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Vibration analysis of continuous maglev guideways with a moving distributed load model

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Abstract. A model of moving distributed load with a constant speed is established for vertical vibration analysis of a continuous guideway in maglev transportation system. The guideway is considered as a continuous structural system and the action of maglev vehicles on guideways is considered as a moving distributed load. Vibration of the continuous guideways used in Shanghai maglev line is analyzed with this model. The factors that affect the vibration of the guideways, such as speeds, guideway's spans, frequency and damping, are discussed.

1. Introduction

The first commercial maglev transportation system in the world was completed in Shanghai in 2002. It will improve the development of high-speed maglev system greatly. But the total cost of the system is a bit high compared with other transportation system. One of the main reasons for this is that the demand for the rigidity of guideways is very strict in order to limit its vibration. And the cost of guideways has occupied 60%-70% in the whole investment of the system. So it becomes more and more necessary to study the vibration of the guideway and make it optimized from the point of view of vibration.

When studying the vibration of the maglev guidways, some researchers directly transplant the analytical models and methods which are matured in the traditional rolling train system into the analysis of the maglev system. This brings great convenience to the studying, but will cause problems at the same time. In studying the vibration of the rolling train the bridge has been considered as discrete finite elements and the interaction between the vehicles and bridge has been simplified as single or several concentrated forces. In the maglev system it is evident that the interaction between the vehicles and guideways is continuous. The guidways' mass and rigidity are evident continuous, too. It will inevitably cause some system errors to dividing them into discrete finite elements

In this paper the continuous structure model and continuous distribution of the interaction between the vehicles and the guideways are adopted. The model of moving distributed load with constant speed is established for vertical vibration analysis of double-span continuous guideway in maglev system. With this model the vibration characteristics of the double-span continuous guideways is analyzed. The whole studying is founded on the reality of Shanghai maglev line, so the result has practical value.

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Comparing with some more complicated methods [1,2] the model established in this paper need less detail information of the maglev vehicles and can give the guideways' response rapidly, so it can be conveniently used in comparison and selection of the cross-section parameters in guideway design.

2. Model of moving distributed load with constant speed

For the beam with constant cross section which subjected to distributed dynamic load, supposing that the rigidity of the cross section is EI, the span is l, the mass of unit length is m and the distributed load acting on it is $q\left(x,t\right)$, according to the theory of structural dynamic mechanics the vibration equation of the beam is [3]

$$EI\frac{\partial^4 y}{\partial x^4}+c_s I\frac{\partial^5 y}{\partial t\partial x^4}+m\frac{\partial^2 y}{\partial t^2}+C(x)\frac{\partial y}{\partial t}=q(x,t) \tag{1}$$

The displacement of the guideway $y\left(x,t\right)$ can be considered as the superposition of all the modes,

$$y(x,t)=\sum_{i=1}^{\infty}A_i(t)\phi_i(x) \tag{2}$$

Substituting equation (2) into equation (1) and adopting the assumption of proportional damping, the fundamental equation of every mode can be obtained

$$\ddot{A}_i(t)+2\xi_i\omega_i\dot{A}_i(t)+\omega_i^2 A_i(t)=f_i(t) \tag{3}$$

Where

$$\left.\begin{aligned}
&f_i(t)=\frac{\bar{P}_i(t)}{\bar{m}_i}\\
&\bar{m}_i=m\int_0^l\phi_i^2(x)dx\\
&\bar{P}_i(t)=\int_0^l\phi_i(x)q(x,t)dx
\end{aligned}\right\}(i=1,2,...,\infty) \tag{4}$$

The free vibration property of double span continuous beams is derived with analytic method. There are two types of mode. Odd modes are antisymmetric and even modes are symmetric. Supposing that the single span is l, the frequencies of the double-span beam are

$$\omega_i=\begin{cases}\left(\dfrac{i+1}{2l}\pi\right)^2\left(\dfrac{EI}{m}\right)^{1/2} & i=1,3,5\cdots\\[2ex] \left(\dfrac{(0.25+i/2)\pi}{l}\right)^2\left(\dfrac{EI}{m}\right)^{1/2} & i=2,4,6\cdots\end{cases} \tag{5}$$

The functions of the mode shapes are

$$\phi_i(x)=\begin{cases}\sin\dfrac{i+1}{2l}\pi x \quad x\in[0,2l],\ \text{when } i=1,3,5,\cdots\\[2ex] \left.\begin{aligned}&\dfrac{\sin\lambda_i x}{\sin\lambda_i l}-\dfrac{\text{sh}\,\lambda_i x}{\text{sh}\,\lambda_i l} && x\in\left[0,l\right]\\ &\cos\lambda_i(x-l)-\text{ch}\,\lambda_i(x-l)\\ &-\text{ctg}\,\lambda_i l[\sin\lambda_i(x-l)-\text{sh}\,\lambda_i(x-l)] && x\in\left[l,2l\right]\end{aligned}\right\}\text{when } i=2,4,6,\cdots\end{cases} \tag{6}$$

Considering the vehicles used in Shanghai maglev line, the length of the vehicles is always greater than the length of the guideways. Supposing that the length of vehicle is L_0, the weight per unit length is p, the speed of the vehicles is v and the starting time is $t=0$, several key time points can be obtained. They are the time when the head of the vehicle reaches the middle support t_1, the head of the vehicle reaches the right support t_2, the tail of vehicle reaches the left support t_3, the tail of vehicles reaches middle support t_4 and the tail of vehicles reaches the right support t_5 respectively. Their value are $t_1 = l/v$, $t_2 = 2l/v$, $t_3 = L_0/v$, $t_4 = (l + L_0)/v$ and $t_5 = (2l + L_0)/v$.

Considering the difference expression of mode function between the right and the left span the whole process of the vehicle passing the guideway has to be divided into six intervals on the basis of the time points mentioned above. Supposing that there are loadings where $x \in [a,b]$ the dynamical loading in the six intervals can be expressed as

$$q(x,t) = \begin{cases} p & x \in [a,b] \\ 0 & x \notin [a,b] \end{cases} \tag{7}$$

The generalized mass can be obtained from equation (4) and equation (7), that is

$$\tilde{m}_i = \begin{cases} ml & i = 1,3,5,\cdots \\ 2ml & i = 2,4,6,\cdots \end{cases} \tag{8}$$

The generalized loads of antisymmetric modes (when $i = 1,3,5,\cdots$) are

$$\tilde{P}_i(t) = \begin{cases} \dfrac{2pl}{(i+1)\pi}(1-\cos\dfrac{i+1}{2l}\pi vt) & t \in [0,t_2] \\ 0 & t \in [t_2,t_3] \cup [t_5,\infty] \\ \dfrac{2pl}{(i+1)\pi}[\cos\dfrac{i+1}{2l}\pi v(t-t_3)-1] & t \in [t_3,t_5] \end{cases} \tag{9}$$

The generalized loads of symmetric modes (when $i = 2,4,6,\cdots$) are

$$\tilde{P}_i(t) = \begin{cases} \dfrac{p(1-\cos\lambda_i vt)}{\lambda_i \sin\lambda_i l} + \dfrac{p(1-\mathrm{ch}\lambda_i vt)}{\lambda_i \mathrm{sh}\lambda_i l} & t \in [0,t_1] \\ \dfrac{2p}{\lambda_i}\left[(\dfrac{1-\cos\lambda_i l}{\sin\lambda_i l} + \dfrac{1-\mathrm{ch}\lambda_i l}{\mathrm{sh}\lambda_i l})\right] - \dfrac{p}{\lambda_i}\left[\dfrac{1-\cos\lambda_i(2l-vt)}{\sin\lambda_i l} + \dfrac{1-\mathrm{ch}\lambda_i(2l-vt)}{\mathrm{sh}\lambda_i l}\right] & t \in [t_1,t_2] \\ \dfrac{2p}{\lambda_i}(\dfrac{1-\cos\lambda_i l}{\sin\lambda_i l} + \dfrac{1-ch\lambda_i l}{sh\lambda_i l}) & t \in [t_2,t_3] \\ \dfrac{p}{\lambda_i}[\dfrac{1+\cos\lambda_i(vt-L_0)-2\cos\lambda_i l}{\sin\lambda_i l} + \dfrac{1+ch\lambda_i(vt-L_0)-2\mathrm{ch}\lambda_i l}{sh\lambda_i l}] & t \in [t_3,t_4] \\ \dfrac{p}{\lambda_i}[\dfrac{1-\cos\lambda_i\left(2l+L_0-vt\right)}{\sin\lambda_i l} + \dfrac{1-\mathrm{ch}\lambda_i\left(2l+L_0-vt\right)}{\mathrm{sh}\lambda_i l}] & t \in [t_4,t_5] \\ 0 & t > t_5 \end{cases} \tag{10}$$

By substituting equation (8), (9) and (10) into equation (4) and (3) the expression of the vibration differential equation of every mode can be obtained. Equation (3) can be solved by using the Duhamel integral. Then the response of the beam can be obtained by superposing every mode.

3. Vibration analysis of the continuous guideways of shanghai maglev line

The vibration performance of the double-span continuous guideways used in Shanghai maglev line is studied.

The vehicle can include 3, 5 and 8 sections. Figure 1 and 2 shows the displacement and acceleration of the middle point in the left span when the vehicle passes in the design speed, 430km/h.

Where curve *a*, *b* and *c* are the responses of the vehicle with 3 sections, 5 sections and 8 sections respectively. The results of other speeds are similar to this. It can be seen that the graphs of different groups are coincident during the first two and last three intervals. Meanwhile there are differences during the 3rd interval while the guideway is entirely loaded. The feature of the vibration performance of the guideway is similar when different vehicles' organizing of groups passes the guideway. So it can be concluded that different groups of the vehicle have little influence on the vibration performance of the guideway.

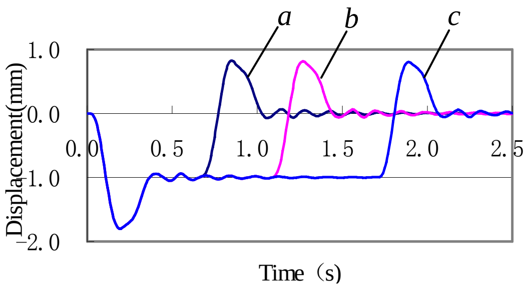

Figure 1. The displacement response of midspan of different groups.

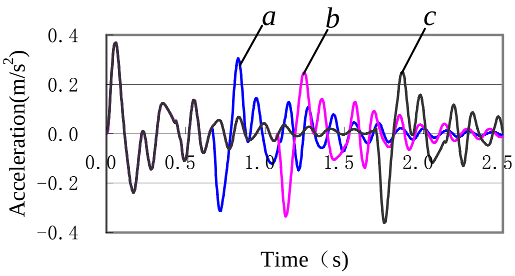

Figure 2. The acceleration response of midspan of different groups.

The vibration performance with the speeds from 100km/h to 600km/h is analyzed. The responses of displacement and acceleration in the middle of the left span with three typical speeds are shown in figure 3 and figure 4. In the figures curve *a*, *b* and *c* are the responses in the speeds of 100km/h, 430km/h and 600km/h respectively. The impact coefficients of different speeds are shown in figure 5.

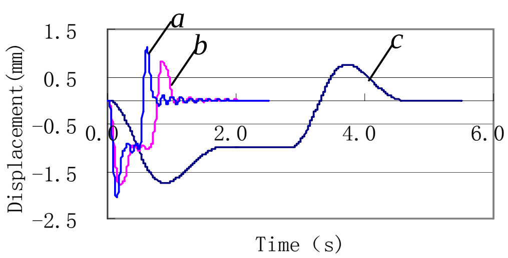

Figure 3. Displacement response under different speeds.

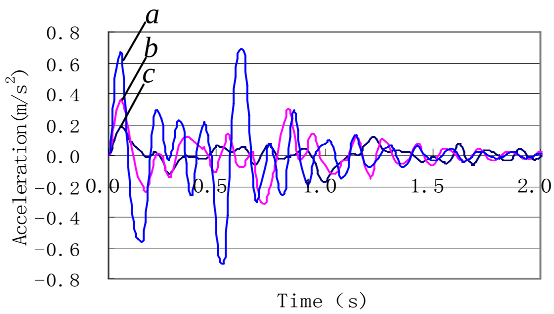

Figure 4. Acceleration response under different speeds.

It can be seen that the change of vehicle speed influences the vibration response of the guideway significantly. The impact coefficient increases with the vehicle speed. There is a valley when the speed is about 400km/h which is near the design speed, indicating that the current guideway works well in regards to dynamics. There is no extreme maximum point when the speed is below 600km/h, indicating that there is no critical speed when the speed doesn't exceed 600km/h.

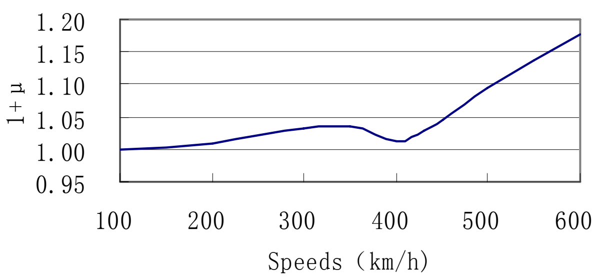

Figure 5. The impact coefficients under different speeds.

In order to discuss the possibility of optimizing the current guideways the dynamic performance of guideways with different spans is studied. The vibration response of different spans with design speed is shown in figure 6 and figure 7. In the figures curve *a*, *b*， *c* and *d* are the responses when the span are 21.672m, 24.768m, 27.864m and 30.960m respectively. Where 24.768m is the span used in Shanghai maglev line. The relation between impact coefficient and span is shown in figure 8. The relation between the first frequency of the guideway and impact coefficient is shown in figure 9.

It can be seen from the results that under design speed the absolute value of the vibration response of the guideway increases when the span increasing. So does the coefficient of impact. It increases slowly when the length of the span is less than 25m. When the length is greater than 25m, the increase becomes steep. However, there is no extreme maximum point, which means that there is no critical span within the considered spans. Even if the span increases to 30.960m the coefficient of impact is only about 1.13 and the maximum displacement is still no more than $l/6400$, where l is the length of single span. The maximum acceleration is no more than 0.06g, where g is the gravitational acceleration. So it can be concluded that there are some margin to extend the guideway span with the current rigidity of the cross section. From the relationship of the first frequency and coefficient of impact it can be found that when the frequency is above 7Hz the coefficient of impact is not sensitive to the change of frequency. When the frequency is below 7Hz the impact coefficient increases with the decrease in frequency. However, the increase is smooth and no extreme is found within the studied range of span.

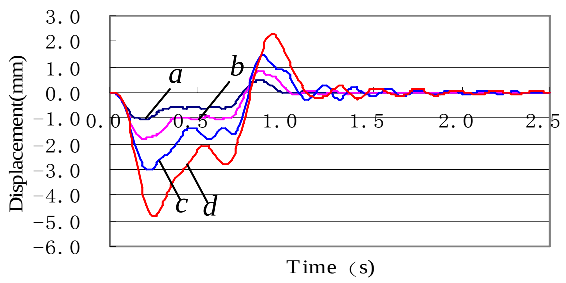

Figure 6. Displacement response of different spans.

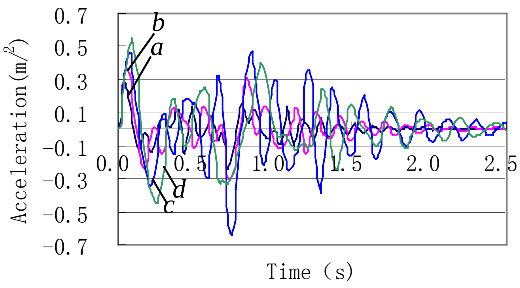

Figure 7. Acceleration response of different spans.

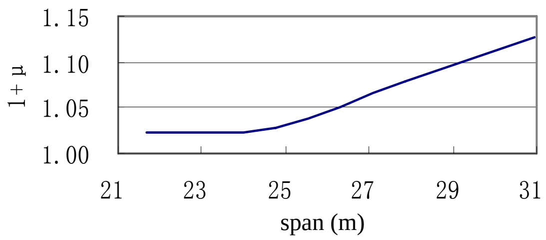

Figure 8. The impact coefficients of different span.

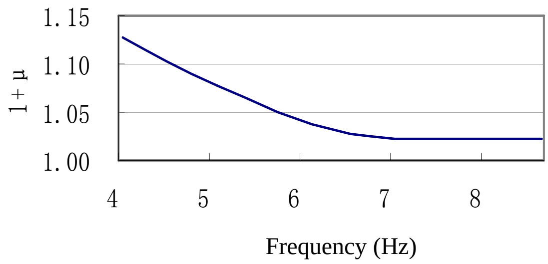

Figure 9. The impact coefficients of different frequency.

The impact coefficient with different damping ratio is shown in figure 10. It can be seen that the dynamic displacement of the guideway decreases when damping ratio increases.

The damping ratio of the maglev guideway is a very complicated factor. In the design of the guideway, it is difficult to control damping ratio only by the internal damping of the material and the external damping of the medium. At the same time, due to the strict limit on the deformation of the supports, it is also difficult to get satisfactory damping

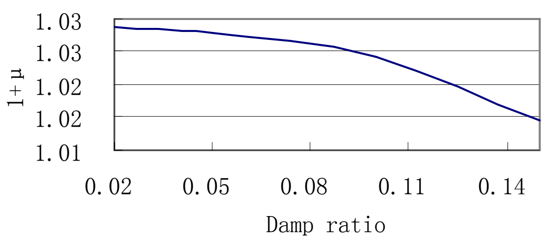

Figure 10. The impact coefficients of different damp ratio.

ratio by using the damping of the supports. It seems that the only feasible method to reduce the vibration response by increasing the damping is installing extra damping devices or filling damping material into the cavum of the guideway.

4. Conclusions

In this paper the model of moving distributed load with constant speed is established for vertical vibration analysis of the continuous maglev guideway. The calculation program is developed. This model need less detail information of the maglev vehicles and can give responses of the guideways rapidly, so it can be conveniently used in comparison and selection of the cross-section parameters in guideway design. With this model and its program the vibration performance of the guideways used in Shanghai maglev line is studied. The main conclusion is as following.

(1) Different groups of vehicle sections don’t influence the vibration performance of the guideway evidently.

(2) The vibration response of the guideway increases with the vehicle speed. The guideway works well under the design speed of the vehicles. There is no critical speed when the speed of the vehicle is no more than 600km/h.

(3) The impact coefficient increase when the span increases and frequency decreases. There is not critical span and critical frequency in the studied scope, which means that there is some margin to extend the span while keeping the current rigidity of cross section.

(4) The increase of the damp ratio can reduce the vibration response of the guideway. The feasible method should be to install extra damper or fill damping material into the cavum of the guideway.

By far there are not so many research results that are meaningful enough in the point of engineering technology based on the vibration of the maglev guideway. In order to prompt the development of maglev transportation system, profound and comprehensive research on this field should be emphasized.

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