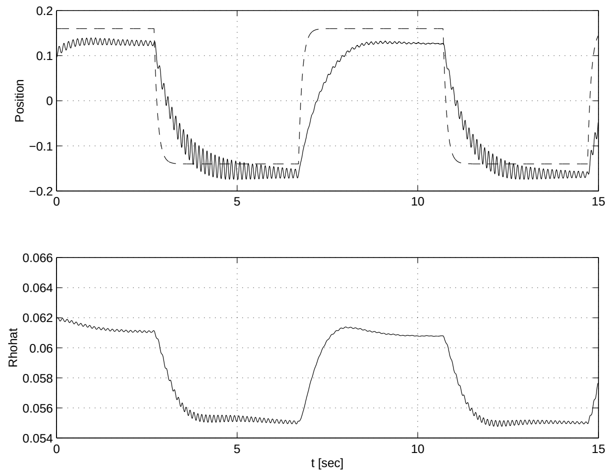

Figure 4.16. Realtime measurement using a square signal as reference, where $(c_1, c_2) = (3, 3)$ and $\gamma = 0.005$

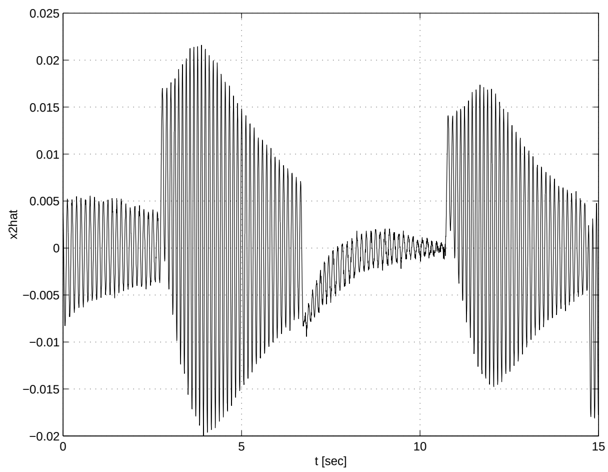

Figure 4.17. Realtime measurement using a square signal as reference, where $(c_1, c_2) = (3, 3)$ and $\gamma = 0.005$. The observer estimates, in mean, a better value of the steel ball velocity than in Figure 4.8.

in the modelling chapter. We should not forget the uncertainty in the parameters a and b due to the way we estimated them.

4.1.5 Changing the stabilizing function

Backstepping technique gives us room to choose these consecutive controllers as we want to. Here we will change the controller (4.11) into

$$\chi^{des} = -c_{1p}z_1 - c_{1i}\int_0^t z_1\,dt - k(z_1+r) \triangleq \alpha \qquad c_{1i}, c_{1p} > 0. \tag{4.17}$$

where the new term will remove the offset of z_1. Following those steps we take in 4.1.4 we arrive at the derivative of the first clf

$$\dot{V} = -c_{1p}z_1^2 - c_{1i}\int_0^t z_1^2\,dt + z_1\varepsilon.$$

Fortunately we do not have to do all the calculations above again. The first and the last terms in the above expression for the derivative are as before, and the new term is quadratic in the error variable z_1. In other words, the derivative of the this clf is guaranteed negative definite as before. In step 2, we will get the following equations in the error variables:

$$\begin{aligned}\dot{z}_1 &= z_2 + \alpha + k(z_1+r) + \varepsilon \\ \dot{z}_2 &= g - \theta\lambda(u+u_0)^2 + c_{1p}(z_2+\alpha+k(z_1+r)) + (k+c_{1p})\varepsilon + c_{1i}z_1.\end{aligned}$$

The reader is by now familiar with the next step. Already designed the clf for the overall error system we computed its time derivative and choose our control input to render this function negative definite. The clf is, as before,

$$V_3 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2kd_1}\varepsilon^2 + \frac{1}{2kd_2}\varepsilon^2$$

and we get the new control input as

$$(u+u_0)^2 = \frac{z_1 + g + c_{1p}(z_2+\alpha+k(z_1+r)) + c_{1i}z_1 + c_2z_2}{\theta\lambda(z_1)}. \tag{4.18}$$

How do we choose (c_{1p}, c_{1i})?

We have to find a way to choose (c_{1p}, c_{1i}) that gives us a hint on how the step response would, approximately, look like. Examining the subsystem

$$\dot{z}_1 = z_2 + \alpha + k(z_1+r) + \varepsilon,$$

we can try to find a solution for it. Laplace-transforming of this equation give us

$$s\tilde{Z}_1 - z_1(0) + c_{1p}\tilde{Z}_1 + \frac{c_{1i}}{s}\tilde{Z}_1 = \tilde{Z}_2 + \frac{\varepsilon(0)}{s+k}$$

where we have made some conciliations for convenience. We even have used the expression $\varepsilon(t) = Ce^{-kt}$ which is the solution of the observation error (4.10c). More manipulations and we arrive at

$$\tilde{Z}_1 = \frac{s}{s^2 + c_{1p}s + c_{1i}}\big[\tilde{Z}_2 + \frac{\varepsilon(0)}{s+k} + z_1(0)\big].$$

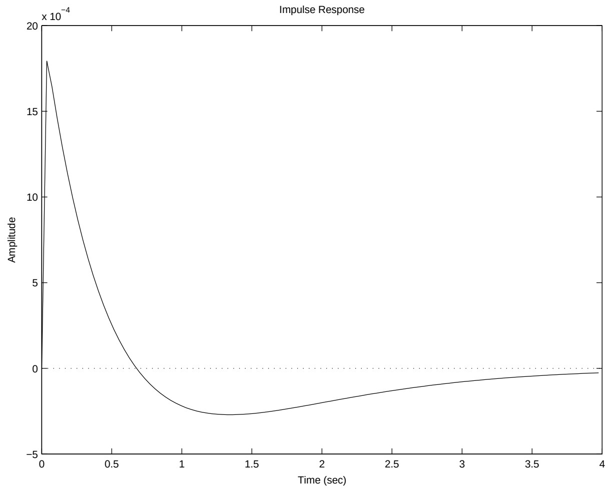

Figure 4.18. Impulse response from transfer function through which the observer error (4.10c) enters the solutions for the first error variable z_1. Here we chose $(c_{1p}, c_{1i}) = (3, 1.5^2)$, $k = 10$ and $\varepsilon(0) = 0.2$.

The dynamics of the term $s/(s^2 + c_{1p}s + c_{1i})$ can be place in $-c_{1p}/2$ if we choose $c_{1i} = (c_{1p}/2)^2$. This choice guarantee the impulse response of this transfer function to be bounded. (A further theoretical explanation can be found in [1] and specially example (2.16)). We neglect the effect of $z_1(0)$ thinking that near the origin, this term is sufficiently small. The term we were worried about was

$$\frac{s}{s^2 + c_{1p}s + c_{1i}} * \frac{\varepsilon(0)}{s + k}$$

and the fact the observer might have offset error in some cases. Although, the impulse response from this term showed to be vanishing rather quickly, as can be seen from Figure 4.18. Therefor the subsystem will remain bounded and converge to the origin if z_2 is stabilized to a neighborhood of the origin, which will be achieved in second step of the backstepping procedure. Note that going through the calculations we done before for the overall clf time derivative, which gave us the expression for the parameter update law, we arrive at the following

$$\dot{\hat{\varrho}} = \gamma z_2[z_1 + g + c_{1p}(z_2 + \alpha + k(z_1 + r)) + c_{1i}z_1 + c_2 z_2]. \tag{4.19}$$

Having these values and the new controller, it is time to do simulations on the system. We used the values $(c_{1p}, c_{1i}, c2) = (7, 3.5^2, 7)$ and $\gamma = 0.0005$. Tests showed that changing $k = 100$ is much more suitable. Although, the tracking have not improved significantly, which can be seen in Figure 4.19, and the parameter update law is as before and has slightly bigger overshot than before, which can be seen in Figure 4.20. In the last one we see even the observer estimate of the velocity

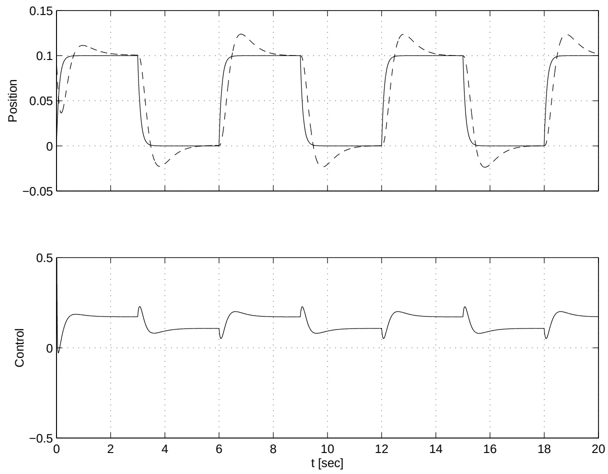

Figure 4.19. Simulation of position tracking, at the top, and control input, at the bottom. The dashed line is x_1 and the solid is r. $(c_{1p}, c_{1i}, c2) = (7, 3.5^2, 7)$, $\gamma = 0.0005$ and $k = 100$. The tracking have not improved compared to Figure 4.12, but still satisfying. The control input is still in the allowed region although slightly higher than before.

which is now much faster than before. Yet, we still believe that this controller is better than the one before and hope that the tests on MagLev test equipment 33-210 will show that.

Now it is time to test the new controller (4.18) on the MagLev system 33-210. Tracking the square-wave like reference signal were very satisfying. This can be seen in Figure 4.21 where, at the top, the reference tracking is visible and, at the bottom, $\hat{\varrho}$ can be seen during the same time cycle. Note that we were able to increase the controller parameters to the new values $(c_{1p}, c_{1i}, c2) = (7, 3.5^2, 7)$, a situation which was not possible before. This might be an indication of the system being time dependent. Some of these simulations do differ in performance depending on how long the system is switched on. It might be the I-part giving the controller the opportunity to counteract perturbations better. The system is very unstable. Therefore the time span in which the controller reacts, is vital in order to not leave the area in which the ball is visible to the photo-sensor. This last test was the best performance we were able to register using adaptive backstepping when the model (4.4) is incorporated to represent the magnetic levitation system.

4.2 Three State Model With No Uncertainties

When we started the work on this thesis the objective was to use the three state model and if possible augment it with further dynamics in order to find a better model for the magnetic levitation system. This way the tests conducted on the system 33-210 would be more successful. Unfortunately these attempts were not

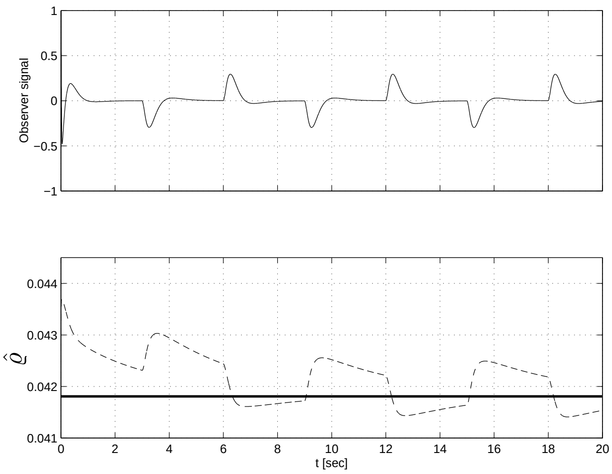

Figure 4.20. Simulation of the observer estimate $\hat{x}_2$ of the ball velocity at the top and the parameter estimate $\hat{\varrho}$, dashed thick line, at the bottom. The solid line represents $1/theta$. $(c_{1p}, c_{1i}, c2) = (7, 3.5^2, 7)$, $\gamma = 0.0005$ and $k = 100$. We see that the observer estimate is fast and rather accurate, but the parameter estimate have not improved compared to 4.13.

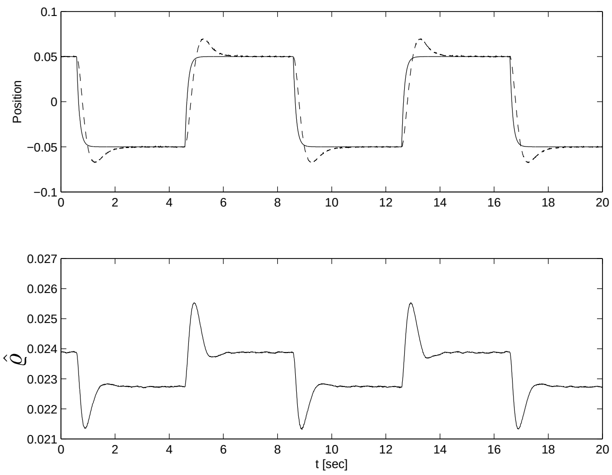

Figure 4.21. Tracking a square-wave reference signal when using the controller (4.18). The level tracking at the top, where the dashed line is x_1. Tracking has improved compared to Figure 4.16 with this new controller, apart from the overshot. The estimate $\hat{\varrho}$ at the bottom.

successful and the problems were to intricate to analyze. Here we will try to briefly mention the work that was done and the problems that occurred.

The model used was, denoting $(x_1, x_2, x_3) = (h, \dot{h}, i^2)$

$$\begin{aligned}
\dot{x}_1 &= x_2 && (4.20a)\\
\dot{x}_2 &= g - \theta_1\lambda(x_1)x_3 && (4.20b)\\
\dot{x}_3 &= -2\theta_2 x_3 + 2\theta_3\sqrt{x_3}u. && (4.20c)
\end{aligned}$$

where

$$\theta_1 = 1/m \qquad (4.21)$$
$$\theta_2 = R/L \qquad (4.22)$$
$$\theta_3 = G/L \qquad (4.23)$$

Following the steps in the two-state-model case, we know that we have to incorporate both observers and parameter update laws. The parameters θ_1 and θ_3 are considered as unknown virtual control coefficients and θ_2 as a unknown parameter. As we saw in the previous section, the parameter update laws for unknown virtual control coefficients do have problems converging to the true values of the parameters. They are only locally convergent not globally. On the other hand the update law for the parameter θ_2 do converge. This is proved in [3]. For conveniens, we will consider the case of full state feedback and no unknown parameters just to demonstrate the work done and the problems which occurred. The error variables were chosen to

$$\begin{aligned}
z_1 &= x_1 - r\\
z_2 &= x_2 - \alpha\\
z_3 &= x_3 - \alpha_1.
\end{aligned}$$

The clf for this system is $V(z_1, z_2, z_3) = (1/2)(z_1^2 + z_2^2 + z_3^2)$ which is very straight forward comparing to the ones found in the former section. The time derivative of this function is negative definite if the overall control law designed as

$$u = \frac{\theta_1\lambda(z_1)z_2 + 2\theta_2(z_3 + \alpha_1) + \dot{\alpha_1} - c_3 z_3}{2\theta_3\sqrt{z_3 + \alpha_1}} \qquad c_3 > 0 \qquad (4.24)$$

This controller would result in the error system

$$\dot{\mathbf{z}} = \begin{pmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & -\theta_1\lambda(z_1) \\ 0 & \theta_1\lambda(z_1) & -c_3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}.$$

The dynamics of this equation can be determined through the zeros of the maximum determinant of this matrix, which is

$$(s + c_1)[(s + c_2)(s + c_3) + (\theta_1\lambda(z_1))^2] + (s + c_3) = 0$$
$$s^3 + (c_1 + c_2 + c_3)s^2 + (1 + c_1c_2 + c_1(c_2 + c_3) + (\theta_1\lambda(z_1))^2)s + c_3 + c_1c_2c_3 = 0.$$

To analyze how we should choose c_i, $i = 1, 2, 3$ we need to find the roots of this equation. We tried to do this by using *Mathematica* but the expressions were very complex and we could not find a reliable expression. Therefore we abandoned the idea of finding a theoretical way of designing the values c_i, $i = 1, 2, 3$ and relayed on trial and error. These tries were unfortunately tedious and not successful.

Also in the cases when we wanted to test how the controller behaves in realtime, we had to implement an observer for at least x_3. The system 33-210 do not give any possibilities to measure the current through the coil. These test did not give any constructive results to tune these parameters. Not knowing if the controller is stabilizing and at the same time to have a observer for the current made these tests very unreliable.

The initial conditions, according to [3], are crucial for the transient performance and also for the stabilization. (For more detailes see [3] section 4.3.2.) Therefore some kind of estimate is needed to initiate the states. Not have been able to measure the current we had to estimate it in some way. We know the current must have a magnitude in order for the ball to levitate at all. Tests made on the system 33-210 when the reference signal is constant and the ball is levitated, we could find the initial condition of the control voltage. Using this value to find the current initial condition showed to be challenging. Equation (4.20c) indicates the connection between the voltage and the current. Although, the initial condition we got from this equation was negative. This is impossible because $x_3 > 0$ due to the chosen variables. Note that using the expression $I = cU + d$, which the manufacturer of the system 33-210 provide, gives a positive value for the initial condition, but yet this value was too big which result in the saturation of the control voltage.

These problems, and the lack of time, forced us to abandon this model and go on with the two state model instead.

4.3 Summary

Backstepping offers a very effective method to stabilize the closed-loop of a nonlinear system when all the states are available for measurement and no uncertainties are present. This was what we find in 4.1. The time-derivative of x_1, the ball distance from the magnet, as an estimate of x_2, the ball velocity, result in a poor performing control law and lack of robustness for measurement disturbances, see 4.1.2. Observer backstepping in 4.1.3 solved this problem and the result were better tracking performance and better robustness. The reduced-order observer proved to be a better choice for estimation of the ball velocity.

A lot of problems occurred when testing this control law in realtime. The adaptive observer backstepping technique in 4.1.4 was able to overcome this problem by adapting the control law to the system used. This adaption were possible by incorporating a parameter update law for the parameter θ, computed in the consecutive backstepping steps along the computation of the control law. In all the cases the control law proved theoretically to be stabilizing according to a control lyapunov function *clf*, which we were able to show that its time derivative was negative definite, when choosing the control law as we did. Though, realtime tests showed the parameter update law to be only locally convergent and that increasing its gain would not push the update law towards a bounded value.

The degree of freedom backstepping offers in the design steps were helpful in finding a better control law which have better performance in realtime. The use of a PI-control law as the first stabilizing function showed to be a much better choice. The control law showed to be faster than before and the tracking error decreases significantly.

The three-state-model were very challenging to use. Mostly because the problems in finding initial conditions which we could emerge from and reach stability. Not having these conditions, the tuning of the controller were difficult. By tuning we mean the choice of values c_i, $i = 1, 2, 3$. The efforts to determine these by the help of the roots of the closed loop system were too complex. These efforts were not successful. Even the use of *Mathematica* did not result in reliable results.

Chapter 5

Conclusions and Future Works

5.1 Conclusions

Choosing between two models with different levels of complexity we found the simpler model to be easier to implement backstepping upon. Even the results are easier to analyze. The implementation of adaptive backstepping on such a model resulted in a very satisfying tracking performance. This although the stability was not global because the parameter estimation presented in [3] for control coefficients showed to be only locally convergent. We even achieved this with smaller control effort than previous works done on the subject. This is just an evidence for the freedom we have choosing the constitutive stabilizing functions. The level of complexity of the control law do increase when the model is more detailed. But the control input do stay within the allowable area as long as we emerge from certain initial states. The importance of the state initial conditions is striking meaning that right initial states are the different between success and failure in stabilizing the closed loop.

5.2 Future Works

In the choice of a model there is a need for some kind of model evaluation. A more extensive work on a time- or frequency domain model evaluation will solve the problem with parameter uncertainties. Specially parameters which are not available for online estimation. A better description for the mutual inductance is needed. A probable measurement of the flux linkage and adding this information to the dynamics of system will improve the performance of the controller. Specially when the steel ball is far from the magnet. How homogeneous is the steel ball has to be counted for. Some of the oscillations do depend on this property of the ball. Specially when we add another steel ball to an already levitated one.

The two-ball system tend to spin about the mass centrum during the levitation process. Choosing a model incorporating the current, a test system which gives the opportunity to measure the coil current is needed. This combination is probably more successful because you do not have to guess the initial condition of the current. You just measure it and this way the tuning of the control parameters will be easier.

Bibliography

[1] T. Glad and L. Ljung. *Reglerteknik. Grundläggande teori.* Studentlitteratur, Lund, Sweden, 2nd edition, 1997. In Swedish.

[2] Hassan K. Khalil. *Nonlinear Systems.* Prentice-Hall, second edition, 1996.

[3] M. Krstić, I. Kanellakopoulos, and P. Kokotović. *Nonlinear and Adaptive Control Design.* John Wiley and Sons, 1995.

[4] Zhong-Hua Li Miroslav Krstić. Opitmal design of adaptive tracking controllers for nonlinear sysems. *Proceedings of the American Control Conference*, pages 1191–1197, 1997.

[5] P. K. Sinha. *Electromagnetic Suspension: Dynamics and Control*, volume 30 of *IEE Contr. Eng. Series.* Prentice-Hall, second edition, 1996.

[6] E. D. Sontag. A "universal" construction of artestin's theorem on nonlinear stabilization. *System and Control Letters*, 13:117–123, 1989.

[7] Melcher James R. Woodson Herbert H. *Electromechanical Dynamics PART 2: Fields, Forces and Motion.* Jon Wisley and Sons, New York, 1968.

[8] Daisuke Miyazaki Zi-Jiang Yang. Adaptive robust nonlinear control of a voltage-controlled magnetic levitation system. *5th IFAC Symposium on Nonlinear Control Systems (NOLCOS 2001)*, July 2001. Saint Petersburg.

[9] Masyuki Minashima Zi-Jiang Yang. Robust nonlinear control of a feedback linearsizable voltage-controlled magnetic levitation system. *T.IEE JAPAN*, 121(7):1203–1211, 2001.