A Simulink simulation framework of a MagLev model

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Abstract: This paper presents a three-degree-of-freedom model of a section of the magnetically levitated train MagLev. The MagLev system dealt with in this article utilizes electromagnetic levitation. Each MagLev vehicle section is viewed as two separate parts, namely a body and a chassis, coupled by a set of springs and dampers. The MagLev model includes the propulsion, the guidance and the levitation systems. The equations of motion are developed. A Simulink simulation framework is implemented in order to study the interaction between the different systems and the dynamics of a MagLev vehicle. The simulation framework will eventually serve as a tool to assist the design and development of the MagLev system in the United States of America.

Keywords: Simulink, simulation framework, MagLev, magnetically levitated train, equations of motion

NOTATION

a	levitation electromagnet pole face length
a_s	guidance electromagnet pole face length
b	levitation electromagnetic circuit depth
b_s	guidance electromagnetic circuit depth
B_R	rotor magnetic field
B_S	stator travelling magnetic field
E_A	stator induced voltage
F_{Air}	aerodynamic drag
F_{CB_X}	chassis/body coupling force along the X axis
F_{CB_Y}	chassis/body coupling force along the Y axis
F_{CB_Z}	chassis/body coupling force along the Z axis
$F_{Following}$	coupling force with the following vehicle
F_{Guid}	lateral guidance force
$F_{Guid/Left}$	left side lateral guidance force
$F_{Guid/Right}$	right side lateral guidance force
F_{Lev}	levitation force
$F_{Preceding}$	coupling force with the preceding vehicle
F_{Prop}	propulsion force
gravity_X	gravity component along the X axis
gravity_Y	gravity component along the Y axis
gravity_Z	gravity component along the Z axis
h	levitation electromagnet window height
h_s	guidance electromagnet window height
I_A	armature (stator) current
I_e	levitation electromagnet excitation current
I_L	left side lateral current
I_R	right side lateral current
m	chassis or body mass
N	number of turns
p	X-component of the chassis or body angular velocity
P_{conv}	converted power
q	Y-component of the chassis or body angular velocity
r	Z-component of the chassis or body angular velocity
R_A	armature resistance
u	X-component of the chassis or body velocity
v	Y-component of the chassis or body velocity
$\boldsymbol{v}$	chassis or body velocity
V_x	forward vehicle speed
V_P	stator per-phase applied voltage
w	Z-component of the chassis or body velocity
w_l	levitation electromagnet window width
$\boldsymbol{W}$	chassis or body angular velocity
w_s	guidance electromagnet window width
X_S	synchronous reactance

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$y_L(t)$	left side lateral gap
$y_R(t)$	right side lateral gap
$z(t)$	levitation gap
μ	ferromagnetic material permeability
μ_0	levitation/lateral air gap permeability
ψ	phase angle between the armature current and induced voltage

1 INTRODUCTION

At the start of the twenty-first century, the air and ground transportation systems are experiencing a major crisis. The ever-increasing density of the traffic is alarming, and new alternatives must be found. The electromagnetic MagLev (magnetically levitated) system is a new solution. The MagLev provides a high-speed ground transportation capability based on contact-less levitation, guidance and the propulsion electromagnetic principle. The MagLev train is fast (speeds up to 500 km/h), highly safe, with a virtually zero probability of derailment (the vehicle wraps around the guideway), and environmentally friendly (no gas emissions, low noise emission, low land consumption).

This paper presents a large-scale model for the MagLev system. Previous research has been conducted in modelling the MagLev vehicle and various subsystems of the MagLev technology. Substantial work has been published on specific studies related to the electromagnetic MagLev technology. In reference [**1**] East and Hayes present a high-level overview of the MagLev system development status. A fair amount of work has also been carried out in understanding and analysing the interactions between the various electromagnetic circuits constituting the propulsion, the guidance and the levitation subsystems and optimizing this circuitry [**2**]. Vehicle/guideway interaction has also been addressed and modelled in references [**3**] and [**4**]. However, all the work mentioned above has only addressed a narrow and a specific element of the MagLev technology.

The work presented in this paper looks at the MagLev system as a whole. The MagLev system could be divided into three main categories:

(a) the MagLev vehicle comprised of one or several sections,
(b) the guideway infrastructure,
(c) the command and control centre (CCC).

The modelling has been performed in a hierarchical fashion, starting from the high-level elements of the system, and proceeding to finer details and modelling of each element. The work is essentially an effort in understanding the MagLev technology presented in references [**5**] and [**6**]. However, the focus is on the MagLev vehicle itself as it operates on a guideway segment. The outcome of the work is a suitable, and more importantly a readily expandable, simulation model of a section of the MagLev system.

2 THE MagLev SYSTEM

The MagLev system comprises the vehicle(s), the guideway and the CCC. The vehicle consists of one or multiple sections and each section is divided into a body and a chassis. The guideway is divided into segments and each of them is powered by an independent substation. For the sake of simplicity, the guideway is considered to be stiff, i.e. no deflection. This assumption obviously obviates the need to analyse the vehicle/guideway interaction. However, future work will include the vehicle/guideway interaction in the model. Such expansion of the model is simplified because of careful decomposition of the MagLev system model into separate modules. In fact, a strict correspondence is sought between a physical element of the system and the Simulink module. Each guideway segment has a specific profile, which is an important parameter in the analysis of the MagLev vehicle dynamic behaviour.

The CCC is a highly complex system that controls and monitors the locations and speeds of all operating MagLev vehicles. It ensures that, at any instant in time, all running vehicles have a clear path to their destinations and that their speeds conform to the operating specifications. In the present model, a simplified version of the command and control centre was used where, for each guideway position, the CCC sends a target speed to each running vehicle. For the sake of clarification, Fig. 1 provides a schematic representation of the MagLev vehicle components.

2.1 The levitation subsystem

The underlying principle behind the levitation subsystem is the instantaneous generation of an attractive force between an electromagnet and a ferromagnetic plate. The levitation subsystem model is derived from the work in references [**6**] and [**7**]. The levitation is modelled as an electromagnetic circuit depicted in Fig. 2 (a side view along the guideway and the vehicle chassis).

The system is essentially composed of an electromagnet, a ferromagnetic material (the guideway plate) and a gap controller. The electromagnet and the gap controller are onboard, and more specifically they are mounted on the vehicle chassis. The controller maintains a constant levitation gap. The current I_e flowing in the electromagnet coil also serves as the excitation current for the propulsion subsystem.

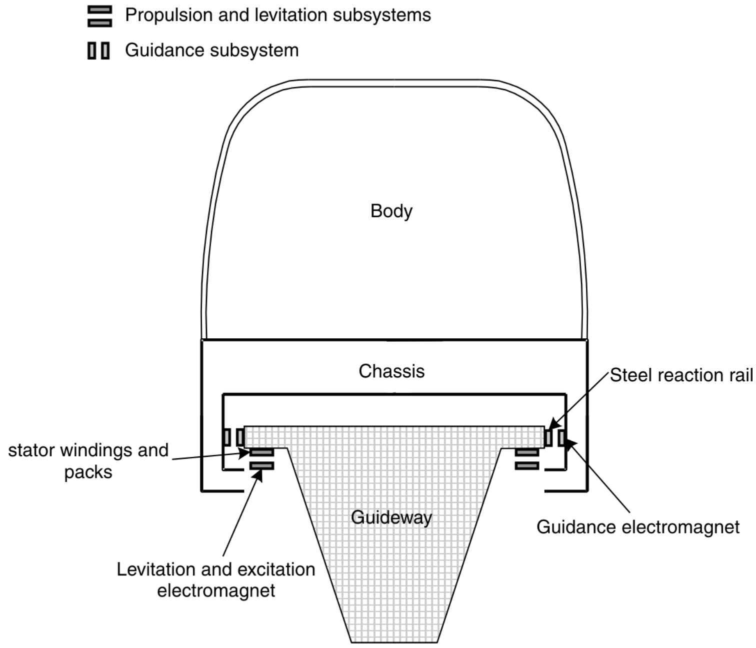

Fig. 1 The MagLev vehicle components

The levitation is on both sides of the vehicle. However, it is assumed that the same current I_e is flowing in both circuits (this implies that the air gap between the vehicle chassis and the guideway is the same on either side, assuming that the weight acting on the chassis is uniformly distributed). The levitation system can thus be modelled as a single circuit with the current I_e being the control measure. The levitation subsystem is independent of the guidance subsystem. The levitation force F_{Lev} generated by the levitation subsystem keeps the vehicle levitated at a certain gap distance from the guideway rail. F_{Lev} is controlled by varying the d.c.

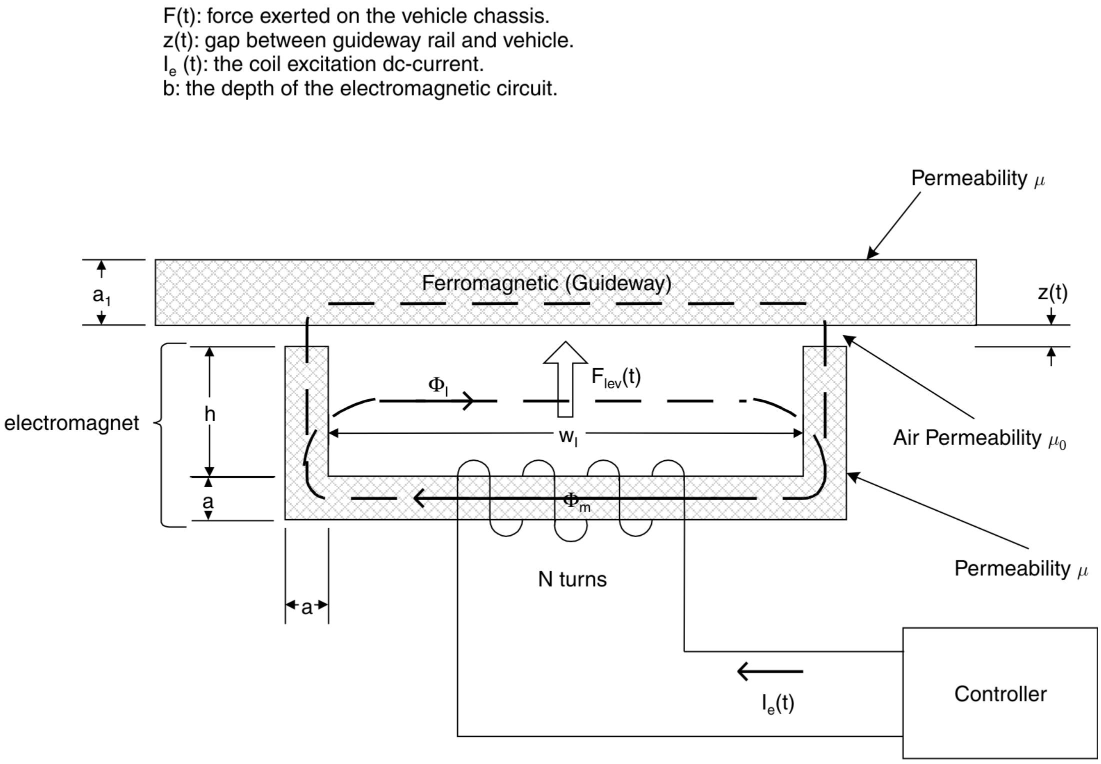

Fig. 2 The levitation circuit

current I_e. The equation describing F_{Lev} as a function of I_e and the vertical gap z is

$$F_{Lev}(t) = \frac{1}{2}\left(\frac{I_e^2}{l}\right)\frac{L}{(1 + z(t)/l)^2} \tag{1}$$

where

$$L = ab\mu_0 \frac{N^2}{2l} \qquad \text{and} \qquad l = \frac{\mu_0}{\mu}(w_l + h + 2a)$$

For further details on the derivation, refer to reference **[8]**.

2.2 The lateral guidance subsystem

The lateral guidance subsystem is based on the same electromagnetic circuit shown in Fig. 2. However, in this case, there are two independent electromagnetic circuits, each controlled by a different current, on each side of the vehicle chassis. The attractive guidance forces are generated through the interaction of the electromagnets and the lateral steel rail. Recall that the generated force acts only in one direction (i.e. attractive); therefore in order to command the MagLev vehicle to move on either side, it is necessary to install a system (such as the one shown in Fig. 2) on each side of the vehicle chassis. The lateral electromagnets and the gap controllers are also onboard. The lateral guidance forces keep the vehicle centred with respect to the guideway track while the vehicle is moving. Denote $F_{Guid/Left}$ as the force generated on the left side of the vehicle chassis and $F_{Guid/Right}$ as the force generated on the right side of the vehicle. The net lateral guidance force F_{Guid} is

$$\begin{aligned} F_{Guid}(t) &= F_{Guid/Right} - F_{Guid/Left} \\ &= \frac{L_s}{2l_s}\left[\frac{I_R^2}{(1 + y_R(t)/l_s)^2} - \frac{I_L^2}{(1 + y_L(t)/l_s)^2}\right] \end{aligned} \tag{2}$$

where

$$L_s = a_s b_s \mu_0 \frac{N^2}{2l_s} \qquad \text{and} \qquad l_s = \frac{\mu_0}{\mu}(w_s + h_s + 2a_s)$$

The various parameters have the same significance as the ones defined for the levitation system. The subscript ‘s’ denotes the ‘side’ and y_R and y_L are the right and left side gaps respectively.

2.3 The propulsion subsystem

Propulsion in the MagLev system is achieved by a linear synchronous motor (LSM). The linear synchronous motor comprises three-phase stator windings mounted on the underside of the guideway track and producing a travelling magnetic field B_S (its velocity being proportional to the frequency of the input signal) along the guideway. The second component of the LSM is the onboard excitation system. The excitation system made of the levitation electromagnets produces an excitation magnetic field B_R. Propulsion is achieved when the excitation magnetic field B_R synchronizes and locks to the travelling magnetic field B_S. As a consequence, the speed of the vehicle is proportional to the input frequency of the three-phase stator windings. The way the LSM works is analogous to an a.c. synchronous machine. The stator of the LSM comprises the three-phase stator winding fitted into stator pack slots. The rotor of the LSM is made up of the onboard electromagnets utilized for the levitation system. From a force point of view, the excitation current I_e and the stator magnetic field B_S produce a force that pulls the vehicle forward. This propulsion force is controlled by changing the magnitude and the phase angle of the armature, i.e. stator, current (equivalently the input voltage). The excitation current I_e is not used for propulsion thrust control purposes and whenever I_e changes, I_A has to be adjusted in order to counterbalance the effect of I_e. The LSM can function in two modes, either as a motor consuming power or as a generator producing power. The mode of operation depends on the load experienced by the MagLev vehicle. In general, if the vehicle is decelerating, the LSM will operate as a generator. If the vehicle is accelerating, the LSM will operate as a motor. It is assumed that the rotor and stator are always synchronized, i.e. the vehicle’s speed corresponds to the frequency applied to the stator. The per-phase equivalent circuit of a synchronous machine is given in reference **[9]**. A set of equations can be derived from the equivalent circuit and various electrical measures can be deduced, such as the armature current I_A, the converted active power P_{conv}, the consumed reactive power, the phase angle between the applied voltage V_P and I_A, etc. It is further assumed that V_P varies linearly with the input frequency **[9]**. The induced voltage E_A is a function of the excitation current I_e and the input frequency (equivalently the vehicle speed).

The converted active power is of particular interest. In fact, P_{conv} and the propulsion force F_{Prop} are related in the following way:

$$F_{Prop} = \frac{P_{conv}}{V_x} \tag{3}$$

where V_x is the forward vehicle speed. P_{conv} can be derived and computed from the per-phase equivalent circuit:

$$P_{conv} = 3E_A I_A \cos\psi \tag{4}$$

where E_A and I_A are the root mean squares of the induced voltage and the armature current respectively and ψ is the phase angle between E_A and I_A.

In the present modelling, and without loss of generality, a MagLev vehicle section is considered to be comprised of one levitation electromagnet, one lateral guidance electromagnet on each side of the vehicle and a three-phase a.c. synchronous machine. In developing the propulsion, the guidance and the levitation subsystems, it is assumed that the electromagnetic circuits never reached saturation and that the flux varied linearly with the applied current (in other words, the material permeability is assumed to be constant).

3 THE MagLev MODEL

A MagLev vehicle section is comprised of two elements: the body and the chassis. These two elements are coupled via springs and dampers. Neighbouring sections belonging to the same vehicle are also coupled via a spring and a damper. Each of the vehicle section bodies and the vehicle section chassis are viewed as a point mass. The equations of motion are developed in the body coordinate system. The chassis and the body each have their own body coordinate system. The flat earth is considered to be an inertial frame and the coordinate transformations from one coordinate system to another are expressed in terms of roll, pitch and yaw. The guideway track has its own coordinate frame representing the curving profile of the guideway with respect to the flat earth. It is assumed that at all times the unit vectors of the guideway coordinate system (characterized by roll, pitch and yaw measures) coincide with those of the body coordinate systems of the body and the chassis. This implies that, since the guideway profile is known, the roll, pitch and yaw of both the vehicle body's body coordinate systems of the vehicle body and the vehicle chassis are known at any guideway position.

The model comprises six degrees of freedom: three translational degrees (X, Y and Z body axes) for the body and three translational degrees for the chassis. The convention used for the body coordinate system is as follows:

1. The X axis is pointing ahead in the direction of travel of the MagLev vehicle; it is tangent to the guideway surface.
2. The Y axis is pointing from the right to the left side of the vehicle; it is also tangent to the guideway surface.
3. The Z axis is normal to the guideway surface and forms a right-handed coordinate system with the X and Y axes.

Denote $\boldsymbol{v}_{\mathrm{B}} = (u, v, w)_{\mathrm{B}}$ and $\boldsymbol{w}_{\mathrm{B}} = (p, q, r)_{\mathrm{B}}$ respectively as the velocity and the angular velocity of the vehicle chassis or body. The subscript 'B' indicates that the vector is expressed in the body coordinate system. The angular velocity is due to the fact that the vehicle is constrained to a curved guideway. In fact, the components p, q and r are readily derived from the forward (along the X axis) speed and the curvature of the guideway, which is given as a parameter.

Expressing Newton's second law in body coordinate systems, the vehicle chassis or body inertial acceleration is

$$(\dot{\boldsymbol{v}})_{\mathrm{B}} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \frac{1}{m}\sum(\boldsymbol{F})_{\mathrm{B}} - (\boldsymbol{w})_{\mathrm{B}} \times (\boldsymbol{v})_{\mathrm{B}}$$

$$= \frac{1}{m}\begin{pmatrix} F_X \\ F_Y \\ F_Z \end{pmatrix} - \begin{pmatrix} qw - rv \\ ru - pw \\ pv - qu \end{pmatrix} \quad (5)$$

where $(\boldsymbol{F})_{\mathrm{B}} = (F_X, F_Y, F_Z)_{\mathrm{B}}$ is the vector force acting on the vehicle chassis or body and m is the mass.

All the forces are expressed in the body coordinate system. In the following sections, a list of the forces acting on the chassis and the body is provided.

3.1 Forces acting on the chassis

The propulsion, guidance and levitation forces all act on the vehicle chassis. The sum of the forces acting along the X axis is

$$F_X = F_{\mathrm{Prop}} + \mathrm{gravity}_X + F_{\mathrm{CB\_}X} - F_{\mathrm{Preceding}} - F_{\mathrm{Following}} - F_{\mathrm{Air}} \quad (6)$$

where

F_{Prop} = propulsion force generated by the LSM [equations (3) and (4)]

$\mathrm{gravity}_X$ = X-component of the force due to gravity, and depends on the chassis weight and the vehicle (or guideway) roll, pitch and yaw*

$F_{\mathrm{CB\_}X}$ = force due to the coupling between the chassis and the body along the X axis; the coupling is modelled as a spring and a damper

$F_{\mathrm{Preceding}}$ = coupling force between the vehicle and the vehicle preceding it; the coupling is modelled as a spring and a damper

$F_{\mathrm{Following}}$ = coupling force between the vehicle and the vehicle following it; the coupling is modelled as a spring and a damper and between neighbouring vehicle sections is only along the X axis

F_{Air} = aerodynamic drag, which varies as the square of the forward speed (other relationships could be applied)

* This is the only occasion where the transformation matrix is used to express the gravitational acceleration vector in the body coordinate.

The sum of the forces acting along the Y axis is

$$F_Y = \text{gravity}_Y + F_{\text{Guid}} + F_{\text{CB\_}Y} \tag{7}$$

where

gravity$_Y$ = Y-component of the force due to gravity and depends on the chassis weight and the vehicle roll, pitch and yaw
F_{Guid} = lateral guidance force [equation (2)]
$F_{\text{CB\_}Y}$ = force due to the coupling between the chassis and the body along the Y axis; the coupling is modelled as a spring and a damper

The sum of the forces acting along the Z axis is

$$F_Z = F_{\text{Lev}} + \text{gravity}_Z + F_{\text{CB\_}Z} \tag{8}$$

where

F_{Lev} = levitation force [equation (1)]
gravity$_Z$ = Z-component of the force due to gravity and depends on the chassis weight and the vehicle roll, pitch and yaw
$F_{\text{CB\_}Z}$ = force due to the coupling between the chassis and the body along the Z axis; the coupling is modelled as a spring and a damper

3.2 Forces acting on the body

The forces acting on the vehicle body are basically the aerodynamic drag, gravity and the forces due to the coupling of the chassis and the body. The sum of the forces acting along the X axis is

$$F_X = \text{gravity}_X - F_{\text{CB\_}X} - F_{\text{Air}} \tag{9}$$

where all the measures have the same definition as in the previous section. The sum of the forces acting along the Y axis is

$$F_Y = \text{gravity}_Y - F_{\text{CB\_}Y} \tag{10}$$

The sum of the forces acting along the Z axis is

$$F_Z = \text{gravity}_Z - F_{\text{CB\_}Z} \tag{11}$$

Having derived all the forces acting on the vehicle, the MagLev model will then be simulated in order to predict the vehicle section's (body and chassis) position, speed and acceleration at any instant in time.

4 THE SIMULATION MODEL

The MagLev system is simulated using the Matlab/Simulink environment. Simulink provides an easy and rapid simulation tool. The MagLev system simulation model is built in a hierarchical fashion. The highest level in the hierarchy is composed of a guideway segment, a vehicle and the CCC. (In a more generalized system, the highest level will contain the CCC, multiple segments, and multiple vehicles.) This level is depicted in Fig. 3. Each of the three elements contained in the highest level is then treated as a separate subsystem (or module).

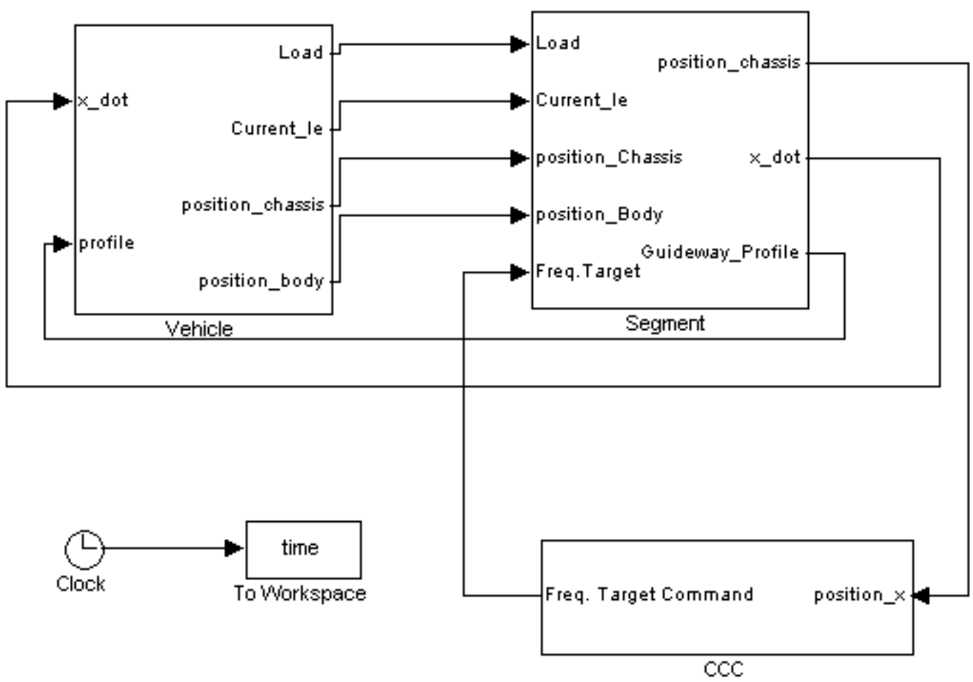

Fig. 3 The MagLev system (the highest level)

Each of the subsystems is further decomposed into multiple modules. For instance, the vehicle subsystem is further decomposed into vehicle sections to form the second hierarchical level.

Once the MagLev modules have been determined, the input and output signals to each module are identified. The input and output signals could be electrical (e.g. current), mechanical (e.g. force) or informational (e.g. vehicle speed) signals. The final step is to interconnect all the modules for each hierarchical level. During the simulation model development, modules were tested separately to verify their functional behaviour. Because of this hierarchical organization and module decomposition, adding new functionality and subsystems to the simulation framework is simplified.

The ultimate goal of the simulation is to run multiple vehicles, with each vehicle being composed of multiple sections, on a complete track made of several segments. Each vehicle section could be running on any guideway segment. For this reason, from a simulation point of view, each vehicle section module should be connected to all guideway segment modules. This clearly leads to an explosion in the number of interconnections with the number of vehicle section and guideway segment modules. This problem is solved by having each vehicle section connected to a single 'virtual' guideway segment. A guideway segment has its profile as its unique attribute. While a vehicle section is physically moving from one guideway segment to another, the profile of the 'virtual' guideway segment will have to be updated to reflect the real physical profile of the guideway segment on which the vehicle section is running. Keeping track and updating the 'virtual' guideway segment profile is performed by the CCC (this is obviously a simulation artefact assigned to the CCC). Each guideway segment has its associated database where information about its profile (in terms of roll, pitch and yaw) is kept. For the sake of illustration, the vehicle section chassis module is shown in Fig. 4. This diagram basically models the dynamic model of the vehicle section chassis presented earlier.

4.1 The input and output parameters

The input parameters are divided into two sets. The first set of parameters refers to all parameter values related to the vehicle and its levitation, guidance and propulsion systems. The values of these parameters are stored in an initialization Matlab file. The second set of input parameters concerns the guideway segment profiles.

A multitude of output parameters are generated by the simulation. All of the parameters could be plotted as a function of time or as a function of the guideway position (the X axis). The output parameters include the vehicle's roll, pitch and yaw, the chassis and body position, speed and acceleration, the lateral guidance force, the levitation and propulsion forces, the excitation current, the vertical and horizontal gaps, the consumed power, etc.

4.2 The use case

The use case simulation consists of running a one-section vehicle on a single guideway segment. The real data for the MagLev parameters were not available, and none of the input parameters chosen (and hence the output parameters) reflected any real physical value. In fact, most of the input parameters were chosen to be values between 0 and 1. However, once the complete MagLev system parameters become available, the framework will permit simple inclusion of this information and the simulation will reflect the behaviour of the actual system.

The following is a collection of some plots generated by the simulation. Figure 5 shows the vehicle roll, pitch and yaw as a function of guideway position. This illustrates how the guideway profile is changing.

Figure 6 shows the active converted power P_{conv} provided by the linear synchronous motor. At 10 s, P_{conv} reaches a 0.4 steady state value before it drops suddenly and becomes negative at about 34 s. The reason for P_{conv} being negative is that at that point the linear synchronous motor is operating as a generator because the vehicle is going downhill (see the pitch variation in Fig. 5).

Finally, Fig. 7 gives an idea of how the lateral guidance force and the left side gap vary. The vehicle starts off with a 0.02 m left side gap and stabilizes at its set point gap, i.e. 0.015 m, after 10 s. At around 17 s, the side gap is perturbed because the vehicle starts suddenly leaning to the right (see the roll curve in Fig. 5). The lateral guidance force increases to bring back the vehicle to its track-centred position before it stabilizes to a non-zero value (in fact, the vehicle remains on a banked guideway).

5 CONCLUSION

The work presented in this paper was an effort in studying the MagLev system as a whole and identifying the various parts constituting such a system. The focus was on modelling and simulating the dynamic behaviour of the MagLev vehicle comprised of the support (or levitation), guidance and propulsion systems. The support and the guidance systems were both modelled as an electromagnetic circuit consisting of an electromagnet, a ferromagnetic guideway plate and an air gap. The vertical and the lateral gaps were controlled by varying the currents flowing into these circuits. The propulsion system, made of a linear synchronous motor, was modelled as an a.c. synchronous machine and all

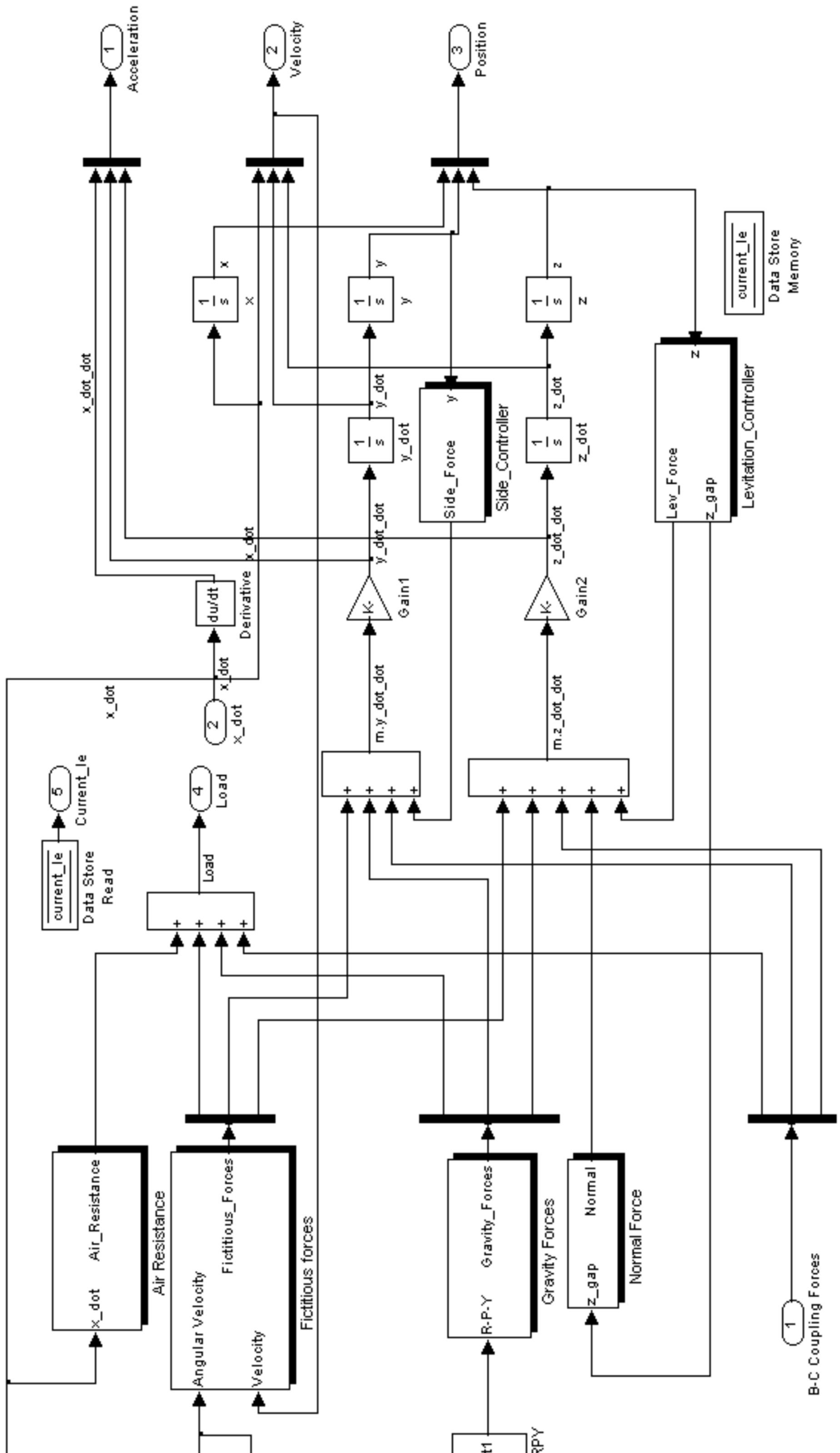

Fig. 4 The vehicle section chassis

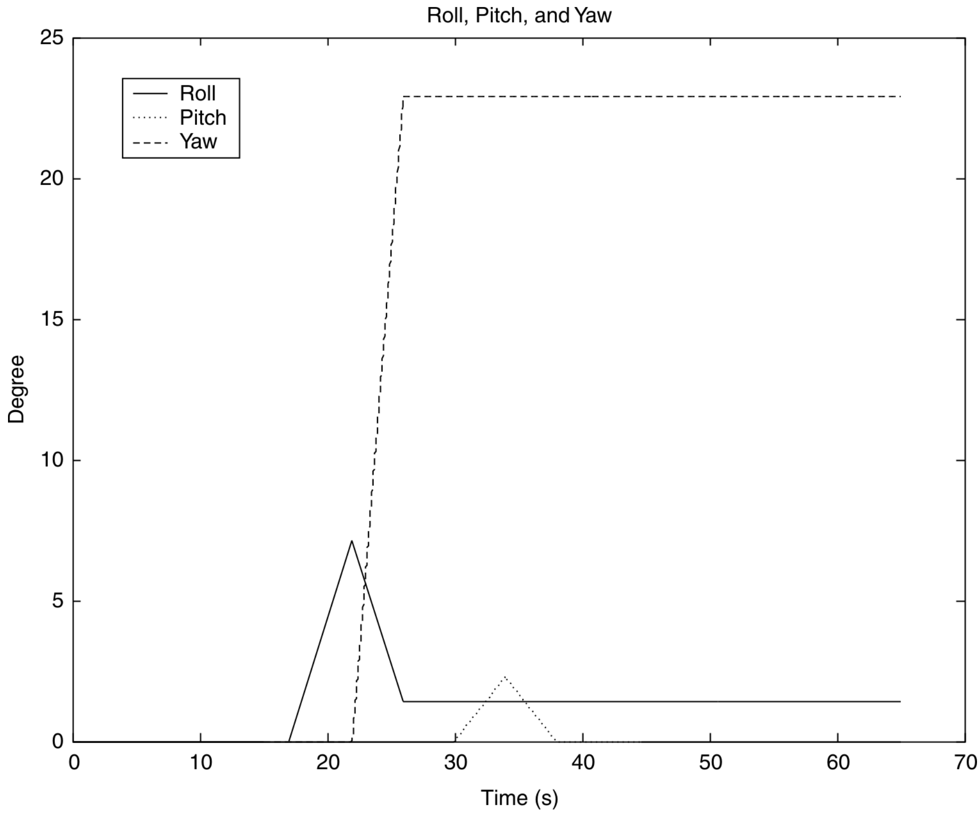

Fig. 5 Roll, pitch and yaw

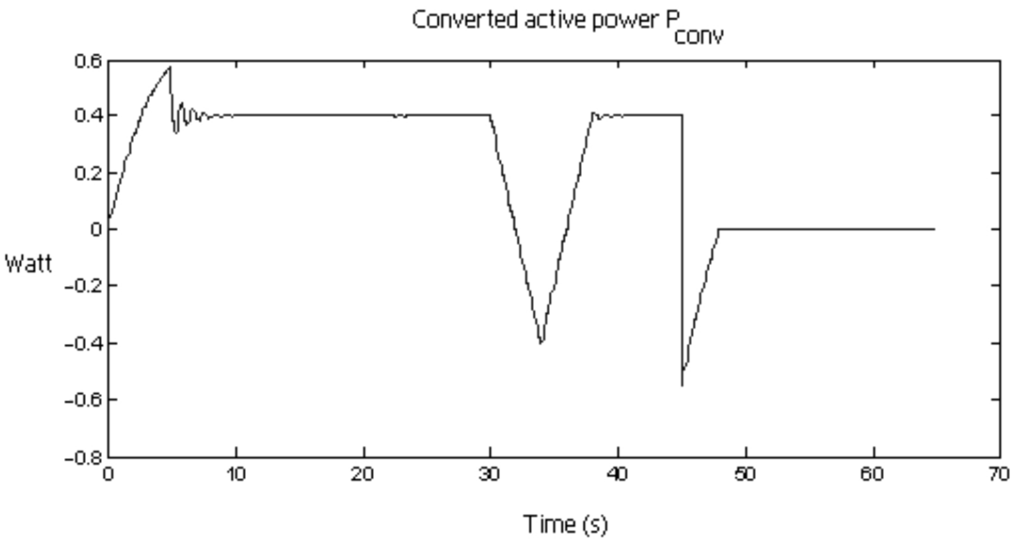

Fig. 6 Converted active power P_{conv}

the electrical measures were derived based on the per-phase equivalent circuit of an a.c. synchronous machine.

The MagLev model simulation was implemented using Matlab/Simulink. The Simulink simulation framework can be used as a design and assessment tool in the continued development of the MagLev system. Several analyses could be carried out, such as a sensitivity analysis, i.e. varying the various input parameters and verifying the vehicle response in terms of ride quality for instance.

Future MagLev work will include the expansion of the model to account for elastic guideways and model the guideway as a Bernoulli–Euler beam subject to the vehicle weight. A more rigorous study on eddy current effects should also be conducted. Finally, implementa-

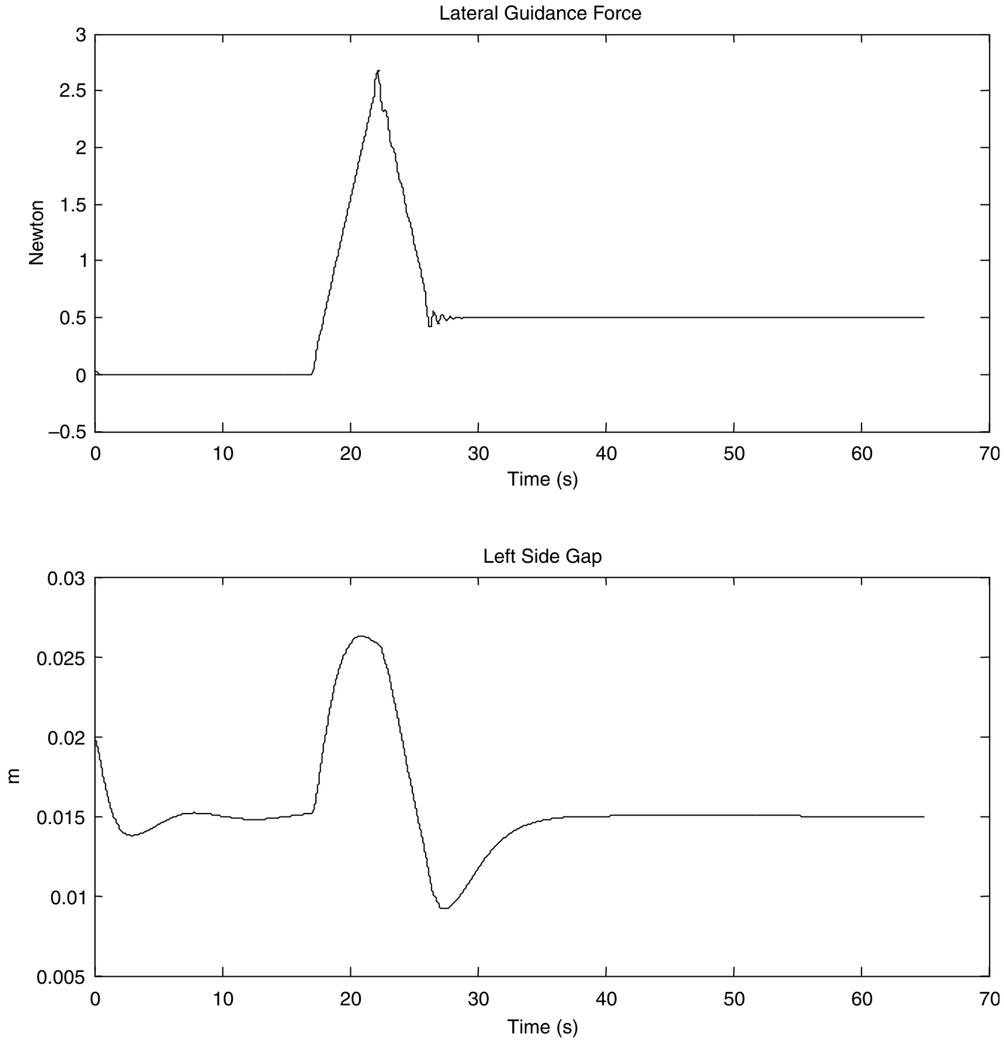

Fig. 7 Lateral guidance force and left side gap

tion of the emergency breaking system (used in the event of a power failure) will also be carried out.

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