

Controller design and test results for a four axis HTS coil based Maglev system [☆]

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Received 4 October 2006; accepted 11 January 2007

Available online 23 January 2007

Abstract

Controller design and experimental results are reported in this paper for a four axis high temperature superconductivity (HTS) coil based electromagnetic levitation (Maglev) system. The HTS coils are made of Bi2223/Ag multifilamentary tapes. It has been experimentally proved that the designed controller works satisfactorily, although the physical parameters of a HTS coil based electromagnet (HTSEM) vary significantly with the frequency of the input voltage. A performance comparison has also been made between the classical lead-lag compensator and the modern \mathcal{H}_∞ loop-shaping controller. It becomes clear that robust control theories are capable of providing a controller with better performances, which is in a good agreement with numerical simulations. Moreover, it implies that the particular parameter variation characteristics can be simply dealt with by the available robust control theories that are naturally existent in a HTSEM.

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PACS: 84.71.Ba; 85.70.Rp; 85.70.Ay; 84.71.Mn

Keywords: Frequency dependent parameter; High temperature superconductivity; Magnetic levitation; Robust control

1. Introduction

Magnetic levitation is an attractive technology, due to its amazing ability in reducing mechanical frictions and system maintenance cost. Among its various industrial applications, magnetically suspended transportation seems most spectacular [1,2,16,18]. The available magnetic levitation methods can be principally divided into two categories.

One of them is usually called electromagnetic levitation system (EMS) in which attracting electromagnetic forces are utilized to suspend a vehicle. Another one is usually called electrodynamic levitation system (EDS) in which large superconductivity magnets are employed to produce repulsive forces in order to suspend a vehicle. While the EMS method has advantages in high stiffness, low magnetic field leakage, high riding comfortability, etc., it suffers from high energy consumption, small realizable working gap, large system weight penalty, etc. On the other hand, the EDS method has advantages in low energy consumption, large realizable working gap, etc., but it suffers from large magnetic field leakage, low stiffness, uncontrollable riding comfortability, etc. [9,11,13].

A possible approach in combining together the advantages of the EDS method and the EMS method is to employ high temperature superconductivity (HTS) coils in a magnetic suspension system. With significant recent advancements in HTS coil production technologies, the possibility of

[☆] This research was supported in part by the National Natural Science Foundation of China under Grant 60574008 and 60625305, the Trans-Century Training Programme Foundation for the Talents by the Ministry of Education, the Specialized Research Fund for the Doctoral Program of Higher Education, P.R.C., under Grant 20050003096, and Tsinghua University under Grant JC2002025 and JC2003058. Helps from the applied superconductivity research center, Tsinghua University, Beijing, PR China, are also highly appreciated.

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realizing this magnetic suspension method becomes higher and higher. However, as the resistance and inductance of a HTS coil based electromagnet (HTSEM) varies significantly with the frequency of the input voltage, as well as the amplitudes of the direct and alternative currents, it has been widely wondered whether HTSEMs can be directly used as an actuator in electromagnetic levitations (Maglev) [3,12].

It is worthwhile to point out that the existence of frequency dependent physical parameters is not limited to HTSEMs. For example, in human cerebral autoregulation, as well as the earth's nutation and precession, similar phenomena have also been observed [7,8]. But limited to the authors' knowledge, there is still not a systematic way for the analysis and synthesis of a dynamic system with frequency dependent parameters. In fact, even an openly published research work can hardly be found which directly or indirectly deals with these issues.

Note that modern robust control theory is capable of dealing with not only parametric uncertainties, but also unmodelled dynamics [5]. On the other hand, without restrictions on complexity, the frequency response of any linear time-invariant (LTI) plant can in principle be well approximated by a transfer function [10]. This implies that if the dynamics of a HTSEM can be well approximated by a finite dimensional transfer function and the approximation error is not too significant to violate the existence condition of a robust controller, then, it is very hopeful that HTS coils can be directly adopted in the control of Maglev systems.

To test the controllability of a HTSEM, a one axis experimental set up has been built in [14,15] which experimentally shows great potentials of the applicability of HTS coils to a Maglev system. However, as the system is not completely without mechanical contact and a Maglev system is very sensitive to mechanical frictions [11], the above declarations need further solidification. For this purpose, a four axis experimental system is constructed in which all possible mechanical contacts have been successfully avoided. In this paper, robust control theory is applied in a conventional way to the control of this four axis HTS coil experimental Maglev system, in which the frequency response deviations are regarded to be unmodelled dynamic uncertainties of HTSEMs from that of a finite dimensional LTI plant due to its frequency dependent parameters. It has been found that the closed-loop system can be robustly and stably suspended by the designed controller. The performances of the designed controller have also been compared with those obtained from classical lead-lag compensations. Experimental results show that robust control theories are able to supply a controller with better robustness and noise attenuation performances, etc., which agrees well with numerical simulations and theoretical analyzes. The results of this research imply that the available robust control theories have capabilities in dealing with the intrinsic model uncertainties of an electromagnet made of HTS coils.

The rest of this paper is organized as follows. In the next section, the experimental set up is described. Modelling and controller design are discussed in Section 3, while experimental results are reported in Section 4. Finally, this paper is concluded by Section 5.

2. System description

In Fig. 1, the experimental set up is given, which has almost the same structure as that of an ordinary Maglev system, except that the electromagnets are made from HTS coils, rather than copper or aluminium coils.

The vehicle is suspended by attracting forces produced by four electromagnets that are made by HTS coils and U-shaped iron cores which are constructed from laminated silicon steel with a thickness of 0.23 mm. These electromagnets are positioned at each corner of the vehicle. Four eddy current sensors are used to detect the air gaps of these electromagnets. Controllers are realized by a digital signal processor (DSP) TMS320C32, and four A/D converters, as well as four D/A converters are employed to convert respectively a measured analog air gap signal to a digital signal which is inputted to the DSP, and the output of a digitally realized controller to an analog input of a power amplifier. Four switched mode power amplifiers are utilized which can almost linearly convert an analog voltage signal to the current of a HTSEM in a frequency range about 0–200 Hz.²

A detailed description is given in [4] on the design and the electromagnetic characteristics of the HTSEM that are made of Bi2223/Ag multifilamentary tapes.

Generally, the vehicle has six degrees of freedom. That is, three translation movements (surge, sway and heave) and three rotation movements (roll, pitch and yaw). As lift, guide and propulsion of a Maglev system are naturally decoupled into three sub-systems which can be independently controlled, only levitation control is dealt with in this paper. When the sway motion and the yaw motion of the vehicle are to be controlled, four additional lateral electromagnets and four additional lateral position sensors are required. Moreover, a linear motor is usually employed in the control of the vehicle's surge motion. Compared with the levitation control, the other two controls are relatively simple and easy [11].

In the remaining of this paper, the electromagnets and the position sensors, as well as the power amplifiers, are numbered in an anti-clock direction beginning from the right-lower corner of Fig. 1.

² It is worthy of mentioning that while the power amplifiers are designed using general principles as those summarized in [6], the current feedback gains are significantly different from those when their load is an ordinary electromagnet with almost the same reluctance and inductance. Similar phenomenon has also been observed when a linear mode power amplifier is adopted. The reasons are still not very clear, but may possibly be due to the frequency dependence of the reluctance and inductance.



Fig. 1. Structure of the experimental system.

Table 1
Physical parameters of the experimental system

Parameter	Value	Unit
Mass of the vehicle	39.3	kg
Length of the vehicle	0.595	m
Width of the vehicle	0.49	m
Length of the electromagnets	0.086	m
Rotary inertia of the vehicle around the axis in the propulsion direction	1.787	kg m ²
Rotary inertia of the vehicle around the axis vertical to the propulsion direction	3.295	kg m ²
Constant current of the electromagnets	3.808	A
Constant air gap of the electromagnets	5.0×10^{-3}	m
Electromagnetic constant of the electromagnets	1.66×10^{-5}	N m ² /A ²
Input-output ratio of the power amplifiers	0.4	A/V
Gain of the first sensor	404.5	V/m
Gain of the second sensor	386.3	V/m
Gain of the third sensor	417.5	V/m
Gain of the fourth sensor	393.0	V/m

The major physical parameters of the experimental HTS coil based electromagnetic levitation system are given in Table 1.

3. Modelling and controller design

A mathematical model is developed for the experimental system using conventional methods adopted in Maglev system designs [11]. More precisely, in developing a nominal model of the HTS coil based experimental Maglev system, the dependence of the resistance and the inductance of the electromagnet on the frequency, the nonlinear effects of the electromagnet's attracting forces as a function of its air gap and current, as well as the nonlinear effects of the air gaps of the electromagnets as functions of the movement parameters of the vehicle, etc., are regarded to be model uncertainties. Under this assumption, through linearizing the nonlinear model obtained from physical laws of the experimental system at its nominal suspension point, a plant

nominal model $P_0(s)$ has been obtained. It has been observed that the nominal model $P_0(s)$ can be written into the following form:

$$P_0(s) = C \begin{bmatrix} g_{01}(s) & & & \\ & g_{02}(s) & & \\ & & g_{03}(s) & \\ & & & \end{bmatrix} B \quad (1)$$

in which C and B are, respectively, 4×3 and 3×4 dimensional constant matrices, while $g_{0i}(s)$, $i = 1, 2, 3$, represent, respectively, the linear part of the dynamics of the vehicle's heave translational movement and two rotational movements, that is, roll and pitch. The above structure of the plant nominal model is due to the symmetric construction of the levitation system which is widely adopted in magnetic suspensions. As the derivations are quite standard and well known, the details are not included. An interested reader is recommended to refer to, for example, [9,11]. On the other hand, a detailed study on the modelling process shows that both matrices B and C are majorly determined by the geometrical parameters and the gains of the eddy current position sensors, which can be measured with a high accuracy. This implies that the model uncertainties of the experimental system are majorly reflected by the modelling errors of $g_{0i}(s)$, $i = 1, 2, 3$ [14].

Substituting the actual physical parameters into the plant nominal model, after some direct algebraic operations, we have that

$$g_{01}(s) = \frac{1}{s^2 - 3920}, \quad g_{02}(s) = \frac{1}{s^2 - 5175}, \quad g_{03}(s) = \frac{1}{s^2 - 4225} \quad (2)$$

$$C = \begin{bmatrix} 1 & 0.245 & -0.2975 \\ 1 & 0.245 & 0.2975 \\ 1 & -0.245 & 0.2975 \\ 1 & -0.245 & -0.2975 \end{bmatrix}, \quad B = \begin{bmatrix} 832.8 & 809.1 & 859.6 & 795.3 \\ 4487.2 & 4359.6 & -4631.4 & -4285.3 \\ -2955 & 2871 & 3050 & -2822.1 \end{bmatrix} \quad (3)$$

From the "sandwich structure" of the nominal model, it becomes obvious that the controller design of the experimental system can be performed on the basis of the analysis and synthesis of three single-input single-output (SISO) systems $g_{0i}(s)$, $i = 1, 2, 3$. This property of the experimental system makes it also possible to compare the performances of the controllers designed by classical and modern control theories. More precisely, assume that a controller $k_i(s)$ has been designed for $g_{0i}(s)$, $i = 1, 2, 3$, then, a controller $K(s)$ for the HTS coil based Maglev system can be obtained as follows.

$$K(s) = B^T (BB^T)^{-1} \begin{bmatrix} k_1(s) & & & \\ & k_2(s) & & \\ & & k_3(s) & \\ & & & \end{bmatrix} (C^T C)^{-1} C \quad (4)$$

It can be simply proved that the multi-input multi-output (MIMO) nominal system is internally stable if and only if all the three SISO nominal systems are internally stable. Moreover, the stability margin of the MIMO system is equal to the smallest one of the three SISO systems. Therefore, in the design of controllers $k_i(s)$, $i = 1, 2, 3$, it is generally appreciable to make the robustness and regulation performances appropriately equal to each other of the individual SISO closed-loop systems [14].

In this paper, controllers $k_i(s)$, $i = 1, 2, 3$, are designed on the basis of the classical lead-lag compensation method and the modern \mathcal{H}_∞ loop-shaping method proposed by McFarlane and Glover [5].

When the lead-lag compensation method is employed, $k_i(s)$, $i = 1, 2, 3$, are designed as follows:

$$\begin{aligned} k_1(s) &= 1.34 \times 10^6 \frac{(1 + 0.32s)(1 + 0.030s)}{(1 + 81s)(1 + 0.0022s)} \\ k_2(s) &= 1.92 \times 10^6 \frac{(1 + 0.31s)(1 + 0.021s)}{(1 + 82s)(1 + 0.0019s)} \\ k_3(s) &= 1.37 \times 10^6 \frac{(1 + 0.35s)(1 + 0.027s)}{(1 + 79s)(1 + 0.0019s)} \end{aligned}$$

A detailed analysis shows that the designed three nominal SISO control systems have almost the same bandwidth, phase margin, and gain margin. Specifically, the bandwidth is approximately 20 Hz, the phase margin is approximately 57°, and the gain margin is approximately 3.3 dB. From classical control theories, satisfactory robustness and regulation performances can be expected for the corresponding controller $K(s)$.

When the \mathcal{H}_∞ loop-shaping method is adopted, the frequency weighting function $w_i(s)$ for $g_{0i}(s)$, $i = 1, 2, 3$, are selected, respectively, as

$$\begin{aligned} w_1(s) &= 1.31 \times 10^6 \frac{(1 + 0.044s)(1 + 0.043s)}{(1 + 36s)(1 + 0.00078s)} \\ w_2(s) &= 1.69 \times 10^6 \frac{(1 + 0.048s)(1 + 0.045s)}{(1 + 36s)(1 + 0.00071s)} \\ w_3(s) &= 1.41 \times 10^6 \frac{(1 + 0.047s)(1 + 0.043s)}{(1 + 37s)(1 + 0.00078s)} \end{aligned}$$

Based on these shaping functions, a controller of degree 6 can be directly obtained for $g_{0i}(s)$, $i = 1, 2, 3$, through employing the M-file *ncfsyn.m* in Matlab with the option *factor* being designated as 1.1. Using Hankel-norm model reduction, the degree of these controllers has been reduced to 4. Numerical simulations show that the reduced controllers have almost the same robustness and regulation performances as the original ones [14]. Moreover, in this design, the obtained performance index are, respectively, 0.3456, 0.3964 and 0.3672.³

³ It is generally recommended that when the \mathcal{H}_∞ loop-shaping method is adopted, a controller with a performance index greater than 0.2–0.3 will work satisfactorily [5].

The obtained reduced order controllers are as follows:

$$\begin{aligned} \bar{k}_1(s) &= 4.24 \times 10^5 \frac{(1 + 0.55s)(1 + 0.086s)(1 + 0.016s)}{(1 + 36s)(1 + 0.062s)(1 + 1.58 \times 10^{-3}s + 7.03 \times 10^{-7}s^2)} \\ \bar{k}_2(s) &= 6.68 \times 10^5 \frac{(1 + 0.49s)(1 + 0.13s)(1 + 0.013s)}{(1 + 36s)(1 + 0.086s)(1 + 1.32 \times 10^{-3}s + 5.12 \times 10^{-7}s^2)} \\ \bar{k}_3(s) &= 5.05 \times 10^5 \frac{(1 + 0.55s)(1 + 0.098s)(1 + 0.014s)}{(1 + 37s)(1 + 0.069s)(1 + 1.52 \times 10^{-3}s + 6.61 \times 10^{-7}s^2)} \end{aligned}$$

while their frequency responses are shown in Fig. 2, together with those of the lead-lag controllers. From this figure, it is obvious that the controllers designed by the classical lead-lag compensation method and the modern robust control theories have similar frequency domain characteristics at both low frequency range and middle frequency range, that is, when the angular frequency is smaller than 600 rad/s. However, their properties differ to each other significantly at high frequency range. In detail, when the angular frequency is greater than 2×10^3 rad/s, the gains of the \mathcal{H}_∞ loop-shaping based controllers roll off very fast with increasing frequency, while those of the lead-lag controller remain high. These imply that the former is at least more robust against plant unmodelled dynamics [5].

Various numerical simulations have been performed to check the performances of the designed controllers. The results show that the \mathcal{H}_∞ loop-shaping based controller is superior to the lead-lag compensation based controller in both robustness and regulation performances [14]. For example, upper and lower bounds have been computed for the structured singular values of the synthesized compensators in the angular frequency range between 10^{-2} rad/s and 10^4 rad/s, under the assumptions that the electromagnetic constants are assumed to have a relative error within 30%, the mass of the vehicle a relative error within 40%, the gain of the power amplifiers a relative error within 10%, and the position for the vehicle's center of gravity differs from its geometric center with an error bounded by 10% of its width and length. The results are shown in Fig. 3. It is clear from this analysis that the modelling uncertainties which can be guaranteed by the \mathcal{H}_∞ loop-shaping based controller, is almost as twice as that by the lead-lag compensation based controller.

4. Experimental results

The designed controller is discretized through the Tustin transformation $z = \frac{2}{T} \frac{s-1}{s+1}$ with a sampling period $T = 0.001$ s. As the bandwidth of the designed closed-loop system is approximately 20 Hz, it is not out of imagination that the performances of the closed-loop system will not be sensibly influenced by sampling.

The vehicle has been stably suspended by both of the designed controllers. To check the robustness and regulation performances of the designed closed-loop system, a step form external disturbance with a magnitude of 2 V has been added separately at the input port of the switched mode power amplifiers, in cases that there is no additional

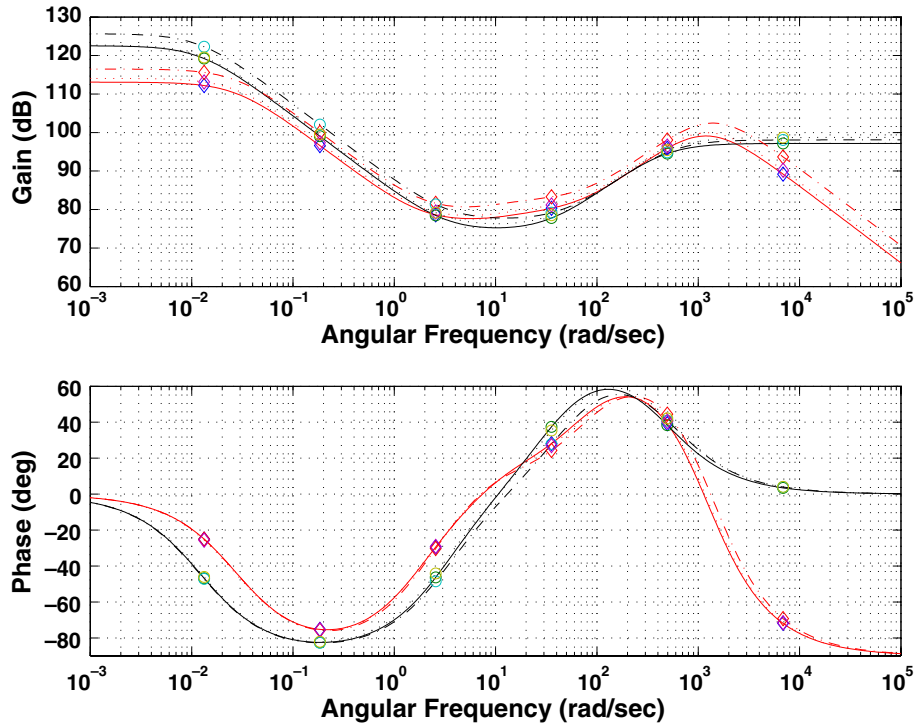


Fig. 2. Frequency responses of the designed controllers. —: controller for $g_{01}(s)$, - - -: controller for $g_{02}(s)$, - · - ·: controller for $g_{03}(s)$; ○: lead-lag controller; ◇: loop-shaping controller.

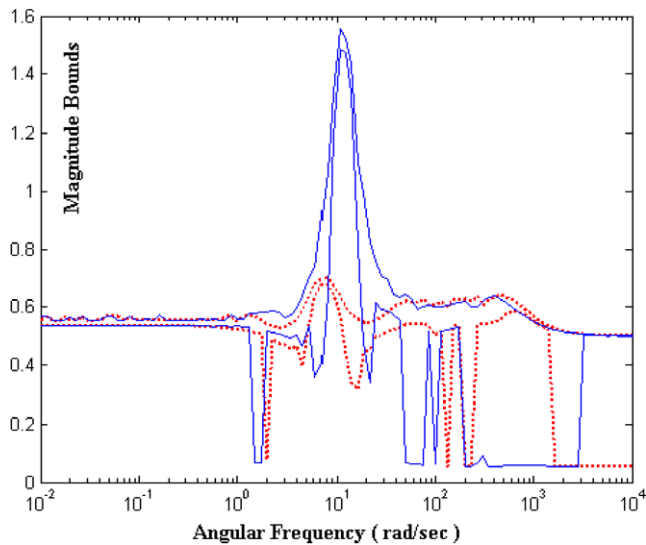


Fig. 3. Upper and lower bounds for the structured singular value of the designed closed-loop system. —: lead-lag controller, · · ·: \mathcal{H}_∞ loop-shaping controller.

load on the vehicle and some loads have been put on the vehicle. It has been confirmed by various experiments that the \mathcal{H}_∞ loop-shaping based controller performs better than the lead-lag compensation based controller. In Figs. 4 and 5, the measured air gap variations are shown when the disturbance is added separately at the input port of

each power amplifier, in cases that no load has been put on the vehicle and a 9 kg weight load (approximately 23% of the vehicle weight) has been put on the vehicle near its geometric center.

It is apparent from these experiments that there are almost no significant differences between the noise attenuation performances, the regulation speeds, as well as the overshoots of the response, etc. of the two designed controllers, when the closed-loop system is working under the situation that the controller is designed. However, when an additional load has been added, the influence of the external disturbance has been attenuated much faster and more smoothly by the \mathcal{H}_∞ loop-shaping based controller, especially the variations of the air gaps of the 2nd and the 3rd HTSEMs. This is especially clear from the 2nd and the 3rd measurements of the upper right and the lower left illustrations in Fig. 5.

On the other hand, it has also been observed from experiments that the robustness against load variations of the \mathcal{H}_∞ loop-shaping based controller is mainly limited by the capacity of the power amplifiers, rather than by the designed controller itself.

5. Concluding remarks

In this paper, experimental results are reported on the applicability of HTS coils to Maglev systems. A comparison has also been made on the robustness and the

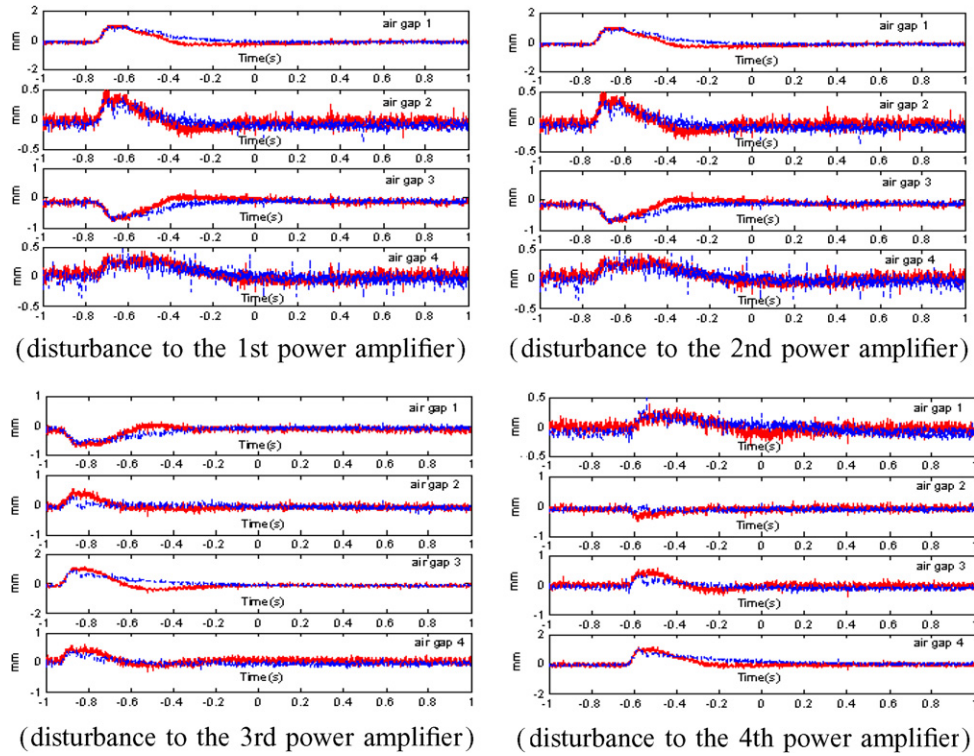


Fig. 4. Response of the nominal closed-loop system. —: lead-lag controller, ···: \mathcal{H}_∞ loop-shaping controller.

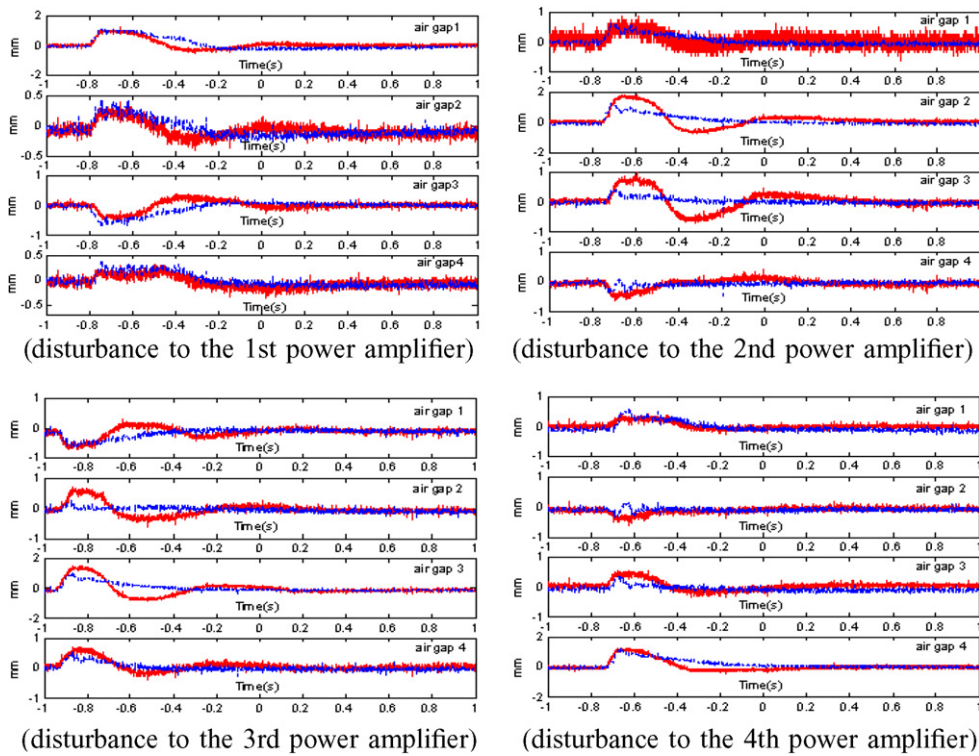


Fig. 5. Response of the closed-loop system with a 9 kg load. —: lead-lag controller, ···: \mathcal{H}_∞ loop-shaping controller.

regulation performances of the controllers designed by modern \mathcal{H}_∞ loop-shaping method and classical lead-lag compensations. Our experience shows that robust control theories can effectively overcome in a systematic

way the intrinsic model uncertainties of a HTSEM resulted from the significant variations of its resistance and inductance with the frequency of its input voltage.

However, it is worthwhile to point out that in order to develop a high efficiency HTS coil based Maglev system, more efforts are required on energy consumption minimizations. To achieve this objective, it appears important to develop a systematic method for control-oriented modeling of a HTSEM. Our primary studies show that while there do exist a finite dimensional transfer function that is capable of representing the dominant dynamics of a HTSEM, its order differs from that of an ordinary electromagnet [17].

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