Multi Degrees of Freedom Maglev System with
Permanent Magnet Motion Control

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Abstract

As magnetic suspension system is supporting system without mechanical contacts, there is a growing demand for magnetic suspension, especially in the field of high accuracy multi-dimensional positioning. Our motivation is to develop a permanent magnet suspension system with novel magnetic suspension technologies. It can realize noncontact positioning and micromanipulation. It can be applied in the fields in which ultra-clean environment is needed to avoid sample contamination, such as semiconductor processing, biotechnology experiments, especially, when the object is moved with micro displacements.

There are many kinds of maglev system, such as superconducting maglev system, electromagnetic maglev system, permanent maglev system and etc. we propose a novel active maglev system with permanent magnets and a motion control mechanism in place of electromagnets and a current control mechanism. Using permanent magnets, the features of this system are effective for saving energy, avoiding heat generation, no mechanical wearing and dust free.

Multi-DOF (degrees of freedom) permanent magnet suspension mechanisms that manipulate the object in the vertical plane and horizontal plane have been developed. The control method is that the position of the suspended object is controlled by means of adjusting the reluctance in a magnetic circuit of the suspension mechanism. In this research, the concrete method for control reluctance is to adjust the air gap between the permanent magnet and the suspended object. As the reluctance is proportional to the air gap in the magnetic circuit, if the air gap is adjusted with the linear actuator, the reluctance in the magnetic circuit can be actively adjusted.

As the first step of multi-DOF micromanipulation, the suspension mechanism of the 1-DOF maglev system is investigated. The principle of suspension is that the suspended object is suspended by an attractive force of a permanent magnet which is driven by an actuator and positioned above the suspended object. The direction of levitation is vertical (both of the magnet and suspension object are only moved in this direction). The suspended object is ferromagnetic material. The equilibrium position of the ferromagnetic body is determined by means of a balance between the gravity force and the attractive force of magnet. The attractive force can be adjusted with adjusting the air gap between the magnet and the ferromagnetic body. The nonlinear attractive force of magnet is linearized and calculated. The model of the suspension system is created and the feasibility of suspension mechanism is analyzed theoretically. The controller of the suspension system is calculated with the LQR (linear quadratic regulator) control law, which ensures the stability of close loop system, and the system is examined by means of numerical simulation and experimental examination.
Secondly, a 2-DOF permanent maglev system, in which three permanent magnets are used to manipulate the object in the vertical plane, is investigated. Based on the suspension principle, the prototype has been built. In this prototype, the suspended object is an iron ball and magnets are driven by actuators to control the position of iron ball. One magnet is located on the upper to suspend the iron ball, and the other two magnets are located on the two sides of the iron ball to control its motion on the horizontal direction.

Fig. 1 shows the outline of 2-DOF maglev system. There are three permanent magnets in this suspension system and each magnet has two poles, N and S, so there are some different arrangements which have different flux distribution. The pole arrangement is named in terms of the magnets polarity, for instance, if the magnet polarities which are facing the iron ball are S, S and S, the name of pole arrangement is called SSS. The poles order of arrangement is the top magnet, left magnet and right magnet. Therefore, the pole arrangement showed in Fig.1, is called SNN. In the 2-DOF maglev system, the total number of pole arrangements is 6 which are SSS, SSN, SNN, NNN, NNS, and NSS. From the performance of magnet field we know that the SSS and NNN have the same flux distribution, the SSN and NNS have the same flux distribution, and the SNN and NSS have the same flux distribution, so we choose the former three arrangements for analysis. The magnet pole arrangement is studied with FEM (finite elements method) analysis. As the result of FEM analysis, the SNN (or NSS) arrangement is used in this system, because this arrangement can provide sufficient force acting on the suspended object and the equilibrium position can be assumed to the center of the magnets.

Next, the horizontal motion model of the system is created, in which the influence of the vertical attractive force is neglected. There are three situations of this system considering the input and output of system, two forces input system, same force input system, and two magnets connecting system. The controllability and observability of these models are analyzed by the linear control theory. Consequently, only the two forces input system can be controlled and observed, so it is applied as the motion model of the suspension system.

The motion equations are obtained based on the free body diagram of the model. In this model, the
magnet force is nonlinear with the air gap length, so these equations are nonlinear. First, these equations are linearized and then the state space equation can be used to express the motion model. In terms of the LQR control law, an optimal controller of suspension system is obtained by choosing the state weighting matrix Q and input weighting matrix R. The system is examined with this controller. However, the dynamic characteristic of step response of the simulation result is different from that of the experimental result. This phenomenon would be caused by the vertical magnet attractive force that had been neglected when the motion model is created. So an improved model is created in which the vertical magnet attractive force is considered. Based on the improved model, a new controller is obtained by means of LQR control law. With the new controller, the simulation result is consistent with the experimental result and the dynamic characteristic verifies that the system is stable.

The displacements of the iron ball in the horizontal direction were measured about reference input signals. The relationship between displacement and reference input is linear within a range of 0.006(m) from left to right of the equilibrium position, so the iron ball can be positioned at any position within this range in the horizontal direction.

Thirdly, a 4-DOF permanent maglev system is developed, in which the iron ball can be manipulated in the horizontal plane, i.e. the iron ball spins along the suspension axis and can be positioned near the center of mechanism. In this system, five permanent magnets are used. One is used to suspend the iron ball and the other four magnets are uniformly distributed in the horizontal plane of the suspended object to make the iron ball rotate and move in horizontal plane. The first step is to make the iron ball rotate along the suspension axis, and the second step is to make the iron ball move in the horizontal plane.

In the 4-DOF system, the arrangement of the polarity of the magnet is SNNNN or (NSSSS) as defined above. The suspended object is an iron ball also, which is a ferromagnetic substance. It must be magnetized so that there is remanent magnetism on the surface of the iron ball. When the iron ball is suspended, the influence of the strongest remanent magnetism decides the vertical direction of suspension. When a magnet located in horizontal plane approaches the iron ball, there is a magnetized point on the surface of iron ball facing the approaching magnet. And when this magnet withdraws and the next magnet approaches, the approaching magnet will attract the magnetized point. The four magnets alternatively approaching to the iron ball make the iron ball rotate along the suspension axis.

In the rotation mechanism, a laser sensor is applied to measure the rotation velocity. The four permanent magnets are driven by four linear actuators moving forward or backward for adjusting the air gap between the permanent magnet and the iron ball to control the torque. The motions make the iron ball rotate. The rotation model has been created and the motion equations have been obtained. The experimental examinations are performed. However, the result indicates that the rotation velocity is not uniform. This problem can be improved by increasing the number of the magnets which are located in the horizontal plane.
With the same construction of the rotation system, the iron ball can also be located at any position in the horizontal plane near the central position of the system. The principle is that through controlling the motion of four magnets which are located in the horizontal plane we can adjust the air gaps between the iron ball and magnet, the attractive forces of four magnets are changed, so the iron ball can be positioned at arbitrary position in the horizontal plane within a small range near the original point. The motion model is created and analyzed. The system can be controlled and observed, therefore, the LQR control law can be used to design the controller of this system. The system examination is performed with this controller. The feasibility of the motion is confirmed in terms of the examination results.

The presented work demonstrates the feasibility of manipulating the suspended object in the vertical plane and horizontal plane. Based on the outcome of experiments, it is concluded that

i. The suspended object is suspended successfully and stably.
ii. The suspended object can be manipulated in the vertical plane.
iii. The suspended object can be rotated and manipulated in the horizontal plane.

Finally, several suggestions are given to continue this research for improving the rotate quality and the robustness of suspension system.
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Chapter 1

Generation introduction

1.1 Overview

1.1.1 Definition and Requirement for suspension mechanism

**DEFINITION** Within an orthogonal coordinate system, every object or body possesses a linear position of three coordinates as well as an orientation of three rotations relative to another body. Generally, six independent variables are necessary and sufficient to define the spatial position fully in any given coordinate system. These are called **degrees of freedom (DOF)**.

**Suspension or bearing** is restriction/confinement of some DOF towards zero deviation, whilst, other DOF are non-restricted as good as possible. This comprised both static load carrying capacity as well as the dynamic compensation of disturbances.

**REQUIREMENT** The most common customarily used bearings are rolling element bearings, and most of the support mechanism just like the rolling mechanism and the slipping mechanism are supported with the mechanical contact force. It is inevitably to cause friction in these contact form mechanisms, so appropriate lubrication is necessary. Moreover, because a mechanical contact part becomes the source of dust, special consideration is needed for using in a clean environment.

1.1.2 Performance of magnetic suspension mechanism

Magnetic suspension mechanism is one kind of the support mechanism in which the object is supported by means of magnetic force, and there is no mechanical contact force when the magnetic suspension mechanism is working. Contactless operation leads to elimination friction, stickiness, wear, and dissipation, hence more accurate, noise free, reliable, long life, high speed, and clean operation at low operating costs even under severe conditions like extreme temperatures, contamination free environments, and Ultra High Vacuum [1]-[6].
1.2 Classification and application of magnetic suspension mechanism

1.2.1 Classification of magnetic suspension system

There is diversiform in magnetic suspension mechanism [2]-[9]. As we know that there are three kinds of material can generate magnetic force, i.e. a permanent magnet, a usual-state conductivity electromagnet and a superconductivity electromagnet, and there are three kinds of material is easy to let current and flux pass, i.e. a superconductor, a metal conductor, and a ferromagnetic substance. Combined these six kinds material, various magnetic suspension system can be constructed. According to the generation principle of the suspension force, magnetic suspension system is classified as

1. Levitation using forces of repulsion between permanent magnets.
2. Levitation using forces of repulsion diamagnetic materials.
3. Levitation using superconducting magnets.
4. Levitation by forces of repulsion due to eddy currents induced in a conduction surface or body.
5. Levitation using the force which acts on a current carrying linear conductor in a magnetic field.
6. Suspension using a tuned LCR circuit and the electrostatic force of attraction between two plates.
7. Suspension using a tuned LCR circuit and the magnetic force of attraction between an electromagnet and a ferromagnetic body.
8. Suspension using controlled DC electromagnets and the force of attraction between magnetized bodies.
9. Suspension using the force of attraction between a permanent magnet and a ferromagnetic body.
10. Suspension using the force of attraction between an electromagnet and a ferromagnetic body.

1.2.2 Application of magnetic suspension system

So far, many applications of magnetic suspension mechanism are commercially available already, e.g. Magnetic levitation conveyance system using forces of repulsion between permanent magnets [16]; the support system of maglev trains using forces of repulsion due to eddy currents induced in a conduction surface or body, such as transformer RAPIDDO of Germany and Shanghai [15]-[16]; the magnetic surfacing experiment line of JR using superconducting magnets [12]-[14]; the magnetic bearing using the force of attraction between an electromagnet and a ferromagnetic body, which is
call the EMS system [17]-[24]; the magnetic bearing using the force of attraction between an electromagnet and permanent magnet [10], [11], and the superconductivity repulsion type thrust bearing using the force of repulsion between superconducting magnet and permanent magnet [30], etc. The most widely used magnetic suspension system is the electromagnet suspension (EMS) system.

1.2.3 Disadvantages of the EMS system

Because the Lorentz force is weak, the lightweight thing is needed to choose as a suspended object. Although, a large suspension force can be acquired through make an electric wire into the shape of a coil, it becomes impossible to disregard the generation of heat when pass the current. Moreover, since the structure becomes complicated it is not easy to constitute a minute suspension mechanism with an EMS system.

In a common EMS system, a bias current is required in order to support the object prudence when the current passed in the coil of an electromagnet. Therefore, the problem of generation of heat by the EMS system cannot be generally disregarded. Moreover, because of the inductance ingredient of the coil of an electromagnet, the iron loss will be generated and there is delay of exciting force. It is difficult to control the EMS system in a high frequency domain.

On the other hands, the volume of the coil is a problem when the miniaturization of a system is considered. In addition, it is restricted to keep away a coil from a suspended object if the leak of flux from a magnetic circuit is taken into consideration. This has been the obstacle of a miniaturization of an EMS system.

In this research, a permanent magnet is used to replace the electromagnet, i.e. the force of attraction between an electromagnet and a ferromagnetic body is used to make the suspension mechanism. Although, it is not a solution of the problem for the miniaturization of a suspension system, it can make the coil keep away from the suspended object, mitigate a power loss and eliminate the influence of generated heat.

1.3 Motivation, goals and control method

1.3.1 Motivation

The purpose of this research is to develop a new active permanent magnetic suspension mechanism in which the suspended object can be manipulated in multi DOF. It can realize noncontact remote control and micromanipulation. It can be applied in such fields in which ultra-clean environment is needed to avoid sample contamination, such as semiconductor processing,
biotechnology experiments, especially, when the object is moved with micro displacement.

1.3.2 Goals

At first, applied the linear control theory examines the feasibility of the permanent magnet suspension system. Based on the considerations of above, the research project will aim at making the suspended object move in multi DOF, i.e. move in vertical plane and in horizontal plane. Furthermore, the accuracy of the motion of the suspended object is considered.

1.3.3 Control method

Adjusting the reluctance in a magnetic circuit of the suspension mechanism is considered to control the position of the suspended object. As a mater of fact, in this research, the concrete method is to adjust the air gap between the permanent magnet and the suspended object. That is to adjust the attractive force with permanent magnet. As the reluctance is proportional to the air gap in the magnetic circuit, if the air gap is adjusted with the linear actuator, the reluctance in the magnetic circuit can be actively adjusted.

Based on adjusting the air gap with the actuator to compose a permanent magnet suspension mechanism, the stabilization of the suspension system has to be considered. Due to that the attractive force works strongly if the permanent magnet is approaching to the suspended object, whereas the attractive force works weakly if the permanent magnet is departing from the suspended object, it is unstable in a permanent magnet and ferromagnetic. If a permanent magnet can be driven more quickly compared with the movement of the suspended body, the restoration force to the suspended body can be gained.

In a word, in such a system, the control object is not the size of the working force but the movement of the permanent magnet and the suspended object. It is the feature of this suspension system. If it is possible that the suspension mechanism which is composed by using this method, generally, seems to be impossible, the similarity suspension mechanism can be composed by using various generation forces. For instance, the suspension mechanism that uses the intermolecular force and the atomic force, etc. can be expected to be composed.

A concrete target of this research is to confirm the possibility of the suspension mechanism of the form to adjust the attractive force by controlling the movement of a permanent magnet with the actuator, and adjusting the air gap with the suspended body as described above.
1.4 Structure of thesis

This chapter that deals with the background and the performance of magnetic suspension mechanism and the goal and motivation of this investigation is the first part of this thesis. The succeeding chapter, chapter 2, treats the principle of permanent magnetic suspension system. The possibility of the permanent magnetic suspension system is verified theoretically and the examination is fulfilled. It is the second party of this thesis. The third part of this thesis, chapter 3, describes a 2-DOF permanent magnetic suspension system in which the suspended object is controlled to move in the vertical plane. The arrangement of the magnet poles is analyzed with the FEM (finite element method) in this system. The controller of this system is designed with LQR (linear quadratic regulator) control law. The numerical simulation and experimental examination is carried out. In chapter 4, a 4-DOF permanent magnet suspension system in which the suspended object is controlled to move not only in the vertical but the horizontal plane. It is designed based on the 2-DOF permanent magnetic suspension system. As usual, this thesis will end with a listing of the main conclusions in chapter 5, and the well meant advice and unfinished aspects that are recommended as topics for further research.
Chapter 2

Permanent magnetic suspension mechanism

In chapter 1, a new type magnetic suspension system in which the permanent magnet is used was proposed. In this chapter, the principle of this suspension system is described and the prototype of this suspension system is constructed based on the principle. Considering the feature of permanent magnet, the suspension feasibility and the experiment of suspension are examined. The attractive force of permanent magnet and the method to adjust the attractive force is analyzed theoretically.

2.1 Permanent magnetic suspension mechanism

2.1.1 Fundamental form of suspension mechanism

An outline of proposed suspension system with a permanent magnet and linear actuator is shown in Fig.2.1. In this system, the suspended object is a ferromagnetic body. It is suspended by an attractive force of a permanent magnet which is driven by an actuator and positioned above the suspended object. The direction of levitation is vertical (the magnet and the object move only in this direction). The equilibrium position of the ferromagnetic body is determined by means of a balance between the gravity force and the magnet force. The attractive force can be adjusted through adjust the air gap between the magnet and the ferromagnetic body.

If the actuator does not actively control the magnet’s position, the levitated object will either fall or adhere to the magnet. However servo-control of the actuator can make this system stable. Because there is a smaller attractive force for a larger air gap between the permanent magnet and object, the actuator drives the magnet upwards in response to object movement from its equilibrium position towards the magnet. Similarly, the actuator drives the magnet downwards in response to object movement away from the magnet. In this way, the object can be stably suspended without contact. In
comparison to the electrical control method of electromagnetic suspension systems, this system is a mechanical control maglev system [7].

![Fig.2.1 Outline of suspension mechanism with air gap control](image)

2.1.2 **Performance of permanent magnet suspension mechanism**

The performance of permanent magnet suspension system shown in Fig.2.1 is:

(i) Suspension is realized by mechanical control

(ii) Since no electromagnet is used, it is not necessary to take into consideration the influence of heat generated with a coil.

(iii) Since the control point and the suspension point are separated, no power loss in suspension point

(iv) Besides magnetic force, electrostatic power, intermolecular force and the force between atoms can be used to constitute the suspension mechanism

In the performance (i), since the suspension mechanism is stabilized by mechanical control, it can only be consisted of a permanent magnet when the equipment has a predictive position control such as a robot manipulator. When the noncontact conveyance mechanism is constituted according to the principle of this mechanism, the wiring for a coil can be omitted compared with the thing using an electromagnet.
In the performance (ii), as we know, if a suspension mechanism is constructed with an electromagnet, it is necessary to feed a passing current continually for supporting the gravity of the suspended object and a bias current for linearization in the coil. Thus, the heat is generated by the resistance of the coil. In this suspension mechanism since no coil is used, the influence of the heat can be disregarded.

In the performance (iii), when the distance between a coil and a suspended object is long, even if the magnetic resistance of the iron core was disregarded for analysis, the influence of the magnetic resistance of the iron core and leak of flux can not be disregarded in practice, thus sufficient suspension power may not be acquired actually. When the suspension system is operated with permanent magnet motion control, if there is a distance between the control part and the suspended object, the magnet can be connected with a long stick to the control part, and the suspension system has no power loss.

In the performance (iv), the suspension mechanism shown in Fig.3.1 enables a non-contact suspension because of the attractive force between suspension mechanism and the suspended object. A movement mechanism is used for stabilization of the suspension system. Therefore, this mechanism does not need to control the generating power itself. Any suspension mechanisms can be theoretically constructed by using a attractive force. Such as static electricity force, intermolecular force and the force between atoms can be applied to construct a small suspension mechanism without limit the suspended object to ferromagnetic substance [28].

2.2 Theoretical examination of feasibility

This section, the suspension mechanism shown in Fig.3.1 is examined theoretically. The suspension mechanism is modeled and a movement equation is drawn. The attractive force near the equilibrium point is linearized and a linear model of suspension system is obtained. The feasibility of suspension mechanism is examined by means of linear control theory. The following two models are considered by whether the influence of the attractive force is taken into consideration to movement of the permanent magnet.

Just like that shown in Fig.2.1, when a permanent magnet is driven an actuator, an attractive force from the suspended object works in addition to the driving force from an actuator. Compared with the mass of the suspended object, if the mass of the permanent magnet is large, the force from the suspended object can be disregarded during the motion of the permanent magnet. At this time, the movement of the permanent magnet can be considered independently with the movement of the suspended object. It can be considered that position control of the movement of the permanent magnet is carried out by the actuator. The input of a suspension system serves as a permanent magnet position.
Since the attractive force equals the weight of the suspended object at the equilibrium position, if the mass of the suspended object becomes large, the attractive force will become large. And when the permanent magnet is driven directly by a motor, it is necessary to take the attractive force into consideration to the movement of a permanent magnet. At this time, it can be considered that movement of a permanent magnet is a power control with the actuator. The input of the suspension system is the forces applied to a permanent magnet, and an output is the position of the suspended object. For analysis Fig 2.2 is used to express the model shown in Fig 2.1.

The sign used in Fig 2.2 is explained below.

- $z_0$: suspended object position
- $z_1$: permanent magnet position
- $m_0$: mass of suspended object
- $m_1$: mass of permanent magnet
- $f_a$: force of the actuator
- $f_m$: attractive force between permanent magnet and suspended object
- $d_{\text{gap}}$: air gap between permanent magnet and suspended object
- $k$: constant of the permanent magnet attractive force
- $k_m$: linearization constant of the permanent magnet attractive force
- $k_1$: spring constant of the permanent magnet support part
ξ: damping coefficient of a permanent magnet support part

Moreover, a minute deviation from an equilibrium position is expressed as ∆.

### 2.2.1 Attractive force between magnet and suspended object

For analyzing the suspension system, an attractive force between the suspended object and the permanent magnet is examined. The attractive force is proportional to the square of the flux which passes the air gap, and the flux is inverse proportional to the length of the air gap. Therefore the attractive force is shown in equation (2.2.1).

\[
f_m = \frac{k}{d_{\text{gap}}^2} \quad (2.2.1)
\]

Assume that at the equilibrium point, the attractive force is \(f_m0\) and the air gap is \(d_{\text{gap}}\). \(\Delta d_{\text{gap}}\) is the minute change of the air gap, so when there is a minute change the air gap becomes \(d_{\text{gap}} + \Delta d_{\text{gap}}\). At this time, the attractive force is

\[
f_m = \frac{k}{(d_{\text{gap}0} + \Delta d_{\text{gap}})^2}
\]

\[
= \frac{k}{d_{\text{gap}0}^2 + 2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2}
\]

\[
= \frac{k(d_{\text{gap}0}^2 - 2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2)}{(d_{\text{gap}0}^2 + 2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2)(d_{\text{gap}0}^2 - 2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2)}
\]

\[
= \frac{k(d_{\text{gap}0}^2 - 2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2)}{d_{\text{gap}0}^4 - (2d_{\text{gap}0}\Delta d_{\text{gap}} + \Delta d_{\text{gap}}^2)^2}
\]

(2.2.3)

Since \(\Delta d_{\text{gap}}\) is very small compared with \(d_{\text{gap}}\), the second order term can be neglected. Then

\[
f_m = \frac{k}{d_{\text{gap}0}^2} - \frac{2k}{d_{\text{gap}0}^3}\Delta d_{\text{gap}} = f_m0 - k_m\Delta d_{\text{gap}} = f_m0 - \Delta f_m
\]

(2.2.4)

If only the minute deviation is considered

\[
\Delta f_m = -k_m\Delta d_{\text{gap}}
\]

(2.2.5)

Usually the spring coefficient at the right side of equation is positive, however, in the magnetic suspension system it is negative (\(k_m>0\)). On the other words, in a magnetic suspension system, the
suspended object is supported by a negative spring constant.

2.2.2 Theoretical analysis

2.2.2.1 Model analysis

From Fig.2.2 the motion equations of the suspended object and the permanent magnet can be obtained respectively. The motion equation of the suspended object is

\[ m_0 \Delta \ddot{z}_0 = f_m - m_0 g \]  
(2.2.6)

And the motion equation of the permanent magnet is

\[ m_1 \Delta \ddot{z}_1 = -\xi_1 \Delta \dot{z}_1 - k_1 \Delta z_1 - f_m + f_a - m_1 g \]  
(2.2.7)

Moreover,

\[ \Delta d_{gap} = \Delta z_1 - \Delta z_0 \]  
(2.2.8)

Linearization is performed near the equilibrium point and a linear model is obtained. Equations (2.2.6) and (2.2.7) are linearized to be the equations (2.2.9) and (2.2.10) respectively. And then the linear control law can be applied in the analysis of suspension model.

\[ m_0 \ddot{z}_0 = k_m (\Delta z_0 - \Delta z_1) \]  
(2.2.9)

\[ m_1 \ddot{z}_1 = -\xi_1 \dot{z}_1 - k_1 \Delta z_1 - k_m (\Delta z_1 - \Delta z_0) + f_a \]  
(2.2.10)

The state variable of this suspension system is

\[ x = (\Delta z_0 \quad \Delta \dot{z}_0 \quad \Delta z_1 \quad \Delta \dot{z}_1)' \]  
(2.2.11)

The force \( f_a \) which acts on the permanent magnet is the input \( u \) of system and the position of the suspended object \( z_0 \) is the output \( y \) of the system. The state space equation is

\[ \dot{x} = Ax + bu \]  
(2.2.12)

\[ y = cx \]  
(2.2.13)

Where

\[ b = (0 \quad 0 \quad 0 \quad 1/m_1)' \]
By means of the linear control theory, the controllability and observability can be determined. In this system, the controllability $C$ and the observability $O$ and their determinant are

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_m}{m_0} & 0 & -\frac{k_m}{m_0} & 0 \\ 0 & 0 & 0 & m_0 \\ -\frac{k_m}{m_1} & 0 & \frac{k_m}{m_1} & -\frac{1}{m_1} \end{bmatrix}$$

$$c = (1 \ 0 \ 0 \ 0)$$

$$C = \begin{bmatrix} b & Ab & A^2b & A^3b \end{bmatrix}$$

(2.2.14)

$$O = \begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' \end{bmatrix}$$

(2.2.15)

$$\det(C) = -\frac{k_m^2}{m_0 m_1^2}$$

(2.2.16)

$$\det(O) = -\frac{k_m^2}{m_0^2}$$

(2.2.17)

It is verified that this suspension system can be controlled and observed [26].

### 2.2.2.2 Calculation of feedback gain

In terms of linear control theory, due to that the system is controlled and observed, the LQR (linear quadratic regulator) full state feedback control law can be used to design the controller of this system.
The control system block diagram is shown in Fig.2.3. In this diagram:

- $r_{ef}$ the reference input of system
- $u$ the input of system
- $z_0$, $z_1$ the iron ball’s position and the permanent magnet’s position respectively.

From this diagram and motion equations we can see that the iron ball’s position can be controlled via control the permanent magnet position. The reference signal and $z_0$ is added to control $z_1$ through the controller.

The cost function is:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left[ x'Qx + u'Ru \right] dt$$

In this equation, $Q$ is the state weighting matrix and $R$ is the input weighting matrix.

$$K = R^{-1}b^TP$$

$P$ is the solution of Riccati Equation. After choose the states weighting matrix $Q$ and input weighting matrix $R$ and solve the Riccati Equation, the feedback gain of system $K$ can be gotten [25].

Choosing weighting matrixes $Q$ and $R$ appropriately, the feedback gain $K$ can be obtained. Using LQR control law to design the controller ensures the stability of close loop of system. So this system must be stable under such feedback gain [27].

### 2.2.3 Examination for permanent magnet suspension system

#### 2.2.3.1 Prototype of suspension system

Based on the principle of suspension system, a prototype is constructed. It is shown in Fig.2.4. It is a prototype of 2-DOF permanent maglev system in which we only consider the vertical direction, that is a suspension mechanism. In this mechanism, the suspended object is an iron ball which has a diameter of 0.025(m) and a weight of 0.063(kg), i.e. $m_0 = 0.063(kg)$. The permanent magnets have cylindrical shape with a diameter of 0.008(m) and a length of 0.01(m). The moving part with the magnet has a weight of 0.375(kg), i.e. $m_1 = 0.375(kg)$. The actuator, VCM (voice coil motor), has a stoke length of 0.015(m) and a propulsive force of 10(N) at a coil current of 2(A). The gap sensor senses the permanent magnet position. The permanent magnet is driven by the VCM and moved up and down to adjust the air gap between the permanent magnet and the iron ball, i.e. adjust the attractive force of the iron ball to balance the gravity of the iron ball.
2.2.3.2 Measurement of the permanent magnet attractive force

The value of the attractive force between the permanent magnet and the iron ball object was measured by the manufactured measurement equipment shown in Fig 2.5. A Lode Cell whose measurement range is 5 kg is used for measurement. The measurement method is that the iron ball is attached on the Load Cell and through moving the permanent magnet to adjust the air gap between the iron ball and the permanent magnet to measure the attractive force under different air gap. In that case, the interval is 0.0005(m) in the range from 0.0005(m) to 0.015(m). The results is shown in the Fig.2.6. From the equation (2.2.1), we know that the attractive force between the permanent magnet and the iron ball is inverse proportion to the square of the air gap length. Therefore, from the relationship curve of the attractive force and the air gap shown in Fig.2.6, the constant of the permanent magnet attractive force can be known, $k \approx 1.291 \times 10^{-5} \text{ (N·m²)}$. Moreover, it turned out that the magnet used for an experiment can take out sufficient support force to support the iron ball during the suspension time.
Fig. 2.5 Photograph of experimental set up

permanent magnet

iron ball

load cell

Fig. 2.6 Relationship between attractive force and air gap
2.2.3.3 Numerical simulation examination

The system configuration is shown by Fig.2.7. The gap sensor sensing the permanent magnet movement, which is located behind the actuator, is an eddy current type. It has a sensing range of 0.01(m) and a resolution of 10(µm). The movement of the iron ball is sensed by 2 axes photo sensor and has a sensing range of 0.02(m) and a resolution of 5(µm). Controller is a digital DSP controller with 12 bit resolution A/D converters and 16 bits D/A converter. Through A/D converters, sensors’ signals are converted to digital value and input to the DSP Controller. Controller computes the current for VCM and through D/A converter the currents are converted to analog value and input to the current amplifier to control the movement of the permanent magnet. Applied the method described in section 2.2.2.2, the feedback gain $K$ is obtained.

$$K = [-853.4210 \quad -31.8049 \quad 406.2074 \quad 1.5533]$$

Applied the feedback gain $K$ in the control system, the numerical simulation of step response is obtained. Fig.2.8 shows the numerical simulation examination of step response of the linear model, and Fig.2.9 shows the numerical simulation examination of step response of the nonlinear model. The step value is 0.001(m). It is added at the time at 1 second. After the step disturbance is added, the results had been recorded for 1 second about the iron ball position and the permanent magnet position. In these two figures, the positive direction is the upward motion direction of the permanent magnet and the iron ball. When the step signal is added, the permanent magnet is moved downward and the air gap length between the permanent magnet and the iron ball becomes small so the
The attractive force becomes large to attract the iron ball move upward. And then the permanent magnet is moved upward also, i.e. they are converged. After the transient time the permanent magnet and the iron ball is balanced at the new position. The step disturbance is eliminated.

The permanent magnet position is performing an operation for stabilization, after moving in the opposite direction. Moreover, compared the transient time of the response showed in Fig.2.8 and Fig.2.9, it can be checked that nonlinearity is in a system. Compared with these two figures, we can see that the transient time of the Fig.2.8 is shorter than that of the Fig.2.9 slightly, i.e. the convergence time is late in the Fig.2.9. That is the difference between the linear model and nonlinear model in the step response.

Furthermore, when the step value is increased, which the simulation results are shown in Fig.2.10 and Fig.2.11, in the linear model, the response is stable, but in the nonlinear model, the system becomes unstable. It turns out that the air gap of a permanent magnet and a suspended object disappeared, and the attractive force will become infinite. Therefore, the data is no meaning. From these results we know that when a big step disturbance is applied the nonlinear control technique have to be considered.

The simulation parameter is shown in table.2.1.

<table>
<thead>
<tr>
<th>Table 2.1 values and symbols of 2-DOF permanent maglev system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball</td>
</tr>
<tr>
<td>Mass of suspension magnet</td>
</tr>
<tr>
<td>VCM spring constant</td>
</tr>
<tr>
<td>VCM damping constant</td>
</tr>
<tr>
<td>Magnet field constant of left magnet</td>
</tr>
<tr>
<td>VCM force constant</td>
</tr>
</tbody>
</table>
Fig. 2.8 Numerical simulation of step response of linear model (step value is $0.001$ m)

Fig. 2.9 Numerical simulation of step response of the nonlinear model (step value is $0.001$ m)
Fig. 2.10 step response of linear model (step value is 0.12(m))

Fig. 2.11 step response of nonlinear model (step value is 0.12(m))
2.2.3.4 Experimental examination

Applied the controller in the experimental mechanism, the suspension is succeeded. Fig.2.12 is the photograph of the permanent magnet and the iron ball during the suspension. The step response and impulse response of the experimental examination are shown in the Fig.2.13 and Fig.2.14 respectively.

![Fig2.12 Photograph during suspension](image)

![Fig2.13 Experimental examination of step response](image)
And we also do the experiment when the mass of iron ball is varied under the same controller, the iron ball can be suspended.

<table>
<thead>
<tr>
<th>Table 2.2 mass of different iron ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball 1</td>
</tr>
<tr>
<td>Mass of the iron ball 2</td>
</tr>
<tr>
<td>Mass of the iron ball 3</td>
</tr>
<tr>
<td>Mass of the iron ball 4</td>
</tr>
<tr>
<td>Mass of the iron ball 5</td>
</tr>
</tbody>
</table>

2.3 Discussion

In this system, the suspension is success. It is controlled by means of adjusting the air gap between the permanent magnet and the suspended object. It can be said that this mechanism in which the main components of the attractive force of a permanent magnet is instable is stabled by means of mechanic control. For this reason, the feature of this system is no heat generation compared with the suspension mechanism using an electromagnet. The model was made about this mechanism and the feasibility of this suspension system was considered from a viewpoint of linear control theory. Modeling was performed and it is checked that the model can be controlled and observed. The numerical simulation and experimental examination is performed. The results verified that the proposed suspension system can be realized.
Chapter 3

Magnetic suspension mechanism with the horizontal motion control

This chapter will describe a magnetic suspension mechanism in which the suspended object is manipulated in the vertical plane. It is a 2-DOF magnetic suspension mechanism and is made based on the suspension system described in the chapter 2.

In this chapter, the motion principle is explained and the arrangement of magnetic polarity is analyzed with FEM method. A simple motion model and an accuracy motion model are made respectively. Based on the motion model the controller of control system is obtained with LQR optimal control method. The simulation and experimental examination results verifies that the model is correct.

Fig.3.1 Outline of suspension mechanism
3.1 Principle of 2-DOF maglev system

The 1-DOF magnetic suspension mechanism principle showed with Fig.3.1 can be simplified to be that shown with Fig.3.2. In Fig.3.2, the actuator showed in Fig.3.1 is omitted, i.e. a permanent magnet is used to represent the permanent magnet and the actuator. The different color means the different polarity of a permanent magnet. We assumed that the red color represents the S polarity of a permanent magnet and the black color is N polarity.

The principle of 1-DOF permanent maglev system in which the suspended object can be moved in the vertical direction has been introduced in chapter 2. Here, we want to move the suspended object in the horizontal direction when it is suspended. That is the suspended object can be moved in two directions, vertical and horizontal, so this system is a 2-DOF permanent maglev system. For this reason, three permanent magnets are used. One is used to suspend the object, and the other two magnets are used to move the object moving in horizontal direction. The outline of 2-DOF permanent maglev mechanism is shown in Fig.3.3. Due to that the suspended object is manipulated in the horizontal direction the attractive force of suspension magnet can be neglected. The system shown in Fig.3.3 can be simplified to be that shown with Fig.3.4

Fig.3.2 Simplified Outline of suspension mechanism
In Fig.3.4, $d_1$ is the air gap between the left magnet and the iron ball and $d_2$ is the air gap between the right magnet and the iron ball. The attractive force of left magnet is $f_{m1}$ and that of right magnet is $f_{m2}$. During the suspension, through adjust the air gap $d_1$ and $d_2$ make the suspended object move in the horizontal direction. The concrete method is to adjusting the reluctance in a magnetic circuit of the suspension mechanism to control the position of the suspended object. It has been described in chapter 1.
3.2 FEM analysis and pole location

A magnet has two poles, N and S and different poles arrangement will have different flux distribution. From Fig.3.3 we know that there three permanent magnets in this system, so it is required to decide magnetic polarity of three permanent magnets which face to the iron ball for stabilization suspension. In the 2-DOF permanent maglev system the total number of polarity combination is 6, SSS, SSN, SNN, NNN, NNS and NSS. The poles order of combination is of suspension magnet, left magnet and right magnet. For instance, if the surface of the iron ball counts S pole at the vertical direction and both N poles at horizontal direction it is called SNN arrangement, i.e. SNN expresses that the polarity of the magnet which faces to the iron ball is that the suspension magnetic pole is S, and left magnetic pole and right magnetic pole are N. From the performance of magnets we know that the former three combinations and later three combinations have the same flux distribution, so we only choose the former three combinations as the object of the analysis.

The magnetic field under different magnetic pole arrangement is studied with FEM (finite elements method). In this research, it is three dimensions and two dimensions analysis to the magnetic field in the suspended object circumference. For SSS arrangement, SNN arrangement and SSN arrangement, the Magnetic field analysis results are shown with the figures from Fig.3.5 to Fig.3.10.

Fig.3.5, Fig.3.6 and Fig.3.7 show the three dimensions magnetic field analysis results about the pole arrangement of SSS, SSN and SNN respectively. In every figure, the upper part shows the magnetism of permanent magnet acting on the suspended object, and the lower part shows the magnetism of the iron ball which is magnetized by the permanent magnet. Different color expresses different strength of the magnetism. The strongest magnetism is expressed with the red color. In the figure of a suspended object, a contour line is used to express magnetism.

Furthermore, Fig.3.8, Fig.3.9 and Fig.3.10 show the two dimensions magnetic field analysis results about the pole arrangement of SSS, SSN and SNN respectively. In every figure, the upper part shows the magnetism and the lower part shows the flux linkage which is expressed with the line, the white part is a magnet, and the round shape near a center is the suspended object.

In these figures, the air gap length between the magnet and iron ball are all 0.005(m) as an initial value. The suspended object was arbitrarily moved within ± 0.003(m) in horizontal direction. Magnetic field analysis was performed. From the analysis results, we can see that near the center of a suspended object either the passing flux oppose or suit for suspension.
Fig. 3.5 Result of magnetic force 3D analysis (SSS)
Fig. 3.6 Result of magnetic force 3-D analysis (SSN)
Fig. 3.7 Result of magnetic force 3D analysis (SNN)
Fig. 3.8 Result of magnetic force and flux diagram 2-D analysis (SSS)
Fig. 3.9 Result of magnetic force and flux diagram 2-Danalysis (SSN)
Fig. 3.10 Result of magnetic force and flux diagram 2-D analysis (SNN)
Fig. 3.11 Result of magnetic field horizontal analysis

Fig. 3.12 Result of magnetic field vertical analysis
Fig.3.11 shows the analysis results of magnetism in horizontal direction which is composed of virtual work and Maxwell stress tensor under different arrangement, whilst, Fig.3.12 shows the analysis results of magnetism in vertical direction.

From a horizontal analysis result, we can see that the arrangement SNN and SSS have almost the same results which are symmetric with the original point, but the results of arrangement SSN is different which is not symmetric with the original point. It turns out that leftward force is acting on the suspended object near the original point in the SSN arrangement.

From the analysis result of the vertical direction, we can see that the difference is remarkable. The magnetism can be checked near the initial value. Furthermore, the magnitude order of the magnetism acted on the suspended object is SNN arrangement, SSN arrangement and SSS arrangement. This is considered to be the difference of magnetic circuit in each magneto arrangement.

Moreover, for explain the choice of the pole arrangement and obtain the relationship between the vertical force and displacement and the relationship between the horizontal force and displacement, the magnetic field analysis is performed. The results are shown from Fig.3.13 to Fig.3.16.

Fig.3.13 shows the magnetic flux distribution of the opposite polarity. Fig.3.14 shows the magnetic flux of the same polarity. The relationship between the vertical force and displacement of the suspended object is shown in Fig.3.15, and the relationship between the horizontal force and displacement of the suspended object is shown in Fig.3.16.

Consequently, the arrangement SNN (or NSS) is chosen in this research, because in this arrangement the vertical force is largest and the horizontal force is symmetric. That means using this arrangement the most weighting suspended object can be suspended and it is easy to control the movement of the suspended object in the horizontal direction.
Fig. 3.13 magnetic flux diagram (opposite poles face to object)

Fig. 3.14 magnetic flux diagram (same poles face to object)
Fig. 3.15 Result of magnetic field for vertical analysis

Fig. 3.16 Result of magnetic field for horizontal analysis
3.3 Prototype of 2-DOF permanent maglev system

According to the principle of the 2-DOF permanent magnet suspension mechanism and the polarity arrangement, a prototype of this system is set up. It is shown in Fig.3.17. In this mechanism, the suspended object is an iron ball which has a diameter of 0.025(m) and a weight of 0.063(kg), i.e. $m_0=0.063$(kg). The permanent magnets have cylindrical shape with a diameter of 0.008(m) and a length of 0.01(m). The moving part with the magnet has a weight of 0.375(kg), i.e. $m_1=0.375$(kg). The actuator, VCM (voice coil motor), has a stoke length of 0.015(m) and a propulsive force of 10(N) at a coil current of 2(A). The gap sensor senses the permanent magnet position. The permanent magnet is driven by the VCM and moved up and down to adjust the air gap between the permanent magnet and the iron ball, i.e. adjust the attractive force of the iron ball to balance the gravity of the iron ball.

From the picture we can see that there are three voice coil actuators which drive the three permanent magnets respectively. Motion of the magnet is sensed by eddy current sensor. An iron ball is the suspended object. When the iron ball is suspended, through control the motion of two horizontal magnets we can make the iron ball move along horizontal direction. So there are two motion directions in this system, vertical motion and horizontal motion, i.e. this system is a 2-DOF permanent magnetic suspension system.

![Prototype of permanent maglev system](image)
3.4 Model analysis

From the prototype and principle of 2-DOF permanent maglev system, the motion model of iron ball and permanent magnets were created. At first, a simple model in which the influence of the attractive force of the suspension magnet was created. For verify the correction of simple model, an accuracy model was created. They are introduced as following.

3.4.1 Simple model of 2-DOF permanent maglev system

3.4.1.1 Description of simple model

From Fig.3-17 we can see that the motions of the iron ball and the magnets divide into two directions, vertical and horizontal, movement. It is consumed that two motions are independent of each other. The analysis of vertical motion has been already investigated in chapter 2. Here, the horizontal motion, which involves the motions of an iron ball and two permanent magnets driven by actuators, are mainly investigated. Horizontal motion model is shown in Fig.3.

As can be seen, the symbols used in the following study are:

- \( z_0 \): displacement of the iron ball
- \( z_1 \): displacements of the left permanent magnet
- \( z_2 \): displacements of the right permanent magnet
- \( m_0 \): mass of the iron ball
- \( m_1 \): mass of the left magnet
- \( m_2 \): mass of the right magnet
- \( f_{a1} \): generating forces of the left actuator
- \( f_{a2} \): generating forces of the right actuator

Fig.3.18 Simple model of horizontal motion
\(f_{m1}\): attractive force between the ball and left magnet
\(f_{m2}\): attractive force between the ball and right magnet
\(d_{01}\): the left side initial air gap length between the iron ball and left magnet
\(d_{02}\): the right side initial air gap length between the iron ball and right magnet
\(d_1\): the left side air gap length between the iron ball and left magnet
\(d_2\): the right side air gap length between the iron ball and right magnet
\(k_{m1}\): the coefficient of the left magnetic field
\(k_{m2}\): the coefficient of the right magnetic field
\(k_{m1}^\prime\): the linear coefficient of the left magnetic field
\(k_{m2}^\prime\): the linear coefficient of the right magnetic field
\(k_1\): the spring coefficient of the left actuator
\(k_2\): the spring coefficient of the right actuator
\(\xi_1\): the damping coefficient of the left actuator
\(\xi_2\): the damping coefficient of the right actuator

The displacements and forces are considered positive when they act in the right side direction. And it is consumed that the friction of the guide of the actuator and viscosity of the air are negligible. The equations of the motion of the suspended object and the magnets are shown with equations (1), (2) and (3).

\[
m_{0}\ddot{z}_0 = f_{m2} - f_{m1} \tag{3.4.1}
\]

\[
m_1\ddot{z}_1 = -\xi_1\dot{z}_1 - k_1z_1 + f_{m1} + f_{a1} \tag{3.4.2}
\]

\[
m_2\ddot{z}_2 = -\xi_2\dot{z}_2 - k_2z_2 - f_{m2} + f_{a2} \tag{3.4.3}
\]

Where \(f_{m1} = \frac{k_{m1}}{d_1^2}\) and \(f_{m2} = \frac{k_{m2}}{d_2^2}\). Due to the relationship between magnetic force and the air gap is nonlinear, the equations (3.1), (3.2) and (3.3) are nonlinear equations. The system is nonlinear system. We will use the linear control theory to analyze the system, so the nonlinear equations must be linearized.

\[
f_{m1} = \frac{k_{m1}}{d_1^2} = \frac{k_{m1}}{(d_{01} + z_0 - z_1)^2} \tag{3.4.4}
\]

\[
f_{m2} = \frac{k_{m2}}{d_2^2} = \frac{k_{m2}}{(d_{01} + z_2 - z_0)^2} \tag{3.4.5}
\]

\[
\therefore \Delta z_1 = z_0 - z_1 \quad \Delta z_2 = z_2 - z_0 \tag{3.4.6}
\]
\[
\therefore f_{m1} = \frac{k_{m1}}{(d_{01} + \Delta z_1)^2}
\]

(3.4.7)

\[
f_{m2} = \frac{k_{m2}}{(d_{01} + \Delta z_2)^2}
\]

(3.4.8)

### 3.4.1.2 Linearization of the motion model

According to the linearization principle [25], equation (3.7) can be linearized to be

\[
f_{m1} = \frac{k_{m1}}{(d_{01} + \Delta z_1)^2}
\]

\[
= f_{m1}\left|\frac{d_i = d_{01} + \Delta z_1}{\Delta z_1 = 0} \frac{\partial f_{m1}}{\partial \Delta z_1} \right|_n = 0
\]

\[
= \frac{k_{m1}}{d_{01}^2} - \frac{2k_{m1}}{d_{01}^3} \Delta z_1
\]

\[
= f_{m01} + k_{m1} (z_0 - z_1)
\]

(3.4.9)

And the equation (3.8) can be linearized to be

\[
f_{m2} = \frac{k_{m2}}{(d_{01} + \Delta z_2)^2}
\]

\[
= f_{m2}\left|\frac{d_i = d_{02} + \Delta z_2}{\Delta z_2 = 0} \frac{\partial f_{m2}}{\partial \Delta z_2} \right|_n = 0
\]

\[
= \frac{k_{m2}}{d_{02}^2} - \frac{2k_{m2}}{d_{02}^3} \Delta z_2
\]

\[
= f_{m02} + k_{m2} (z_2 - z_0)
\]

(3.4.10)

So the motion equations (3.1), (3.2) and (3.3) are changed to be

\[
m_0 \ddot{x}_0 = f_{m02} - f_{m01} + k_{m2} (z_2 - z_0) - k_{m1} (z_0 - z_1)
\]

(3.4.11)

\[
m_1 \ddot{z}_1 = -k_1 \dot{z}_1 - k_{m1} (z_0 - z_1) + f_u
\]

(3.4.12)
\[ m_2 \ddot{z}_2 = -\xi_2 \dot{z}_2 - k_2 z_2 - f_{m2} - k_{m2} (z_2 - z_0) + f_a \]  
(3.4.13)

\[ \therefore f_{m01} \quad \text{and} \quad f_{m02} \quad \text{are constants, and} \]

\[ f_{m02} - f_{m01} << k \quad m_2 (z_2 - z_0) - k_{m1} (z_0 - z_1) \]

\[ \therefore m_0 \ddot{z}_0 = k \quad m_2 (z_2 - z_0) - k_{m1} (z_0 - z_1) \]  
(3.4.14)

\[ m_1 \ddot{z}_1 = -\xi_1 \dot{z}_1 - k_1 z_1 + k \quad m_1 (z_0 - z_1) + f_{a1} \]  
(3.4.15)

\[ m_2 \ddot{z}_2 = -\xi_2 \dot{z}_2 - k_2 z_2 - k_{m2} (z_2 - z_0) + f_{a2} \]  
(3.4.16)

The equations (3.14), (3.15) and (3.16) are linearized motion equations. After linearizing, the horizontal motions can be expressed by the state space model. It is shown with equations (3.17) and (3.18).

\[ \dot{x} = Ax + bu \]  
(3.4.17)

\[ y = cx \]  
(3.4.18)

Where \( x \) is the state variable, \( y \) is the output, \( A \) is the state matrix, \( b \) is the input matrix and \( c \) is the output matrix. Applied the linear control law, we have to check the controllability and observability of this system. Considering the input of this system there are three situations [26].

### 3.4.1.3 Three situations according the input and output of system

In terms of the linear control law, the controllability matrix \( C \) and observability matrix \( O \) are expressed with equations (3.19) and (3.20) respectively.

\[ C = \begin{bmatrix} b & Ab & A^2b & A^3b & A^4b & A^5b \end{bmatrix} \]  
(3.4.19)

\[ O = \begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' & (A')^4c' & (A')^5c' \end{bmatrix} \]  
(3.4.20)

According to the rank of the controllability matrix and observability matrix we can know the controllability and observability of this system under different situation. [26].

**Theorem 1:** if and only if the rank of controllability matrix is equal to the number of state variable, the system is controllable.

**Theorem 2:** if and only if the rank of observability matrix is equal to the number of state variable,
the system is observable.

When the controllability matrix is square matrix, there are another two theorems to check the controllability and observability of the system.

**Theorem 3:** if and only if the determinant of controllability matrix is not equal to 0, the system is controllable.

**Theorem 4:** if and only if the determinant of observability matrix is not equal to 0, the system is observable.

### 3.4.1.3.1 Two forces input system

In the model shown in Fig.3.20, the two magnets are driven by means of different VCM and the current of VCM is fed with different amplifier. That is the motion of two magnets is independent. The propulsion force $f_{a1}$ and $f_{a2}$ are the system inputs, so this system is called **Two Forces Input System**.

When the output of this system is $y=z_0$, in the state space model, where

$$x = \begin{bmatrix} \dot{z}_0 & z_0 & \dot{z}_1 & z_1 & \dot{z}_2 & z_2 \end{bmatrix}^T,$$

$$b = \begin{bmatrix} 0 & 0 & 1/m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/m_2 & 0 \end{bmatrix}^T.$$
In this model, the controllability matrix is \( C_1 \) and the observability matrix is \( O_1 \). The Rank of \( C_1 \) is 6, and the Rank of \( O_1 \) is 4, so this model is controllable but observable.

However, when the output \( y = [z_0 \ z_1 \ z_2]' \), the output matrix becomes

\[
c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

At this time, the controllability matrix is \( C_2 \), and the observability matrix is \( O_2 \). The Rank of \( C_2 \) and \( O_2 \) are all equal to 6, so the system is controllable and observable.
3.4.1.3.2 Same force input system

In the model shown with Fig.3.21, the magnets are driven by two VCM respectively, but the current of VCM is fed only with one amplifier. That means the same force is generated with two VCM, i.e. \( f_{a1} = f_{a2} = f_a \). We call this system **Same Force Input System**. When the output of this system is \( y = z_0 \) in the state space model, where

\[
x = \begin{pmatrix} \dot{z}_0 & z_0 & \dot{z}_1 & z_1 & \dot{z}_2 & z_2 \end{pmatrix}^T
\]

\[
A = \begin{bmatrix}
0 & -\frac{2k_{m1}}{m_0d_{01}^2} & -\frac{2k_{m2}}{m_0d_{02}^2} & 0 & \frac{2k_{m1}m_2}{m_0d_{01}^3} & 0 & \frac{2k_{m2}m_1}{m_0d_{02}^3}
1 & 0 & 0 & 0 & 0 & 0 & 0
0 & \frac{2k_{m1}}{m_1d_{11}^2} & -\frac{\xi}{m_1} & -\frac{2k_{m2}}{m_1d_{12}^2} & \frac{k_1}{m_1} & 0 & 0
0 & 0 & 1 & 0 & 0 & 0 & 0
0 & \frac{2k_{m1}}{m_2d_{21}^2} & 0 & 0 & \frac{\xi}{m_2} & \frac{2k_{m2}m_1}{m_2d_{22}^2} & -\frac{k_2}{m_2}
0 & 0 & 0 & 0 & 0 & 1 & 0
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
b = (0 \ 0 \ 1/m_1 \ 0 \ 1/m_2 \ 0)^T
\]

\[
c = (0 \ 1 \ 0 \ 0 \ 0 \ 0)
\]

\[u = f_a\]

In this model, the controllability matrix is \( C_3 \), and the observability matrix is \( O_3 \). The Rank of \( C_3 \) and \( O_3 \) are all equal to 4 this system is uncontrollable and unobservable.

However, when the output \( y = [z_0 \ z_1 \ z_2]^T \), the output matrix becomes

\[
c = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

The controllability matrix is \( C_4 \), and the observability matrix is \( O_4 \). The Rank of \( C_4 \) and \( O_4 \) are all equal to 6. But when \( m_1 = m_2 \), \( k_{m1} = k_{m2} \) and \( d_{01} = d_{02} \), the rank of \( C_4 \) is 4 and the rank of \( O_4 \) is 6. Therefore, this system is uncontrollable but observable.
3.4.1.3.3 Two magnets connecting system

In the model shown with Fig.3.22, the two magnets are dependent, i.e. they are connected together and driven by means of one VCM. Such a system is called Two Magnets Connecting System. In this system $m=m_1=m_2$, $z_1=z_2$. When the output of this system is $y=z_0$, in the state space model, where

$$x = \begin{pmatrix} \dot{z}_0 & z_0 & \dot{z}_1 & z_1 & \dot{z}_2 & z_2 \end{pmatrix}'$$

$$A = \begin{bmatrix}
0 & -\frac{2k_{m1}}{m_0d_{01}} & -\frac{2k_{m2}}{m_0d_{02}} & 0 & \frac{2k_{m1}}{m_0d_{01}} & 0 & \frac{2k_{m2}}{m_0d_{02}} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{2k_{m1}}{md_{01}} & -\frac{\xi}{m} & -\frac{2k_{m1}}{md_{01}} & -k & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{2k_{m2}}{md_{02}} & 0 & 0 & -\frac{\xi}{m} & -\frac{2k_{m2}}{md_{02}} & -k \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}$$

$$b = \begin{pmatrix} 0 & 0 & 1/m_1 & 0 & 1/m_2 & 0 \end{pmatrix}'$$

$$c = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$u = f_a$$

In this case, the determinant of the controllability matrix $C_5$ and observability matrix $O_5$ are all equal to 0, so the system is uncontrollable and unobservable.
However, when the output $y=[z_0 \ z_1 \ z_2]'$, the output matrix becomes

$$c = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The rank of controllability matrix $C_6$ is 4, and the rank of observability matrix $O_6$ is 6. This system is uncontrollable but observable.

### 3.4.1.4 Determination the control plant of 2-DOF maglev system

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
<th>controllability</th>
<th>observability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two forces input sytem</td>
<td>One output</td>
<td>controllable</td>
<td>unobservable</td>
</tr>
<tr>
<td></td>
<td>Three output</td>
<td>controllable</td>
<td>observable</td>
</tr>
<tr>
<td>Same forces input system</td>
<td>One output</td>
<td>uncontrollable</td>
<td>unobservable</td>
</tr>
<tr>
<td></td>
<td>Three output</td>
<td>uncontrollable</td>
<td>observable</td>
</tr>
<tr>
<td>Two magnets connecting system</td>
<td>One output</td>
<td>uncontrollable</td>
<td>unobservable</td>
</tr>
<tr>
<td></td>
<td>Three output</td>
<td>uncontrollable</td>
<td>observable</td>
</tr>
</tbody>
</table>

Table 3.1 shows the controllability and observability of the 2-DOF permanent maglev system under different input and output situations. According the linear control theory we have to choose one situation of them as a control plant which must be controllable and observable. From the table 3.1, we can see that only one situation, two forces input and three outputs, is controllable and observable. Thus, this situation is chosen as the plant in the research.
Fig. 3.23 Iron ball’s initial position and final position free body diagram

Fig. 3.24 Accuracy model of horizontal motion
3.4.2 Accuracy model of 2-DOF permanent maglev system

In the last section, a simple model in which the influence of suspension attractive force is neglected is introduced. For examining this influence, an accuracy model is created. A free body diagram of the iron ball is drawn when the iron ball is at the center position and is deviated to the right (it is symmetric with the free body diagram when the iron ball is moved to the left). It is shown in Fig.3.23. The upper part of this figure is the iron ball free body diagram at the center position, initial position. The lower part is the iron ball free body diagram when the iron ball is moved to the right direction.

3.4.2.1 Description of accuracy model

In the Fig.3.23 and Fig.3.24

- $z_0$: displacement of the iron ball
- $z_1$: displacements of the left permanent magnet
- $z_2$: displacements of the right permanent magnet
- $m_0$: mass of the iron ball
- $m_1$: mass of the left magnet
- $m_2$: mass of the right magnet
- $f_{a1}$: generating forces of the left actuator
- $f_{a2}$: generating forces of the right actuator
- $f_{m1}$: attractive force between the ball and left magnet
- $f_{m2}$: attractive force between the ball and right magnet
- $d_{01}$: the left side initial air gap length between the iron ball and left magnet
- $d_{02}$: the right side initial air gap length between the iron ball and right magnet
- $k_{m1}$: the coefficient of the left magnetic field
- $k_{m2}$: the coefficient of the right magnetic field
- $k_{m1}'$: the linear coefficient of the left magnetic field
- $k_{m2}'$: the linear coefficient of the right magnetic field
- $k_1$: the spring coefficient of the left actuator
- $k_2$: the spring coefficient of the right actuator
- $\xi_1$: the damping coefficient of the left actuator
- $\xi_2$: the damping coefficient of the right actuator
- $f_m$: the attractive force of suspension magnet.
- $f_{mv}$: the vertical component of $f_m$.
- $f_{mh}$: the horizontal component of $f_m$.
- $d_0$: the initial air gap between suspension magnet and iron ball.
α: the angle between $f_m$ and $f_{mv}$.

$m_{0g}$: the gravity force of iron ball

$d_{01}$: left air gap of the final position between the iron ball and the left magnet

$d_{02}$: right air gap of the final position between the iron ball and the right magnet

\[
tg \alpha = \frac{f_{mb}}{f_{mv}} \approx \frac{z_0}{d_0} \quad (3.4.21)
\]

\[
f_{mb} = -\frac{m_{0g}}{d_0} z_0 = \frac{k_m}{d_{02}^2} - \frac{k_m}{d_{01}^2} \quad (3.4.22)
\]

The motion equations of the maglev system are written as equation (3.23), (3.24) and (3.25).

\[
m_0 \ddot{z}_0 = f_{m2} - f_{m1} - f_{mh} \quad (3.4.23)
\]

\[
m_1 \ddot{z}_1 = -\xi_1 \ddot{z}_1 - k_1 z_1 + f_{m1} + f_{a1} \quad (3.4.24)
\]

\[
m_2 \ddot{z}_2 = -\xi_2 \ddot{z}_2 - k_2 z_2 - f_{m2} + f_{a2} \quad (3.4.25)
\]

### 3.4.2.2 Linearization of the motion model

As described in the last section, the equation (3.23), (3.24) and (3.25) are nonlinear equations. We have to linearize them. The linearized motion equation are shown with equation (3.26), (3.27) and (3.28).

\[
m_0 \ddot{z}_0 = k_{m2} (z_2 - z_0) - k_{m1} (z_0 - z_1) - \frac{m_{0g}}{d_0} z_0 \quad (3.4.26)
\]

\[
m_1 \ddot{z}_1 = -\xi_1 \ddot{z}_1 - k_1 z_1 + k_{m1} (z_0 - z_1) + f_{a1} \quad (3.4.27)
\]

\[
m_2 \ddot{z}_2 = -\xi_2 \ddot{z}_2 - k_2 z_2 - k_{m2} (z_2 - z_0) + f_{a2} \quad (3.4.28)
\]

### 3.4.2.3 Controllability and observability

After linearized the motion equations, the accuracy model can be expressed with state space model shown as equations (3.17) and (3.18). Where

\[
x = (\dot{z}_0 \quad z_0 \quad \dot{z}_1 \quad z_1 \quad \dot{z}_2 \quad z_2)'
\]
In the accuracy model, we only consider controllability and observability under the two forces input and three outputs situation. In this case, the rank of controllability matrix $C_7$ and observability matrix $O_7$ are all equal to 6, so the system can be controlled and observed.

$$\begin{align*}
A_1 &= 
\begin{bmatrix}
0 & -k_{a1} - k_{a2} - g/m_0 & 0 & k_{a1} & 0 & k_{a2} \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & k_{a1}/m_1 & -\xi_1/m_1 & -k_{a1}/m_1 & k_{a2}/m_1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & k_{a2}/m_2 & 0 & 0 & -\xi_2/m_2 & -k_{a2}/m_2 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{align*}$$

$$b = \begin{bmatrix}
0 & 0 & 1/m_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/m_2 & 0
\end{bmatrix}^T$$

$$c = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

$$u = (f_{a1} \ f_{a2})^T$$

In the accuracy model, we only consider controllability and observability under the two forces input and three outputs situation. In this case, the rank of controllability matrix $C_7$ and observability matrix $O_7$ are all equal to 6, so the system can be controlled and observed.
3.5 Controller design

For realize the 2-DOF permanent magnet suspension system, the controller of this system have to be designed. The control system block diagram is shown in Fig.3.25. In this diagram

Ref is the reference input of system

\( u_1, u_2 \) are the inputs of system

\( z_0 \) is the iron ball’s displacement.

\( z_1 \) is the left magnet’s displacement

\( z_2 \) is the right magnet’s displacement

The relationship among the iron ball’s position and the two permanent magnets’ position is complicated. From this diagram and motion equations we can see that the iron ball’s position can be controlled via control the two permanent magnets’ position. Two magnets are coupled that means these two permanent magnets’ position effect each other. The reference signal and \( z_0 \) are divided into two parts to control \( z_1 \) and \( z_2 \) through the controller. The parameter of this system is shown in table.3.2.

<table>
<thead>
<tr>
<th>Table 3.2 values and symbols of 2-DOF permanent maglev system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball</td>
</tr>
<tr>
<td>Mass of the left magnet</td>
</tr>
<tr>
<td>Mass of the right magnet</td>
</tr>
<tr>
<td>Left VCM spring constant</td>
</tr>
<tr>
<td>Right VCM spring constant</td>
</tr>
<tr>
<td>Left VCM damping constant</td>
</tr>
<tr>
<td>Right VCM damping constant</td>
</tr>
<tr>
<td>Magnet field constant of left magnet</td>
</tr>
<tr>
<td>Magnet field constant of right magnet</td>
</tr>
<tr>
<td>VCM force constant</td>
</tr>
</tbody>
</table>

3.5.1 Controller design method

There many methods to design the controller for a linear system. In terms of linear control theory, due to this system is controllable and observable, the LQR (linear quadratic regulator) full state feedback control law can be used to design the controller of this system.

The cost function is

\[
J = \frac{1}{2} \int_0^t \left[ x'Qx + u'Ru \right] dt
\]  \hspace{1cm} (3.5.1)
In this equation, $Q$ is the state weighting matrix and $R$ is the input weighting matrix.

$$K = R^{-1}b^TP$$  \hspace{1cm} (3.5.2)

$P$ is the solution of Riccati Equation. After choose the states weighting matrix $Q$ and input weighting matrix $R$ and solve the Riccati Equation, the feedback gains of system $K$ can be gotten [26].

Using LQR control law to design the controller ensures the stability of close loop of system. So this system must be stable under such feedback gain [27].

### 3.5.2 Choice of weighting matrix

From the Fig.3.20, we know that $z_1$, $z_2$ and $z_0$ are the displacement, therefore, $\dot{z}_1$, $\dot{z}_2$ and $\dot{z}_0$ are the velocity of horizontal magnets and iron ball. Based on the state space equation, we choose the weighting matrix $Q$ and $R$.

$$Q = \text{diag}(1 \ 100000 \ 1 \ 100000 \ 1 \ 100000)$$

$$R = \text{diag}(1 \ 1)$$

Using MATLAB software, we calculate the feedback gain $K$.

$$K = k_o \begin{cases} i = 0, 1 \\ j = 0, 1, 2, 3, 4, 5 \end{cases}$$  \hspace{1cm} (3.5.3)

### 3.5.3 Controller design for simple model

It is assumed that the initial left and right air gap length are 0.008(m). For simple model of 2-DOF permanent maglev system, the feedback gain $K_1$ is

$$K_1 = \begin{bmatrix} 830.3169 & 21.6494 & -384.2543 & -1.5271 & -58.7643 & -0.0669 \\ -830.3169 & -21.6494 & 58.7643 & 0.0669 & 384.2543 & 1.5271 \end{bmatrix}$$

### 3.5.4 Controller design for accuracy model

The initial air gap length is also 0.008(m). For accuracy model of 2-DOF permanent maglev system the feedback gain $K_2$ is

$$K_2 = \begin{bmatrix} 107.0762 & 8.0692 & -349.1031 & -1.4878 & -23.6131 & -0.0276 \\ -107.0762 & -8.0692 & 23.6131 & 0.0276 & 349.1031 & 1.4878 \end{bmatrix}$$
3.6 System examination

Controller is a digital DSP controller with 12 bit resolution A/D converters and 16 bits D/A converter. Through A/D converters, four sensors’ signal are converted to digital value and input to the DSP Controller. Controller computes the current for VCM and through D/A converter the currents are converted to analog value and input to the current amplifier to control the movement of magnets.

Using the feedback gain, which is gotten with the LQR linear control method, the system is examined with numerical simulation and the experimental examination.

3.6.1 System examination for simple model

The numerical simulation and experimental examination have been done for simple model. It includes step response, frequency response and white noise response which is only applied in the numerical simulation.
3.6.1.1 Simulation examination for simple model

The step response, frequency response and the step response in which the white noise is added have been done for the simple model. They are introduced in this section.

3.6.1.1.1 Step response

The initial air gap $d_{01}=d_{02}=0.008$ (m) and the step input of 0.001 (m) is added as the reference signal. The numerical simulation result is shown in Fig.3.27.

![Fig.3.27. Step response of numerical simulation](image)

In this figure, the right movement direction is positive. There is no overshoot, and the system is converged in the 0.1(s). From the figure, we can also see that when the step input is added, two horizontal magnets are moved towards the left. The right air gap becomes smaller and the left air gap becomes larger, i.e. the attractive force of right magnet is larger than that of left magnet. The iron ball is moved to the right, at the same time two horizontal magnets are moved to the right. At last, the iron ball and two magnets are balanced in the new position. At the new position, the left air gap length equals the right air gap length, i.e. the attractive force of the right magnet is equal to that of the left magnet. Thus through control the motion of two permanent magnets, we can control the motion of the iron ball in the horizontal direction.
3.6.1.1.2 Frequency response

The frequency response is shown in the Fig. 3.28. The frequency response shows the gain margin and phase margin of this system. In this system, the gain margin is 5 (dB), and the phase margin is 52 degrees. System has no resonant point.

Fig. 3.28 Frequency response of simulation for simple model
3.6.1.1.3 Step response when the white noise is added

The measurement noise and disturbance inputs are often modeled as white noise when evaluating the control system performance. Thus, a band-limited white noise disturbance is added when the step signal is a reference input. The result is shown in Fig. 3.29. The parameter of white noise is shown in table 3.3.

<table>
<thead>
<tr>
<th>Table 3.3 Parameter of white noise disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise power</td>
</tr>
<tr>
<td>Sample time</td>
</tr>
<tr>
<td>Seed</td>
</tr>
</tbody>
</table>

From Fig. 3.29, we can see that when the disturbance is added, the dynamic characteristic of this system is not influenced. Only the position of the iron ball and two permanent magnets is influenced. It turns out that there are some fluctuations of the position of the iron ball and permanent magnets caused by the white noise. This phenomenon can be controlled by multiplying by the proportionality gain. It is better to perform intensive control so that a gain can be freely set up in the deflection of the suspension object.
3.6.1.2 Experimental examination for simple model

Experimental examination was carried out on the system and the 2-DOF magnetic levitation was succeeded. The Fig.3.30 shows the 2-DOF permanent maglev system during the suspension. The step response and frequency response is introduced.

Fig.3.30 Photo of 2-DOF permanent maglev system during operation

Fig.3.31. Step response of experimental examination for simple model
3.6.1.2.1 Step response

The initial position is that the ball is located in the center of magnets. The air gap between horizontal magnets is set to 0.008(m) through adjust the position of the horizontal magnets. A step input of 0.001(m) was added as the reference signal of the iron ball. The experimental examination of step response is shown in Fig.31. In this figure, the response curves show that after the step input is applied, when the system reach the stable state, the iron ball is moved to the right and two horizontal magnets are moved to the left compared with their initial position. So at the new position, the left and right air gap length is different with that at initial position. It is different with the simulation results. And in the transient process, the overshoot is large compared that with the simulation results.

Besides the different in dynamic characteristic, the final position of the iron ball and two permanent magnets are different. Fig.3.32 shows the motion of iron ball and two permanent magnets in the simulation and experimental examination. In this figure, (a) is about the motion in the simulation of step response, and (b) is about the motion in the experimental of step response. When a positive step input is added, the two magnets are all moved to the left to make the iron ball move to right, and then they are all moved to the right. At the final position, in the simulation examination, the air gap is not changed but changed in the experimental examination.

![Diagram of motion in simulation and experiment](image-url)

(a). motion in the simulation (b). motion in the experiment

Fig.3.32 Motion of iron ball and two magnets in simulation and experimental examination
This phenomenon is mainly caused by the attractive force of suspension magnet. This can be seen from the free body diagram of the iron ball when the iron ball is diverted from the center of the system. It is shown in the Fig.3.33.

From this figure, we can know that when the iron ball is located at the center of the system, the attractive force of the suspension magnet is equal to the gravity of the iron ball. When the iron ball is offset from the center position, the direction attractive force is not opposite to the gravity. There are an angle between the attractive force and the opposite direction of gravity. Therefore, the attractive force will generate two component forces, one is in the opposite direction of gravity and another is in the horizontal direction. If the iron ball is moved to right, the direction of the horizontal component is left, vice versa. Therefore, we create an accuracy model of this system to eliminate this phenomenon. The experimental examination will be introduced in the following section.
3.6.1.2.2 Frequency response

Fig.3.34 shows the frequency response of experimental examination for simple model. From it, the resonant point and bandwidth can be seen. In this system, the resonant point is at $2\pi$ (rad/sec), and the bandwidth is 4 (rad/sec).

![Frequency response of experimental examination for simple model](image)

3.6.2 System examination for accuracy model

The examination results of the simple model shows a difference between the simulation examination and experimental examination. For explain this difference, an accuracy model has been created and analyzed. Hence, the examination for the accuracy model is performed. The examination includes two aspects, simulation examination and experimental examination.

3.6.2.1 Simulation examination for accuracy model

In this section, the step response and frequency response of the accuracy model is introduced.

3.6.2.1.1 Step response

A feedback gain $K_2$ for the accuracy model has been obtained by means of LQR control law in the section 3.5. The simulation examination of the accuracy model is carried out with the feedback gain and $K_2$. The step responses of simulation are shown in Fig.3.35 and Fig.3.36 respectively.
At first, the feedback gain $K_1$, the feedback gain for simple model, was used to check the accuracy model. The step response of simulation examination is shown in Fig.3.35. In this figure, when the iron ball is moved to the right, at the equilibrium position, the two magnets are moved the left compared with that at the initial position. It is the same with the step response of the experimental examination for the simple model. Compared this figure with Fig.3.31, the step response of the experimental examination for simple model, these two figures confirm that the simple model is correct.

Secondly, the feedback gain $K_2$ is applied in the accuracy model. The step response is shown in Fig.3.36. From this figure, we can see that the displacement of the iron ball is larger than the displacement of the magnets. Therefore, the right air gap is slightly smaller than the left one. This is caused by means of the attractive force of the suspension magnet. Moreover, when the iron ball is moved to the right, at the equilibrium position, the two magnets are moved to the right compared with that at the initial position.
Fig. 3.35 Simulation examination of accuracy model with $K_1$

Fig. 3.36 Simulation step response of accuracy model with $K_2$
3.6.2.1.2 Frequency response

The frequency response of the accuracy model is shown in fig.3.37. In this figure, there is a resonant point at 34.6 (rad/sec), and the bandwidth is 8.19 (rad/sec).

Fig.3.37 Frequency response of simulation for accuracy model with $K_2$
3.6.2.2 Experimental examination for accuracy model

In this section, the step response and frequency response of the experimental examination are performed for the accuracy model to examine characteristic of the accuracy model.

3.6.2.2.1 Step response

Using the feedback gain $K_2$, the experimental step response for the accuracy model was obtained under the same condition with that for the simple model. The result is shown in Fig.3.38. Compared this with the Fig.3.36, the simulation result, we can find that there are some fluctuation when the system converged, whilst, in the experimental examination, the impact of step input on the system is larger than that in the simulation examination. This phenomenon is caused by the linearization, i.e. this system is nonlinear system but for analyzing this system we linearized the system. The other aspects of experimental step response for accuracy model confirm to that of the simulation step response for accuracy model. It is verified that the accuracy model is correct and the simple model is enough for manipulate the suspended object to move in the horizontal direction.

![Experimental step response of accuracy model with $K_2$](image.jpg)
3.6.2.2 Frequency response

Fig.3.34 shows the frequency response of experimental examination for simple model. From it, the resonant point and bandwidth can be seen. In this system, the resonant point is at 22.9 (rad/sec), and the bandwidth is 10.3 (rad/sec).

Fig.3.39 Experimental frequency response of accuracy model with $K_2$.
3.7 Measurements of the motion of the iron ball and magnets

The 2-DOF permanent magnet suspension system is built for manipulate the suspended object in the horizontal direction, so the measurements of the motion of the iron ball and magnets when the iron ball is moved in the horizontal direction are performed with the simple model of system. They are shown in the Fig.3.40 and Fig.3.41. Fig.3.40 shows the relationship between the displacement of the iron ball and two magnets under different reference input.

![Fig.3.40 Relationship of displacement of iron ball and horizontal magnets with reference input](image)

In this figure, the origin is the initial position. About the Y axis, the positive direction is the right direction and the negative direction is the left direction of movement. From this figure we can see that, when the reference input value is positive, the iron ball moves to the right and two horizontal magnets move to the left at the equilibrium position. The left air gap length is larger than the right air gap length, i.e. the attractive force of the right magnet is larger than that of the left magnet. When the reference input value is negative, the iron ball moves to the left and two horizontal magnets move to the right at the equilibrium position. The right air gap length is larger than the left air gap length, i.e. the attractive force of left magnet is larger than that of the right magnets. It confirms to the experimental examination results for simple model. From this figure, we can also find that within the range of -3(mm) to 3(mm) of the reference input, the motion of the iron ball in horizontal direction is
linear with the reference input, so the iron ball can be located at any position in the horizontal direction within that range. The manipulation is successful. The photos in which the iron ball is manipulated to deviate from the center position are shown with Fig.3.42 and Fig.3.43.

Furthermore, as we know, while the iron ball deviates from the central position, the attractive force of the vertical magnet will generate a horizontal component, which acts on the iron ball to make a balance between the left and right attractive force. From Fig.3.40, we can get the component of the attractive force of the suspension magnet. In this system, $k_m = 6.63 \times 10^{-6} \text{N/m}$. The horizontal component force of the vertical magnet attractive force can be calculated with the data shown in Fig.3.40 (here the calculation is omitted). It is verified that the equation (13) is correct.

The motion of the iron ball and the suspension magnet in the vertical direction is measured while the iron ball is moved in the horizontal direction. The relationship between the vertical displacement of the iron ball and suspension magnet and reference input is shown in Fig.3.41. In this figure, however, the displacement in the vertical direction is very small compared with that in the horizontal direction. We think that it is another reason for the fluctuation in dynamic characteristic of the experimental step response.

![Fig.3.41 Relationship of displacement of iron ball and suspension magnet with reference input](image-url)
Fig. 3.42 Photo of the system when the iron ball is manipulated to the right direction

Fig. 3.43 Photo of the system when the iron ball is manipulated to the left direction
3.8 Discussion

In this chapter, a 2-DOF permanent magnet suspension mechanism is created based on the 1-DOF system which is introduced in the last chapter. In this system, the suspended object, the iron ball is manipulated in the horizontal direction while it is suspended. The system is consisted of three permanent magnets controlled by VCM respectively and the suspended object. The position of magnets is sensed by eddy current sensors and that of the suspended object is sensed by an optical sensor. The system is controlled by means of adjusting the air gap between the permanent magnet and the suspended object.

For manipulate the suspended object to move in the horizontal direction, firstly, the arrangement of the polarity of three magnets, SNN (or NSS), is chosen with the FEM analysis from six situations according the magnetic field distribution. From this arrangement, the most large support force for supporting the suspended object will be generated and the force acted on the suspended object in horizontal direction is symmetrical. It is clear that this arrangement is easy for control the motion of the suspended object in vertical and horizontal direction.

Secondly, the prototype is constructed according the arrangement of polarity and principle. Based on the prototype a simple model in which the suspension magnet attractive force is neglected is created. Consequently, from the analysis result of linear control theory, two inputs and three outputs system is chosen as the control plant which can be controlled and observed for this system. The controller of this system is designed according the LQR full state feedback control law. Due to that there is some difference between the simulation examination and experimental examination for simple model, an accuracy model is created and examined. From the examining results for the accuracy model, we can say that the simple model is correct and enough for manipulation.

Finally this system is operated successfully. The suspended object can be manipulated in vertical and horizontal direction. Within a range of -3(mm) to 3(mm) of the reference input, the relationship of the displacement of the suspended object and two horizontal magnets with the reference input is linear. Therefore, the suspended object can be located at any position in horizontal direction within the range of the reference input.

However, during the suspended object is moved in horizontal direction, there is a tendency to be unstable in the DOF that is perpendicular to the motion direction in the horizontal plane. For solving this problem and make the suspended object move in other DOF, a new permanent maglev system has been developed. It is introduced in the following chapter.
Chapter 4

4-DOF Magnetic suspension mechanism
with permanent magnet motion control

So far, the suspended object can be manipulated in the vertical plane via the 2-DOF permanent magnet suspension mechanism that is described in the chapter 3. In that system, there is a tendency to be unstable in the other DOF when the suspended object is moved in the horizontal direction. For solving this problem, a new magnetic suspension mechanism in which the suspended object can be manipulated not only in the vertical plane but in the horizontal plane is developed. Firstly, a new permanent magnet suspension mechanism is built in which the suspended object is suspended and rotated along the suspension axis. The principle of this mechanism is introduced. The rotation model is created and the experimental examination verified that the feasibility of this mechanism. Secondly, using this mechanism, the suspended object can be located in the horizontal plane within a small range while it is suspended. Therefore, this system is a four-DOF permanent magnetic suspension system. The principle is introduced and the model is created. The simulation result verified the feasibility of this system.

4.1 Construction of mechanism

A new permanent maglev system is developed. The construction of this system is shown with Fig.4.1 and Fig.4.2. Fig.4.1 is an illustration of this mechanism in which the suspended object is an iron ball, and five permanent magnets are used to manipulate the suspended object. One of them is the suspension magnet located at the above of the iron ball, and the others four magnets are uniformly located in the horizontal plane around the iron ball. The iron ball is suspended in the center of horizontal plane of this mechanism, and the magnets are driven by means of the VCM actuators. The VCM actuators are fixed at the point that has the same distance to the center of mechanism. Fig.4.2 is the photo of this prototype in which the iron ball is omitted.
Fig. 4.1 Illustration of mechanism

Fig. 4.2 Photo of Prototype
From these figures we can see that there are five VCM actuators that drive the five permanent magnets respectively. Positions of the magnets and iron ball are sensed by means of eddy current sensors. When the iron ball is suspended, through control the motion of the upper magnet the height of iron ball can be adjusted to make the iron ball and four horizontal magnets in the same horizontal plane. At the same time, through control the motion of the horizontal magnets to make the iron ball rotate along suspension axis and position the iron ball in the horizontal plane within a range. In the following sections, the principle, model analysis and experimental examination of suspension, rotation and position will be introduced.

4.2 Suspension

The rotation of the suspended object and positioning the suspended object in the horizontal plane are performed while the suspended object is suspended. Therefore, the first step is to make the iron ball suspend.

4.2.1 Principle

The suspension mechanism of this system has the same construction with the 2-DOF permanent maglev system which has been introduced in chapter 2. Therefore, they have the same principle. An outline of proposed suspension system with a permanent magnet and linear actuator is shown in Fig.4.3. A ferromagnetic body is suspended by an attractive force from a permanent magnet, which is driven by an actuator positioned above it. The direction of levitation is vertical; the magnet and the object move only in this direction. The equilibrium position of the ferromagnetic body is determined by means of a balance between the gravity force and the magnet force. If the actuator does not
actively control the magnet’s position, the levitated object will either fall or adhere to the magnet. However servo-control of the actuator can make this system stable. Because there is a smaller attractive force for a larger air gap between the permanent magnet and object, the actuator drives the magnet upwards in response to object movement from its equilibrium position towards the magnet. Similarly, the actuator drives the magnet downwards in response to object movement away from the magnet. In this way, the object can be stably suspended without contact. In comparison to the electrical control method of electromagnetic suspension systems, this system is a mechanical control maglev system [7].

4.2.2 Model analysis

Although the principle of this system is the same with that of 2-DOF permanent maglev system, different permanent magnets make the attractive force between the suspended object and the magnet be different.

![Graph](image.png)

**Fig4.4. Relationship between suspension attractive force and air gap**

4.2.2.1 Attractive force examination

In this suspension system, cylindrical neodymium permanent magnets are used for control the attitude of the suspended object. The diameter of magnet is 0.008 m and its height is 0.008 m also. The suspended object is an iron ball whose diameter is 0.025 m and its mass is 0.063 kg. The attractive force between the magnet and iron ball is measured under different air gap. It is shown in Fig4.4, from which we can see that the attractive force, \( f_m \) is inverse proportional with the square of air gap. It is nonlinear force.
\[ f_m = \frac{k_m}{d_{\text{gap}}^2} \] (4.2.1)

Where, \( K_m \) is the constant of attractive force. It can be obtained from the Fig.2. In this case, \( K_m=16.9 \times 10^{-6} \) Nm\(^2\). When the suspension system is balanced, i.e. the attractive force is equal to the mass of the iron-ball, the air gap is 0.005 m. so far, used such a magnet, the feasibility of suspension is confirmed.

![Fig.4.5 Model of suspension system](image)

### 4.2.2.2 Suspension model

Fig.4.5 shows the model of suspension mechanism. Where,

- \( z_0 \): displacement of the iron ball
- \( z_1 \): displacement of permanent magnet
- \( m_0 \): mass of the iron ball
- \( m_1 \): mass of the magnet
- \( f_a \): generating force of the actuator
- \( f_m \): attractive force between iron ball and magnet
- \( d_{\text{gap}} \): air gap length between iron ball and magnet
- \( k_m \): the coefficient of magnetic field.
- \( k_1 \): the spring coefficient of the actuator
- \( \xi_1 \): the damping coefficient of the actuator.

Thus the motion equations of iron ball and magnet during suspension are:
Equations (4.2) and (4.3) are linearized to be equations (4.4) and (4.5) respectively. And then the linear control law can be applied in the analysis of suspension model.

$$m_0 \ddot{z}_0 = k_m (z_1 - z_0)$$  \hspace{1cm} (4.2.4)

$$m_1 \ddot{z}_1 = -\xi \dot{z}_1 - k_1 z_1 - k_m (z_1 - z_0) + f_u$$  \hspace{1cm} (4.2.5)

In this research, a LQR full state feedback control law is applied. The state space equation of suspension model is shown with equation (6).

$$\dot{x} = Ax + bu$$  \hspace{1cm} (4.2.6)

$$y = cx$$  \hspace{1cm} (4.2.7)

Where

$$x = \begin{pmatrix} z_0 \\ \dot{z}_0 \\ z_1 \\ \dot{z}_1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/m_1 \end{pmatrix}^T$$

$$A = \begin{bmatrix}
-\frac{k_m}{m_1} & 0 & 0 & 0 \\
0 & -\frac{k_m}{m_0} & 0 & 0 \\
k_m & 0 & -\xi & 0 \\
m_1 & 0 & 0 & -k_1 - \xi \frac{1}{m_1} \\
\end{bmatrix}, \quad c = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$u = f_u$$

The determinant of System controllability matrix $C$ and observability matrix $O$ are shown with equations (7) and (8).

$$\det(C) = -\frac{k_m^2}{m_0^2 m_1}$$  \hspace{1cm} (4.2.8)

$$\det(O) = -\frac{k_m^2}{m_0^2}$$  \hspace{1cm} (4.2.9)

Thus the suspension system can be controlled and observed.
4.2.3 Controller design

Based on the controllability and observability of this suspension mechanism the LQR control law is applied to design the controller of it. The control block diagram is shown in Fig.4.6. The cost function of system is shown with equation (4.10).

\[ J = \frac{1}{2} \int_0^t \left[ x'Qx + u'Ru \right] dt \]  \hspace{1cm} (4.2.10)

State weighting matrix

\[ Q = diag(100000 \ 1 \ 100000 \ 1) \]

Input weighting matrix

\[ R = (1) \]

The feedback gain

\[ K_c = (1084.2 \ 35.2 \ 637 \ 5.6) \]

4.2.4 Examination

A step disturbance is applied to exam the suspension mechanism with simulation and experimental method. The step value is 0.001(m). Table 4.1 shows the symbols and values of this suspension mechanism.
Table 4.1 values and symbols of suspension mechanism

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball</td>
<td>$m_0$</td>
<td>63.7g</td>
</tr>
<tr>
<td>Mass of suspension magnet</td>
<td>$m_1$</td>
<td>120.5g</td>
</tr>
<tr>
<td>VCM spring constant</td>
<td>$k_1$</td>
<td>0</td>
</tr>
<tr>
<td>VCM damping constant</td>
<td>$\xi_1$</td>
<td>0</td>
</tr>
<tr>
<td>Magnet field constant of left magnet</td>
<td>$k_{m1}$</td>
<td>1.69*10^{-5}Nm^2</td>
</tr>
<tr>
<td>VCM force constant</td>
<td>$k_a$</td>
<td>5N/A</td>
</tr>
</tbody>
</table>

4.2.4.1 Simulation examination

![Fig. 4.7 Step response of simulation examination](image)

Fig. 4.7 Step response of simulation examination

Fig.4.7 shows the step response of simulation examination result. From this figure we can see that the step disturbance is added at 1(s). Converge time is 0.1(s) and there is no overshoot in the dynamic characteristic.

4.2.4.2 Experimental examination

Fig.4.8 shows the configuration diagram of suspension system. From it, we can see that the gap sensor, which is located upper of the actuator and senses the permanent magnet movement, is an eddy current type. It has a sensing range of 10(mm) and a resolution of 10(µm). The movement of the iron ball is sensed also via an eddy current sensor and its resolution is 5(µm). Controller is a digital DSP controller with 12 bit resolution A/D converters and 16 bits D/A converter. Through A/D
converters, two sensors’ signal are converted to digital value and input into DSP Controller. Controller computes the current for VCM. Through D/A converter, the currents is converted to analog value and input into current amplifier to control the movement of magnet.

When the feedback gain is applied, the iron ball is suspended successfully. The result is shown in Fig.4.9. This figure is a time history of the displacement of iron ball and magnet in which the sample time is 0.005(s), and the interval is 2(s)
4.3 Rotation

Through control the motion of magnets which located in the horizontal plane the suspended object can be rotated along the suspension axis during suspension. This section describes the rotation principle, rotation model and examination.

![Image of spinning mechanism](image-url)
4.3.1 Rotation principle

One of the purposes of this investigation is make a rotation control mechanism in which the suspended object is rotated along the suspension axis when it is suspended. The image of rotation is shown in Fig.4.11. For simplification, only the magnets that are located in the horizontal plane and iron ball are shown in it. This figure is a planform of the system in which the suspension magnet is omitted. From it we can see the location of four magnets and iron ball. The iron ball is located in the center position, and four magnets are located perpendicularly each other around the iron ball in the horizontal plane. When the magnet approaches the iron ball alternatively, the iron ball can be rotated along the suspension axis.

Due to the suspended object is an iron ball, which is a ferromagnetic substance, there is remanent magnetism on the surface of it [Appendix]. However, when the iron ball is suspended, the influence of remanent magnetism is mostly in the vertical direction of suspension. It keeps the iron ball suspended. When the magnet in horizontal approaches the iron ball, there is a magnetization point on the surface of iron ball facing the approaching magnet. And when this magnet withdraws and the next magnet approaches, the approaching magnet will attract the magnetization point, so the iron ball can be rotated. In Fig.4.11, the approaching order of magnets is magnet a, b, c and d. It is one circle.

4.3.2 Rotation model and control

![Diagram](attachment:rotation_model.png)

a. Configuration of the magnets and remanent point
From the rotation principle and prototype of this system, a rotation model is created. It is shown in Fig.4.12. There are two parts in this figure, figure a shows the configuration of the magnets and remanent point from which we can get the relationship between the air gap and the distance from the magnets to the remanent point, and figure b shows the relationship between the attractive forces and rotation angle $\theta$, where

- $A$: the remanent point
- $\theta$: the rotation angle of the iron ball
- $\alpha$: the angle between $d_1^\prime$ and OA
- $\beta$: the angle between $d_2^\prime$ and OA
- $\gamma$: the angle between $d_3^\prime$ and OA
- $\lambda$: the angle between $d_4^\prime$ and OA
- $z_1$: displacements of the permanent magnet 1
- $z_2$: displacements of the permanent magnet 2
- $z_3$: displacements of the permanent magnet 3
- $z_4$: displacements of the permanent magnet 4
- $m_0$: mass of the iron ball
- $m_1$: mass of the magnet 1
- $m_2$: mass of the magnet 2
- $m_3$: mass of the magnet 3
\( m_4 \): mass of the magnet 4
\( f_{a1} \): generating forces of the actuator 1
\( f_{a2} \): generating forces of the actuator 2
\( f_{a3} \): generating forces of the actuator 3
\( f_{a4} \): generating forces of the actuator 4
\( f_{m1} \): attractive force between the remanent point and magnet 1
\( f_{m2} \): attractive force between the remanent point and magnet 2
\( f_{m3} \): attractive force between the remanent point and magnet 3
\( f_{m4} \): attractive force between the remanent point and magnet 4
\( d_0 \): the initial air gap length between the iron ball and magnets
\( r \): the maximum value of the displacement of the magnets
\( d_1 \): the air gap length between the iron ball and magnet 1
\( d_2 \): the air gap length between the iron ball and magnet 2
\( d_3 \): the air gap length between the iron ball and magnet 3
\( d_4 \): the air gap length between the iron ball and magnet 4
\( d_1' \): the distance between the remanent point and magnet 1
\( d_2' \): the distance between the remanent point and magnet 2
\( d_3' \): the distance between the remanent point and magnet 3
\( d_4' \): the distance between the remanent point and magnet 4
\( k_1 \): the coefficient of the magnetic field of magnet 1
\( k_2 \): the coefficient of the magnetic field of magnet 2
\( k_3 \): the coefficient of the magnetic field of magnet 3
\( k_4 \): the coefficient of the magnetic field of magnet 4
\( f_{m1}' \): the component of \( f_{m1} \) along the tangent direction of remanent point
\( f_{m2}' \): the component of \( f_{m2} \) along the tangent direction of remanent point
\( f_{m3}' \): the component of \( f_{m3} \) along the tangent direction of remanent point
\( f_{m4}' \): the component of \( f_{m4} \) along the tangent direction of remanent point
\( f_{m1}'' \): the component of \( f_{m1} \) in the Y direction
\( f_{m2}'' \): the component of \( f_{m2} \) in the X direction
\( f_{m3}'' \): the component of \( f_{m3} \) in the Y direction
\( f_{m4}'' \): the component of \( f_{m4} \) in the X direction

From this figure, the rotation equation of the iron ball and permanent magnet can be obtained.

\[
\dot{\theta} = (f_{m2}' - f_{m4}' + f_{m3}' - f_{m1}')R \tag{4.3.1}
\]

\[
I = m_0 R^2 \tag{4.3.2}
\]
\[ f_{m_1} = f_{m_1} \cos(\alpha - \frac{\pi}{2}) = f_{m_1} \sin\alpha = f_{m_1} \frac{d_1 + R}{d_1} \sin(\theta) \] 
\[ (4.3.3) \]

\[ f_{m_2} = f_{m_2} \cos(\beta - \frac{\pi}{2}) = f_{m_2} \sin\beta = f_{m_2} \frac{d_2 + R}{d_2} \cos(\theta) \] 
\[ (4.3.4) \]

\[ f_{m_3} = f_{m_3} \cos(\gamma - \frac{\pi}{2}) = f_{m_3} \sin\gamma = f_{m_3} \frac{d_3 + R}{d_3} \sin(\theta) \] 
\[ (4.3.5) \]

\[ f_{m_4} = f_{m_4} \cos(\lambda - \frac{\pi}{2}) = f_{m_4} \sin\lambda = f_{m_4} \frac{d_4 + R}{d_4} \cos(\theta) \] 
\[ (4.3.6) \]

### 4.3.3 Examination

#### 4.3.3.1 Simulation examination

In this investigation, the open loop control with which the requirement of rotation can be satisfied is applied. Therefore, it is only needed to input a current to make the magnets move to and fro. Table 4.2 shows the symbols and values of this suspension mechanism.

<table>
<thead>
<tr>
<th>Table 4.2 values and symbols of suspension mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball</td>
</tr>
<tr>
<td>Mass of suspension magnet</td>
</tr>
<tr>
<td>Initial air gap length</td>
</tr>
<tr>
<td>Maximum displacement of magnet</td>
</tr>
<tr>
<td>Radium of iron ball</td>
</tr>
<tr>
<td>VCM spring constant</td>
</tr>
<tr>
<td>VCM damping constant</td>
</tr>
<tr>
<td>Magnet field constant of left magnet</td>
</tr>
<tr>
<td>VCM force constant</td>
</tr>
</tbody>
</table>

Fig.4.13 shows the simulation results of the iron ball rotation and magnets movement. We assumed that the direction is positive direction when the magnet moves towards the iron ball. From this figure, we can see that when the magnets moved periodically the iron ball can be rotated along the suspension axis.
4.3.3.2 Experimental examination

Fig. 4.14 shows the configuration of spinning mechanism. Only one horizontal magnet motion control is shown in this figure. The other three magnets’ motion has the same control system with this figure. The magnet position is sensed by an eddy current sensor. The sensor’s signal is converted to digital signal through A/D converter and then is input into DSP controller. Controller computes the current for VCM and the signal is converted into analog value through D/A converter, and then the value is input a current amplifier to drive the VCM. The magnet is driven by VCM actuator to move straightly towards or away to the iron ball. In this investigation, the suspension and rotation control are independent each other.
Fig. 4.15 shows the control system of the experiment. In this system, four input signals have the same period and the input signal phase difference of each VCM is $\pi/2$ (rad) that keeps the approaching order of magnets.
The experiment is carried out with the control system. The iron ball is rotated successfully. The frequency of reference input signal is 1 Hz, the rotation velocity time history is shown in Fig.4.16. The rotation velocity is measured by means of a laser feed monitor. Fig.4.17 shows the rotation velocity at the first 2 second of the Fig.4.16. The average rotation velocity of the iron ball is about 1 r/s.

From Fig.4.16 and Fig.4.17, we can see that the velocity of rotation is not uniform. That is caused
by open loop control as well as weak remanent on the surface of the iron ball. The cycle of magnet movement is difficult to select when the rotation is started. A feed back control of the iron ball angle is necessary for improving the rotation.

4.4 Position the suspended object in horizontal plane

With the same construction and arrangement of magnets polarity, the suspended object can be manipulated in the horizontal plane while it is suspended. It can be positioned at an arbitrary position in horizontal plane within a small range. In this section, the principle, motion model analysis, controller design and examination are described.

4.4.1 Principle

Fig.4.18 is the planform of this system in which the suspension magnet is omitted. In this figure, Y axis is perpendicular to X axis, so they can be regarded as the coordinate of this figure. The gravity point of the iron ball is located at the original point. Y axis and X axis divided the horizontal plane into four equal parts, part , part , part and part . Magnet 1 and 3 can be moved along the Y axis whilst magnet 2 and 4 can be moved along X axis. Through control the motion of four magnets to adjust the air gap between the iron ball and magnet, the iron ball can be positioned at
arbitrary position in the horizontal plane within a small range near the original point. For instance, if the air gap between magnet 1 and the iron ball and the air gap between magnet 2 and the iron ball are adjust to smaller, and at the same time, the air gap between magnet 3 and the iron ball and the air gap between the magnet 4 and the iron ball are adjusted to be larger, the iron ball can be manipulated to move into part . As the same reason, the iron ball can be positioned in the part , part and part in the horizontal plane. The eddy current sensor 1 and 2 sense the position of the iron ball in the horizontal plane.

If the iron ball is moved along the X axis or Y axis this system can be regarded as a 2-DOF system, however, this system can solve the problem which has been discussed in chapter 3. That means when the iron ball is move along the X axis, it is stable in the Y axis, vice versa.

Fig. 4.19 model of positioning the suspended object in horizontal plane

4.4.2 Model analysis

Fig.4.19 shows the motion model of the iron ball and magnets when the iron ball is positioned in the horizontal plane.

$z_0$: displacement of the iron ball
$z_0_x$: component of displacement of the iron ball in X direction
$z_0_y$: component of displacement of the iron ball in Y direction
$z_1$: displacements of the permanent magnet 1
$z_2$: displacements of the permanent magnet 2
$z_3$: displacements of the permanent magnet 3
$z_4$: displacements of the permanent magnet 4
$m_0$: mass of the iron ball
$m_1$: mass of the magnet 1
$m_2$: mass of the magnet 2
$m_3$: mass of the magnet 3
$m_4$: mass of the magnet 4
$f_{a1}$: generating forces of the actuator 1
$f_{a2}$: generating forces of the actuator 2
$f_{a3}$: generating forces of the actuator 3
$f_{a4}$: generating forces of the actuator 4
$f_{m1}$: attractive force between the ball and magnet 1
$f_{m2}$: attractive force between the ball and magnet 2
$f_{m3}$: attractive force between the ball and magnet 3
$f_{m4}$: attractive force between the ball and magnet 4
$d_{01}$: the initial air gap length between the iron ball and magnet 1
$d_{02}$: the initial air gap length between the iron ball and magnet 2
$d_{03}$: the initial air gap length between the iron ball and magnet 3
$d_{04}$: the initial air gap length between the iron ball and magnet 4
$k_{m1}$: the coefficient of the magnetic field of magnet 1
$k_{m2}$: the coefficient of the magnetic field of magnet 2
$k_{m3}$: the coefficient of the magnetic field of magnet 3
$k_{m4}$: the coefficient of the magnetic field of magnet 4
$k_{m1}'$: the linear coefficient of the magnetic field of magnet 1
$k_{m2}'$: the linear coefficient of the magnetic field of magnet 2
$k_{m3}'$: the linear coefficient of the magnetic field of magnet 2
$k_{m4}'$: the linear coefficient of the magnetic field of magnet 2
$k_1$: the spring coefficient of the actuator 1
$k_2$: the spring coefficient of the actuator 2
$k_3$: the spring coefficient of the actuator 2
$k_4$: the spring coefficient of the actuator 2
$\xi_1$: the damping coefficient of the actuator 1
\( \xi_2 \): the damping coefficient of the actuator 2
\( \xi_3 \): the damping coefficient of the actuator 2
\( \xi_4 \): the damping coefficient of the actuator 2
\( f_{m1x} \): the component of \( f_m \) in the X direction
\( f_{m2y} \): the component of \( f_m \) in the Y direction
\( f_{m3x} \): the component of \( f_m \) in the X direction
\( f_{m4y} \): the component of \( f_m \) in the Y direction

\( f_{m10} \): attractive force between the magnet 1 and the iron ball at the initial position
\( f_{m20} \): attractive force between the magnet 2 and the iron ball at the initial position
\( f_{m30} \): attractive force between the magnet 3 and the iron ball at the initial position
\( f_{m40} \): attractive force between the magnet 4 and the iron ball at the initial position

From Fig.4.19, the motion equations of the iron ball and magnets can be obtained.

\[
m_0 \dddot{x}_{0x} = f_{m2} - f_{m4} - f_{m1x} - f_{m3x}
\]
\[
m_0 \dddot{y}_{0y} = f_{m1} - f_{m3} - f_{m2y} - f_{m4y}
\]
\[
m_1 \dddot{z}_1 = -\xi_1 \ddot{z}_1 - k_1 z_1 + f_{m1} + f_{a1}
\]
\[
m_2 \dddot{z}_2 = -\xi_2 \ddot{z}_2 - k_2 z_2 + f_{m2} + f_{a2}
\]
\[
m_3 \dddot{z}_3 = -\xi_3 \ddot{z}_3 - k_3 z_3 + f_{m3} + f_{a3}
\]
\[
m_4 \dddot{z}_4 = -\xi_4 \ddot{z}_4 - k_4 z_4 + f_{m4} + f_{a4}
\]

The linearized motion equations are:

\[
m_0 \dddot{x}_{0x} = m_4 \dddot{z}_4 - k_{m4} (z_4 - z_{0x}) - k_{m2} (z_{0x} - z_2) - (f_{m10} / d_{01} + f_{m30} / d_{03}) z_{0x}
\]
\[
m_0 \dddot{y}_{0y} = m_3 \dddot{z}_3 - k_{m3} (z_3 - z_{0y}) - k_{m1} (z_{0y} - z_1) - (f_{m20} / d_{02} + f_{m40} / d_{04}) z_{0y}
\]
\[
m_1 \dddot{z}_1 = -\xi_1 \ddot{z}_1 - k_1 z_1 + \dot{f}_{a1}
\]
\[
m_2 \dddot{z}_2 = -\xi_2 \ddot{z}_2 - k_2 z_2 + \dot{f}_{a2}
\]
\[
m_3 \dddot{z}_3 = -\xi_3 \ddot{z}_3 - k_3 z_3 + \dot{f}_{a3}
\]
\[
m_4 \dddot{z}_4 = -\xi_4 \ddot{z}_4 - k_4 z_4 + \dot{f}_{a4}
\]
The state space equation is

\[ \dot{x} = Ax + bu \]  

(4.4.13)

\[ y = cx \]  

(4.4.14)

Where

\[ x = \begin{bmatrix} z_{0x} & \dot{z}_{0x} & z_{0y} & \dot{z}_{0y} & z_1 & \dot{z}_1 & z_2 & \dot{z}_2 & z_3 & \dot{z}_3 & z_4 & \dot{z}_4 \end{bmatrix} \]

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2(k_{1x} + k_{1y})}{m_0} & f_{x0} & f_{x0} & 0 & 0 & 0 & 0 & 0 & k_{1x}/m_0 & 0 & k_{1y}/m_0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2(k_{2x} + k_{2y})}{m_0} & f_{x0} & f_{x0} & 0 & 0 & 0 & k_{2x}/m_0 & 0 & k_{2y}/m_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{3x}/m_0 & 0 & -k_{3x} - \frac{k_{2x}}{m_0} - \frac{k_{2y}}{m_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{k_{4x}}{m_0} & 0 & 0 & 0 & 0 & -k_{4x} - \frac{k_{2x}}{m_0} - \frac{k_{2y}}{m_0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{k_{5x}}{m_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{5x} - \frac{k_{2x}}{m_0} - \frac{k_{2y}}{m_0} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ b = \begin{bmatrix} 0 & 0 & 0 & 0 & 1/m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/m_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/m_4 & 0 & 0 & 0 \\ c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ u = \begin{bmatrix} f_{a1}' & f_{a2}' & f_{a3}' & f_{a4}' \end{bmatrix} \end{bmatrix} \end{bmatrix} \]

The rank of controllability matrix and observability matrix of this state space equation is 12, so the system is controllable and observable

**4.4.3 Controller design**

The LQR (linear quadratic regulator) full state feedback control law is used to design the
controller of this system. The cost function is:

\[ J = \frac{1}{2} \int_0^t \left[ x' Q x + u' R u \right] dt \] (4.4.15)

\[ Q = \text{diag}\{100000 \ 1 \ 100000 \ 1 \ 100000 \ 1 \ 100000 \ 1 \ 100000 \ 1\} \]

\[ R = \text{diag}\{1 \ 1 \ 1 \ 1\} \]

It is assumed that the initial air gap length is 0.008(m). The feedback gain \( K \) is

\[
K = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
627.2 & 21.3 & 0 & 0 & 0 & 0 & 0 & -473.3 & -4.8 & -0.74 \\
0 & -627.2 & -21.3 & 0 & 0 & 0 & 148.5 & 0.74 & 0 & 0 \\
-627.2 & -21.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 148.5 \\
0 & -627.2 & -21.3 & 0 & 0 & 0 & 0 & 0 & 0 & 148.5 \\
0 & 0 & -627.2 & -21.3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -627.2 & -21.3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -627.2 & -21.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -627.2 & -21.3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -627.2 & -21.3 & 0 & 0
\end{bmatrix}
\]

4.4.4 Examination

A step disturbance is applied to exam the suspension mechanism with simulation method. The step value is 0.001(m). Table 4.3 shows the symbols and values of this mechanism.

<table>
<thead>
<tr>
<th>Table 4.3 values and symbols of suspension mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the iron ball ( m_0 )</td>
</tr>
<tr>
<td>Mass of suspension magnet ( m_1 )</td>
</tr>
<tr>
<td>VCM spring constant ( k_l )</td>
</tr>
<tr>
<td>VCM damping constant ( \xi_l )</td>
</tr>
<tr>
<td>Magnet field constant of left magnet ( k_{ml} )</td>
</tr>
<tr>
<td>VCM force constant ( k_a )</td>
</tr>
</tbody>
</table>

4.4.4.1 Simulation examination

Fig.4.20 shows the step response of simulation examination result. From this figure we can see that the step disturbance is added at 2(s). Converge time is 0.1(s) and there is no overshoot in the dynamic characteristic.
4.5 Discussion

In this chapter, as the one step of the development of the noncontact manipulation mechanism, a noncontact spinning mechanism with linear actuators and permanent magnets has been proposed. The mechanism consists of five linear actuated magnets. And the remanent magnetization of the iron ball is utilized. In this mechanism, the suspended object is suspended successfully; it can be rotated along the suspended axis when it is suspending; and it is verified theoretically that the suspended object can be positioned at the horizontal plane near the center position of the mechanism. This mechanism is a 4-DOF permanent maglev system.

As further works, the angle control of the iron ball maybe realized as shown in Fig.4.21. The adjustments of the positioning of the permanent magnets make the iron ball to be positioned in arbitrary angle.
Fig. 4.21 Angle control diagram

Fig. 4.22 Photo of the device
Chapter 5

Conclusion and recommendation

5.1 Conclusion

A magnetic suspension system is the mechanism in which a suspended object can be supported with noncontact magnetism, i.e. there is no mechanical contact part. So the advantage of such a system is that lubrication is unnecessary, no generation of dust and nonlinear friction. It is excellent in maintenance and makes high-speed rotation be possible.

In a magnetic suspension mechanism, the popular control method is that through control the current passed in the coil of electromagnet control the attractive force actively. This research described the suspension mechanism which controls attractive force through control the reluctance of magnetic circuit.

In this investigation, a permanent magnet suspension system in which the motion of the permanent magnet is controlled by the actuator to adjust the air gap between the permanent magnet and the suspended object is developed. In this system a permanent magnet is used to replace the electromagnet to make the suspension system. Compared with the electromagnet suspension system, there is no heat generation; it is easy for tele-manipulation via a connecting bar and need not the volume of coil space.

5.1.1 One-DOF suspension system

The first step of this research is to build a one-DOF suspension mechanism which makes the suspension object be suspended. The feasibility of suspension is verified through the theoretical and experimental examination. The suspension is successful.

In this system, the nonlinear magnetism is linearized. From a viewpoint of linear control theory, it is possible that one-DOF permanent magnet suspension system can be realized. Through control the
position of permanent magnet, the one-DOF permanent magnet suspension system is operated successfully. The controller is designed with an optimal control method, LQR control method. The simulation results and experiment results show that the suspended object can be suspended stably.

5.1.2 Two-DOF suspension system

Based the one-DOF permanent magnet suspension system, a prototype of 2-DOF permanent magnet suspension mechanism in which three permanent magnets are used to manipulate the object moving in the vertical plane has been built. One magnet is located on the upper to suspend the suspended, and the other two magnets are located on the two sides of the suspended object in the horizontal plane to control its motion in the horizontal direction.

From the results of FEM analysis the SNN (NSS) magnetic polarity arrangement is determined, due to this arrangement can provide sufficient force acting on the suspended object and the equilibrium position can be assumed to the center of the magnets. It is the optimal arrangement according the distribution of magnetic field for suspension and motion in horizontal direction.

A simple model and an accuracy model of horizontal motion are created respectively. In the linearized model, considering the input and output of the model, there are six situations. According the results of controllability and observability analysis of the model, only one situation, two forces input and three outputs system, can be controlled and observed. So in terms of the linear control theory, the LQR full states feedback control law can be applied to design the controller of this system in which the two forces input and three outputs situation is the control plant.

The examination results of the simple model show that due to the suspension attractive force is neglected when the simple model is created there are some difference between the simulation examination and the experimental examination. For explain this phenomena the accuracy model is created. The examination results verified that the simple model and accuracy model are all correct and the suspended object can be manipulated in the vertical plane.

The displacement of the suspended object in horizontal direction is measured with a reference input signal. The relationship between the displacement of the suspended object and reference input is linear within a range of 0.006 (m) from left to right of the equilibrium position, so we can locate the suspended object at any position within this range in the horizontal direction.

Consequently, it is carried out that a 2-DOF magnetic suspension mechanism could be actually constituted by using a permanent magnet with its motion control. It can be said that it is succeeded in development of the 2-DOF permanent magnetic suspension mechanism with a motion control of a permanent magnet.
However, when the suspended object is moved in the horizontal direction, there is an unstable tendency on the perpendicular direction to the motion direction in the horizontal plane. A new permanent maglev mechanism is developed to solve this problem.

### 5.1.3 Four-DOF suspension system

Based on the 2-DOF permanent maglev system, a 4-DOF permanent maglev system has been developed in which there are 5 permanent magnets which is driven by 5 VCM actuators respectively to control the suspended object moving in 4-DOF. One permanent is positioned above the suspended object to suspend it as well as the others are located at the horizontal plane symmetric to the suspended object. Through control the motion of these permanent magnets the suspended object can be rotated along the suspension axis and moved along two axes on which four horizontal magnets are located.

The arrangement of the polarity of the magnet is SNNNN or (NSSSS). As mentioned above, the suspended object is a ferromagnetic substance. The performance of the ferromagnetic substance is availed to make the suspended object suspended and rotate along the suspension axis, i.e. the ferromagnetic substance can be magnetized so there is remanent magnetism on the surface of it. However, when the suspended is suspended, the influence of remanent magnetism is mostly in the vertical direction of suspension, which keeps the suspended object suspension. When the magnet located in horizontal plane approaches the suspended object, there is a magnetization point on the surface of iron ball facing the approaching magnet. And when this magnet withdraws and the next magnet approaches, the approaching magnet will attractive the magnetization point. When the magnet approaches the suspended object alternatively, the suspended object can be rotated along the suspension axis.

In the rotation mechanism, a laser sensor is applied to measure the rotation velocity and the four permanent magnets are driven by four linear actuators moving forward or backward for adjusting the air gap between the permanent magnet and the suspended object i.e. to adjust the torque, which makes the suspended object rotate, acting on the suspended object. The experimental examinations are performed. Although the rotation velocity is not uniform, it can be improved via increasing the number of permanent magnet located in the horizontal plane.

With the same construction of rotation mechanism, the iron ball can also be moved along other 2-DOF in the horizontal plane. The principle of this is the same as that of 2-DOF permanent maglev system which is introduced above. The motion model is created and analyzed. The examination that confirms the feasibility of the motion is performed. The problem mentioned in the 2-DOF permanent maglev system can be solved in the 4-DOF permanent maglev system.
5.2 Recommendation

The presented work demonstrates the large potential in multi-DOF permanent maglev system that still exists. Even though the manipulation and rotation are successful in this research, the high quality of the rotation speed and precious of manipulation are expected. In order to enlarge the potential field of applications, the existing set up should be improved on the following points:

Further improvement of the stability and robustness

Improvement of the quality of rotation velocity

Improvement of precious of the manipulation

Cost reduction

As a first step the application of H-infinity control law is recommended in the existing set up. Using this control law the robustness of the system can be improved. Secondly, for improving the quality of rotation velocity, the number or permanent magnet used in the system should be increased. Fig.6.1 shows the planform image of the permanent magnets and suspended object locations in the horizontal plane. The fluctuation of the rotation velocity can be reduced by using this structure. On
the other hand, the suspended object can be manipulated in the horizontal plane while it is spinning along the suspension axis. Finally, as mentioned in the section 4.6, the angle control can be realized with this structure.
Appendix

Magnetic materials

A number of substances are strongly attracted by magnets and can be magnetized. These include iron, nickel, and cobalt, and all those alloys that contain a proportion of these metals. They are classified to soft and hard magnetic materials.

Magnetically soft materials can be magnetized very easily, but the magnetism induced in them is only temporary. They include Stalloy, an alloy of iron with 4% silicon used to make the cores of electromagnets and transformers, and the materials used to make ‘iron’ nails and paper clips. Stroking a magnet over a steel pin from one end to the other will weakly magnetize the steel pin. This is because very large numbers of iron atoms (domains) of the steel become aligned in the same direction.

Magnetically hard materials can be permanently magnetized by a strong magnetic field. Steel and special alloys such as Alcomax, Alnico, and Ticonal, which contain various amounts of aluminum, nickel, cobalt, and copper, are used to make permanent magnets. The strongest permanent magnets are ceramic, made under high pressure and at high temperature from powders of various metal oxides. Iron is an example of a natural hard magnetic material. Its magnetic properties are due to its atomic structure. The electrons in the outer orbit of an iron atom behave as an electric charge and produce a strong magnetic field. In a magnetized piece of iron millions of individual iron atoms, called a domain, are aligned in the same direction. The domains have a north and a south pole.

Substances differ in the extent to which they can be magnetized by an external field (susceptibility). Materials that can be strongly magnetized, such as iron, cobalt, and nickel, are said to be ferromagnetic. This is due to the formation of areas called domains in which atoms, weakly magnetic because of their spinning electrons, align to form areas of strong magnetism. Magnetic materials lose their magnetism if heated to the Curie temperature. Most other materials are
**Paramagnetic**, being only weakly pulled towards a strong magnet. This is because their atoms have a low level of magnetism and do not form domains. **Diamagnetic** materials are weakly repelled by a magnet since electrons within their atoms act as electromagnets and oppose the applied magnetic force. **Anti-ferromagnetic** materials have a very low susceptibility that increases with temperature; a similar phenomenon in materials such as ferrites is called **ferrimagnetism**.

Form of magnetism that can be acquired in an external magnetic field and usually retained in its absence, so that ferromagnetic materials are used to make permanent magnets. A ferromagnetic material may therefore be said to have a high magnetic permeability and susceptibility (which depends upon temperature). Examples are iron, cobalt, nickel, and their alloys. Ultimately, ferromagnetism is caused by spinning electrons in the atoms of the material, which act as tiny weak magnets. They align parallel to each other within small regions of the material to form domains, or areas of stronger magnetism. In an unmagnetized material, the domains are aligned at random so there is no overall magnetic effect. If a magnetic field is applied to that material, the domains align to point in the same direction, producing a strong overall magnetic effect. Permanent magnetism arises if the domains remain aligned after the external field is removed. Ferromagnetic materials exhibit hysteresis. (http://www.tiscali.co.uk/)

![Magnetization curve for a ferromagnetic specimen and an associated Hysteresis loop](image)

As an example a general BH-curve of ferromagnetic material is given in Fig.appendix.1. One can distinguish the coercivity $H_c$ and the remanence $B_r$. Note that, although this cannot be seen from Fig.appendix.1, the slope in the saturated region $S$ is still about equal to $\mu_0 = 1$ and not zero.
Bibliography


for Magnetically Levitated Stage” 8th International Symposium on Magnetic Bearing, 2002, Page(s): 151-156


[21] Samir Mittal, Chia-Hsiang Menq “Precision Motion Control of a Magnetic Suspension Actuator Using a Robust Nonlinear Compensation Scheme” Mechatronics, IEEE Transaction on, VOL. 2, NO.4, December 1997 Page(s):268-280


Relevant papers of this research

Publication for Journal

[1] Tianshi Cui, Koichi Oka “2-DOF Maglev System with Permanent Magnet Motion Control” Control System Technology IEEE Transactions on volume (under review)


Publication for International Conference


Publication for Domestic Conference


[3] Cui Tianshi, Koichi Oka “2 DOF Permanent Maglev System With LQR Controller” The 14th FAN (Fussy, Artificial Intelligence, Neural Networks and Computational Intelligence) symposium’04 in Kochi, Japan 2004 pp.193-194

[4] Yusuke Fujiwara, Koichi Oka, Cui Tianshi “Maglev System With Permanent Magnet Motion Control” 2004 Shikoku-section Joint Convention of the Institutes of Electrical and related Engineers,

[5] Yusuke Fujiwara, Cui Tianshi, Chen Li, Koichi Oka “Manipulation by Linear Driving Permanent Magnet-Rotation Control of Ironball” The 17th Symposium on Electromagnetics and Dynamics, Kochi, Japan 2005 pp.231-236


[7] Yusuke Fujiwara, Cui Tianshi, Koichi Oka “Magnetic Levitation System with Permanent Magnet Motion Control-Development of Spinning Mechanism” The 14th FAN (Fussy, Artificial Intelligence, Neural Networks and Computational Intelligence) symposium'04 in Kochi, Japan 2004 pp.133-136
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