Robust Control Design for Maglev Train with Parametric Uncertainties Using μ - Synthesis

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***Abstract*—**The magnetic suspension systems that they are basis of maglev trains divided in two classes: electrodynamic suspension (EDS) and electromagnetic suspension (EMS). EDS is based on repulsive forces acting on a magnet and is inherently stable system and even has well robustness in many cases with open loop control. But EMS is based on attractive forces acting on a magnet, is inherently unstable system that without feedback control has a poor performance. So, we must use feedback and we need to an exact mathematical model of plant to synthesis the feedback control system. This model should contain different uncertainties to make it more similar to actual model. Therefore, control system should have robust stability and performance under model uncertainties. Above desires will be accessible with a controller in μ framework.

In this paper, we assume that the suspension system is EMS and perturbations of the model parameters are considered as the source of uncertainty. Since we can represent these perturbations in state space parameters (A, B, C, D) ,uncertainty will be structured.

***Keywords*—** Robust control, μ technique, maglev train.

I. INTRODUCTION

MAGNETIC levitation (maglev) is an innovative transportation technology that via replacement of mechanical components by electronics overcomes the technical restrictions of wheel on rail technology. Compared with traditional railways maglev systems have high speed, high safety, less pollution, low energy consumption and high capacity. Since the magnetic suspension system used in these trains is unstable, we must use feedback. In addition we must pay attention to robustness of response in control design. This means that the system should have robust stability and performance under model uncertainties. In this study we consider the perturbations of the model parameters as the source of uncertainty. Since we can represent these perturbations in state space parameters, uncertainty will be structured. At each part of system that exist uncertainty it can be considered uncertainty as a Δ block about certain parameters. In this manner each of such blocks has one input and one output. Putting in order all Δ blocks will forms uncertainty set .In this way all Δ blocks are considered out of plant and obtained uncertainty set has special structure: it is block-diagonal and blocks on diagonal are small Δs that in fact pulled out from interior of plant. Thus, if we have n blocks, then $\Delta = diag(\Delta_1,....,\Delta_n)$. We consider uncertainties in normalized form in formalization of standard conditions at robust design i.e. $\|\Delta_j\|_\infty \le 1$.

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II. μ ROBUST CONTROL

Robust performance problem is described with general framework shown in Fig. 1. P is nominal model and represents system interconnections and K is controller .y is measurement outputs, u is control inputs, d is external inputs and disturbances , e is error signal and z and w are the inputs and outputs same uncertainties.

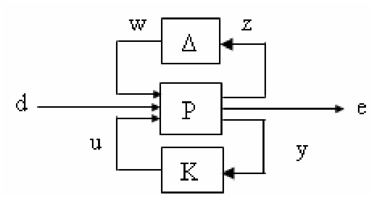
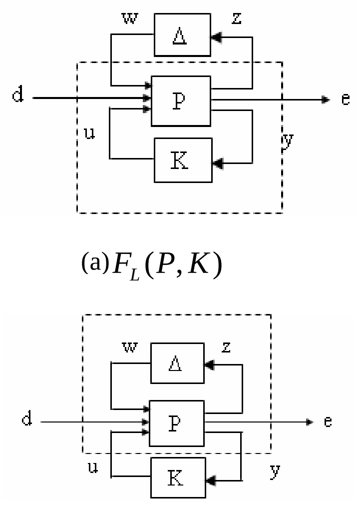


Fig. 1 μ Framework

This framework can be shown as combination of analysis and synthesis problems by linear-fractional transformation (LFT) definition.



(a) $F_L(P,K)$

(b) $F_U(P,\Delta)$

Fig. 2 Combination of analysis and synthesis problems

In robust analysis transfer function F_L from $[w \quad d]$ to $[z \quad e]$ is shown by lower LFT:

$$M = F_L(P,K) = P_{11} + P_{12}K(I - P_{22})^{-1}P_{21} \tag{1}$$

(To define P_{ij} s, we have $\begin{cases} e = P_{11}d + P_{12}u \\ y = P_{21}d + P_{22}u \end{cases}$)

And in robust synthesis F_U form d to e is shown by upper LFT:

$$F = F_U(M,\Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \tag{2}$$

(To define M_{ij} s, we have $\begin{cases} z = M_{11}\omega + M_{12}d \\ e = M_{21}\omega + M_{22}d \end{cases}$)

It will be clear that

$$F_L[F_U(P,\Delta),K] = F_U[F_L(P,K),\Delta] \tag{3}$$

Design specification is to obtain stabilizing controller K such that (for all $\Delta \in B\Delta$,that $B\Delta = \{\Delta \in \Delta : \sigma(\Delta) \le 1\}$) closed loop system be stable and satisfies:

$$\|F_U[F_L(P,K),\Delta]\|_\infty \le 1 \tag{4}$$

Structured singular value that provides suitable test for robust stability and performance is defined as:

$$\mu_\Delta(M) = \begin{cases} 0 \quad if \quad \det(I - M\Delta) \ne 0 \qquad \forall \Delta \in B\overline{\Delta} \\ \{\inf_{\Delta \in B\Delta} \overline{\sigma}(\Delta(j\omega)) \quad : \quad \det(I - M\Delta) = 0\}^{-1} \end{cases} \tag{5}$$

We can consider system performance by external disturbance input. For this purpose we augment fictitious performance uncertainty block to analysis structure. Then extended uncertainty set is defined as:

$$\Delta^* = diag(\Delta, \Delta_P) \tag{6}$$

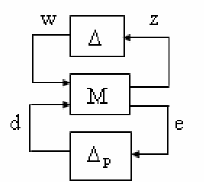


Fig. 3 Representation of system with augmented uncertainty

Definition of the nominal and robust stability and performance are expressed as:

Nominal Stability (NS): M is (interior) stable

Nominal Performance (NP): $\mu_{\Delta_P}(M_{22}) < 1 \quad \forall \omega$

Robust Stability (RS): $\mu_\Delta(M_{11}) < 1 \quad \forall \omega$

Robust Performance (RP): $\mu_{\Delta^*}(M) < 1 \quad \forall \omega$

The purpose of μ synthesis is to minimize peak of value $\mu_{\Delta^*}(\cdot)$ of closed loop transfer function $F_L(P,K)$ upon stabilizing controller K:

$$\min_{K \text{ stabilizing}} \max_{\omega} \quad \mu_{\Delta^*}\{F_L(P,K)(jw)\} \tag{7}$$

There isn't any optimum solution for this minimizing problem, but D-K iteration procedure that compounds μ analysis and H_∞ synthesis often has good results. This procedure attempts to solve

$$\min_{K \text{ stabilizing}} \max_{D(S) \in D; stable, \min. phase} \|D(F_L(P,K)(jw)D^{-1}\|_\infty \tag{8}$$

that $D = \{D | D\Delta = \Delta D\}$.

This problem can be solved repeatedly and by a consecutive solution upon one of the variables D and K and fixing another variable. It's worthy of mention that matrix D is assumed equal to I at first iteration because of we don't access to D.

III. Maglev Train Model

In this paper a model is used with two degrees of freedom with one car body and one magnet. Degrees of freedom are translational displacement at the center of mass of the car body and bogie. To balance the car we should assume the car has two bogies but a part of this system is only considered due to avoidance complexity and high amount of equations because of rotational displacement equation at car body will be added and also the number of equations related with bogies and magnets will be two time as much .

The system contains two suspension systems: magnet suspension and suspension caused by spring and dashpot of bogie. Movement equations satisfy Newton's low; f is magnetic force and all of the parameters introduced in Table I.

$$\begin{aligned} m_s \ddot{v}_s + c_s(\dot{v}_s - \dot{v}_p) + k_s(v_s - v_p) = 0 \\ m_p \ddot{v}_p + c_s(\dot{v}_p - \dot{v}_s) + k_s(v_p - v_s) = f \end{aligned} \tag{9}$$

In magnetic suspension model the attractive force between the pole surface and the ferromagnetic plate (Fig. 4) is:

$$F = \frac{1}{2\mu_0} B_m{}^2 (2A_m) \tag{10}$$

That is obtained with regard to relationships in [1] as

$$F = \frac{n_m \mu_\circ ab}{4h^2}(N_c I_c + N_t i)^2 \tag{11}$$

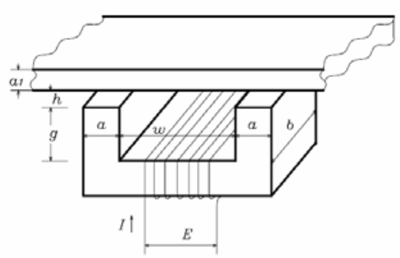


Fig. 4 Electromagnetic and ferromagnetic plate configuration

The relationship between voltage and trim current is derived from Kirchhoff's voltage low:

$$u = R_c i - N_t \frac{d(B_m ab)}{dt} = 0 \quad (12)$$

That with replacement of B_m from [1] we have:

$$u = R_c i + \frac{\mu_0 abN_t^{\ 2}}{2h}\frac{di}{dt} - \frac{\mu_0 abN_t(N_c I_c + N_t i)}{2h^2}\frac{dh}{dt} \quad (13)$$

The magnetic force equation is linearized at nominal air gap where $h = h_0$, $i = i_0 = 0$

$$F \approx \frac{M_1}{2} N_c I_c - \frac{M_1}{h_0} N_c I_c (h - h_0) + M_1 N_t i \quad (14)$$

And the dynamic magnetic force is expressed as

$$f \approx -\frac{M_1}{h_0} N_c I_c (h - h_0) + M_1 N_t i \quad (15)$$

The voltage law is also linearized at the nominal air gap, where $h = h_0$, $i = i_0 = 0$, $\dot{h} = \dot{h}_0 = 0$, $\dot{i} = \dot{i}_0 = 0$

$$\dot{i} \approx M_2 u - M_3 i + M_4 \dot{h} \quad (16)$$

TABLE I
PARAMETERS OF MAGLEV MODEL

parameter	**symbol**
Primary suspension (bogie-magnets) mass	m_p
Secondary suspension (car body) mass	m_s
Vertical displacement of bogie	v_p
Vertical displacement of the car body at the center of gravity	v_s
Secondary damping	c_s
Secondary stiffness	k_s
Permeability of air	μ_0
Flux density across the air gap	B_m
Face area of each magnet pole	A_m
Number of magnets in each module	n_m
Refer to fig.3	a
Refer to fig.3	b
Magnetic air gap	h
Nominal air gap	h_0
Number of turns of coils in each magnet providing a constant force	N_c
Constant current	I_c
Number of turns of coils in each magnet providing trim current	N_t
Trim current	i
Voltage	u
Resistance of trim coil	R_c
Wave number	r_f
Roughness	h_d
Gaussian white noise with zero mean	w_d
Vehicle velocity	V
Beam vertical displacement	v_g
Fundamental frequency	ω

It is assumed that the roughness caused by pier elevation difference and creep deformations will be a non-white stationary random process, which is modeled as the response of a first order filter to a stationary white excitation:

$$\frac{1}{r_f}\frac{dh_d(x)}{dx} + h_d(x) = w_d(x) \quad (17)$$

That it is in time domain as:

$$\frac{1}{r_f V}\dot{h}_d(t) + h_d(t) = w_d(t) \quad (18)$$

The guide way model is expressed in this from:

$$\ddot{v}_g + \omega^2 v_g = 0 \quad (19)$$

That v_g is the bending of the guide way under the train and ω is depended on beam bending rigidity, mass of beam and distance of two spans of beam.

In this manner deviation of nominal air gap is obtained with this equation:

$$h - h_0 = h_d + v_p - v_g \quad (20)$$

IV. CONTROLLER DESIGN

The parameters $k_s, c_s, m_s, m_p, \ldots$ are not constant due to various factors such as train's load change, tolerance of resistance of magnetic suspension system, etc. and contain uncertainties. Range of uncertainties that be considered for each parameter is in this form:

$$m_p = \overline{m}_p(1 + 0.25\delta_{m_p}) \quad (21)$$

$$m_s = \overline{m}_s(1 + 0.5\delta_{m_s}) \quad (22)$$

$$k_s = \bar{k}_s(1+0.4\delta_{k_s}) \quad (23)$$
$$c_s = \bar{c}_s(1+0.3\delta_{c_s}) \quad (24)$$
$$M_3 = \bar{M}_3(1+0.5\delta_{M_3}) \quad (25)$$
$$r_f = \bar{r}_f(1+0.005\delta_{r_f}) \quad (26)$$
$$\omega = \bar{\omega}(1+0.45\delta_{\omega}) \quad (27)$$
or
$$\omega^2 = \bar{\omega}^2(1+0.75\delta_{\omega^2}) \quad (28)$$
$$V = \bar{V}(1+0.25\delta_V) \quad (29)$$

All δ s are in the interval [-1, 1].

We form uncertainties as the matrices by using definition of LFT for example for k_s we have:

$$k_s = \bar{k}_s(1+0.4\delta_k) \rightarrow M_{k_s} = \begin{bmatrix} \bar{k}_s & 0.4\bar{k}_s \\ 1 & 0 \end{bmatrix} \quad (30)$$

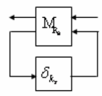


Fig. 5 Uncertain k_s as LFT

Thus the linear system interconnection will be shown in Fig. 6.

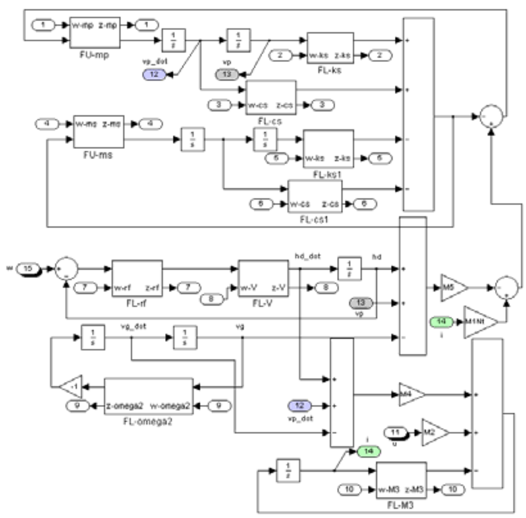


Fig. 6 Interconnection structure of linear system

We form this system by "sysic" program in μ-tools toolbox and then engage in design of controller. Three types of controller LQG, H_∞ and μ are designed for this system. To evaluate the robustness of resulting closed loop system, frequency analysis for any of them will be shown in separate curvatures.

V. Simulation Results

The maglev train system has two measurement outputs; car body vertical acceleration and air gap between bogie and guide way and two inputs; voltage and roughness input. The values of the parameters are given in Table II. By matrices like shown in Fig. 4 weights of uncertainties are considered in system interior structure and as a result the uncertainties will be considered out of plant in normalized form (such as Fig. 1).

TABLE II
Values of Model Parameters

symbol	value	units
m_p	500	kg
m_s	500	kg
c_s	10^4	Ns/m
k_s	10^5	N/m
μ_0	$4\pi \times 10^{-7}$	weber/Am
A_m	0.04	m^2
n_m	12	
a	0.05	m
b	0.1	m
h_0	0.01	m
N_t	96	
R_c	2	ohm
r_f	0.01	m^{-1}
V	400	m/s
ω	34.5	rad/s

At first, we engage in design of LQG controller. We form closed loop system by using of lower LFT and then by closed loop system and uncertainties as upper LFT we do μ synthesis. For each of these definition: nominal and robust stability and performance (in section 2) we draw related curvatures. In each case that the peak value of curvature is smaller than 1 can be resulted that we reach to desired characteristic. For LQG control, bode diagram of controller and nominal performance, robust stability and robust performance tests in sequence are shown in Figs. 7 to 10. It is clear that the system has only suitable nominal performance and does not satisfy robust stability and performance characteristics.

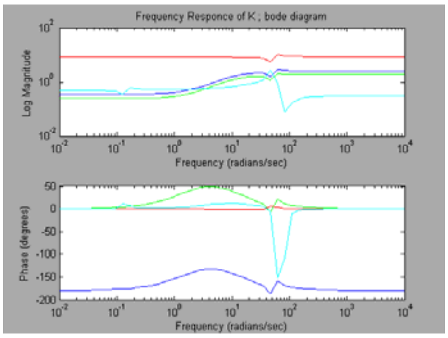


Fig. 7 Bode diagram of LQG controller

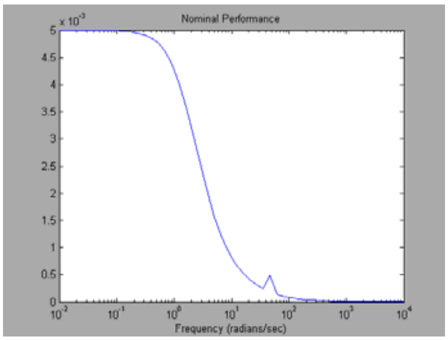


Fig. 8 Nominal performance test of closed loop system with LQG controller

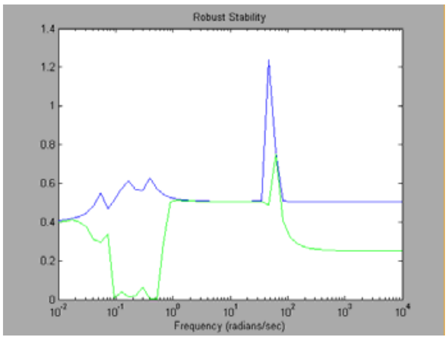


Fig. 9 Robust stability test of closed loop system with LQG controller

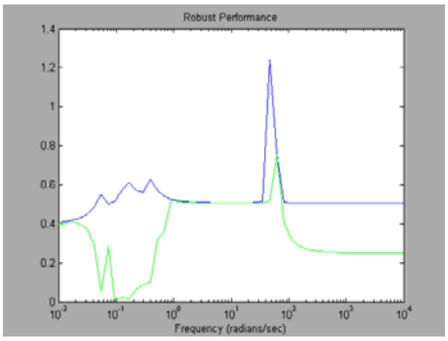


Fig. 10 Robust performance test of closed loop system with LQG controller

At next stage we engage in design of H_∞ controller. The results are shown in Figs. 11 to 14 in sequence like LQG controller. In this case the system has suitable nominal performance, too but because of the peak value of $\mu_{\Delta_P}(M_{22})$ and $\mu_{\Delta^*}(M)$ is equal to 1, we don't attain robust stability and performance.

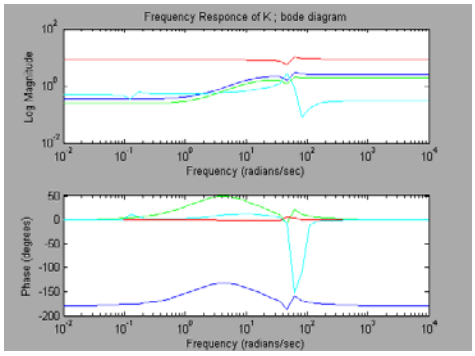


Fig. 11 Bode diagram of H_∞ controller

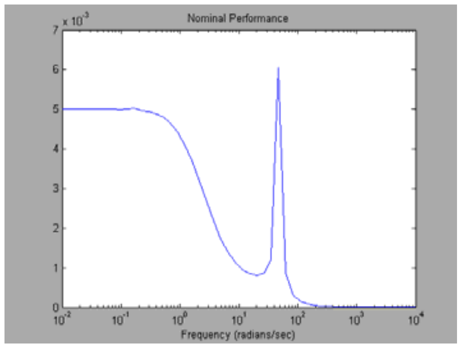


Fig. 12 Nominal performance test of closed loop system with H_∞ controller

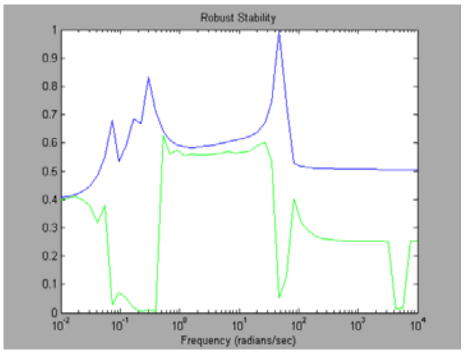


Fig. 13 Robust stability test of closed loop system with H_∞ controller

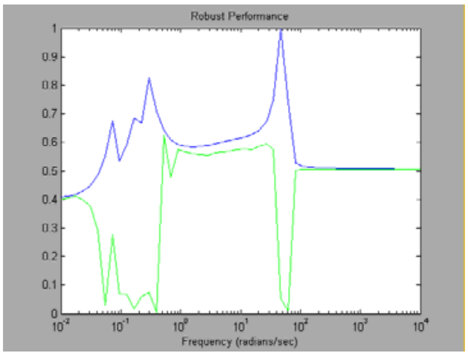


Fig. 14 Robust performance test of closed loop system with H_∞ controller

At the end, we design μ controller for this system. By using of D-K iteration procedure we will reach bode diagram of controller, nominal performance, robust stability and robust performance tests shown in Figs. 15 to 18 at third iteration. With regard to the peak values of last three curvatures are smaller than 1, this type of controller provides suitable nominal performance, robust stability and robust performance conditions.

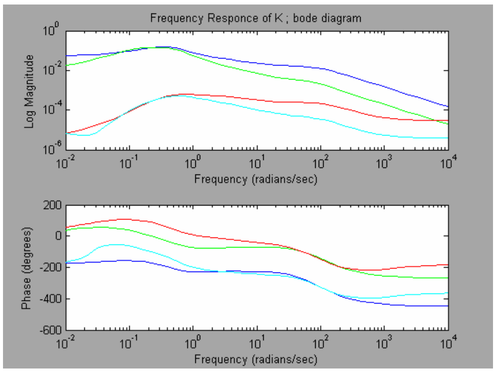


Fig. 15 Bode diagram of μ controller

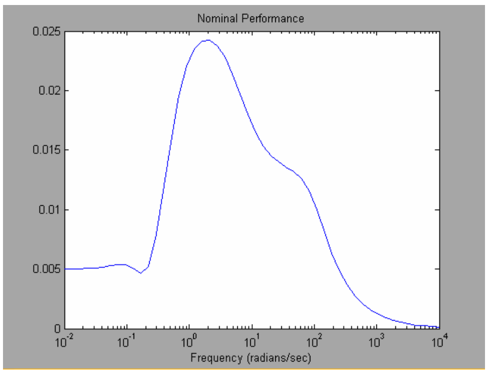


Fig. 16 Nominal performance test of closed loop system with μ controller

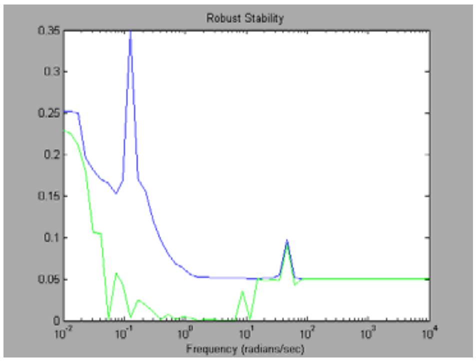


Fig. 17 Robust stability test of closed loop system with μ controller

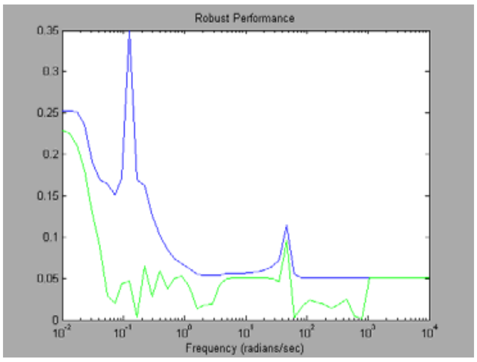


Fig. 18 Robust performance test of closed loop system with μ controller

VI. Conclusion

For maglev train system that mentioned in this paper with available dynamic equations and with structured uncertainty as uncertainty in model parameters, LQG and H_∞ controller don't satisfy the robustness specification for stability and performance, but μ controller attains suitable robustness in stability and performance at third iteration of D-K iteration procedure.

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