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Electro-magnetic and electro-dynamic sustentation comparison

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ABSTRACT: The aim of the present analysis is to compare electro-dynamic (EDS) and electromagnetic (EMS) sustentation systems. For the electro-dynamic solution, the simplified model will be a sine wave MMF generated by a DC conducting layer in front of a semi-infinite conducting space. For the electro-magnetic solution, an electromagnet in front of iron will be considered. In a second step a more precise model is presented, allowing a sensitivity analysis of the main design parameters: speed, air gap, conducting layer thickness, pole pitch, etc

1 SIMPLIFIED METHODOLOGY

1.1 Electro-dynamic sustentation

The system is defined as two semi-infinite spaces air and conducting material with a sine tangential magnetic field moving at the surface (Fig 1). The main variables are:

$\vec{V}$ vector potential

Such as for the magnetic flux density B:

$$\vec{B} = r\vec{o}t\vec{V} \qquad [1]$$

In air :

$$\vec{\Delta}\vec{V} = 0 \quad \text{Laplace's law} \qquad [2]$$

In a material with constant resistivity (ρ) and permeability (μ):

$$\vec{\Delta}\vec{V} = \frac{\mu}{\rho}\frac{\partial \vec{V}}{\partial t} \quad \text{Poisson's law} \qquad [3]$$

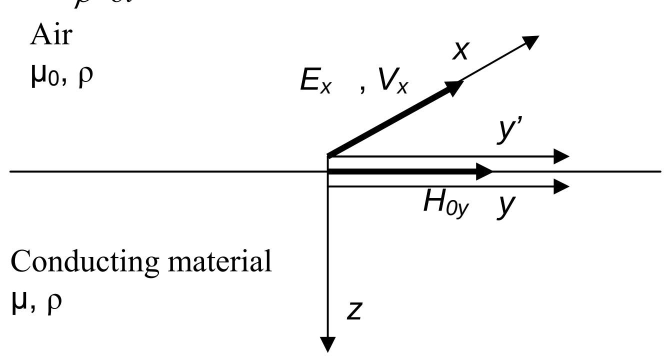

Figure 1. EDS semi-infinite spaces.

Two half-spaces, air and conducting material, are separated by a plane.

On the surface, on the air side, an ideal conducting layer creates an alternating magnetic field in the *y* direction at the level z = 0:

$$H_{y'0} = \hat{H}_0 \sin\frac{\pi y'}{\tau} \quad \text{with } \tau = \text{pole pitch}$$

If the excitation layer moves at the speed v in the y' direction, it comes:

$$y = y' + vt$$

$$H_{y0} = \hat{H}_0 \sin\frac{\pi(y+vt)}{\tau} = \hat{H}_0 \sin\left(\frac{\pi y}{\tau} + \frac{\pi v}{\tau}t\right)$$

$$= \hat{H}_0 \sin\left(\frac{\pi y}{\tau} + \omega t\right)$$

With $\omega = \frac{\pi v}{\tau}$

It is possible to associate complex numbers to sine expressions:

$$\underline{H}_{y0} = \hat{H}_0 e^{j(\pi y/\tau + \omega t)} = \hat{H}_0 \underline{\Omega} \qquad [4]$$

The vector potential is co-linear with the electric field E, in the x direction. By analogy, the vector potential can be written as:

$$\vec{V} = \vec{i}\, V_x(y,z,t)$$

Equation [3] becomes:

$$\Delta V_x = \frac{\mu}{\rho}\frac{\partial V_x}{\partial t} = \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}$$

In a complex form :

$$\Delta V_x = \frac{\mu}{\rho}\frac{d[V_x(z)\underline{\Omega}]}{dt} = \frac{d^2[V_x(z)\underline{\Omega}]}{dy^2} + \frac{d^2V_x(z)}{dz^2}\underline{\Omega}$$

$$j\omega\frac{\mu}{\rho}V_x(z)\underline{\Omega} = \frac{d^2V_x(z)}{dz^2}\underline{\Omega} - \frac{\pi^2}{\tau^2}V_x(z)\underline{\Omega}$$

$$\frac{d^2V_x(z)}{dz^2}\underline{\Omega} - \left(\frac{\pi^2}{\tau^2} + j\frac{\omega\mu}{\rho}\right)V_x(z)\underline{\Omega} = 0$$

Defining:

$\underline{\xi} = \sqrt{\left(\frac{\pi^2}{\tau^2} + j\frac{\omega\mu}{\rho}\right)}$ the solution is:

$$V_x(z) = K_1 e^{\underline{\xi}} + K_2 e^{-\underline{\xi}}$$

The vector potential cannot be infinite for z = ∞, thus:

$$V_x(z) = K_2 e^{-\underline{\xi}}$$

$$V_x(y,z,t) = K_2 e^{-\underline{\xi}}\underline{\Omega}$$

From [1]:

$$B_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} = \frac{\partial V_x}{\partial z}$$

$$B_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = -\frac{\partial V_x}{\partial y}$$

$$B_y = -\underline{\xi}K_2 e^{-\underline{\xi}}\underline{\Omega}$$

$$B_z = -j\frac{\pi}{\tau}K_2 e^{-\underline{\xi}}\underline{\Omega}$$

At the limit (z = 0), according to [3], $H_y = \hat{H}_0\underline{\Omega}$, so:

$$K_2 = -\frac{\mu\hat{H}_0}{\underline{\xi}}$$

$$B_y = \mu\hat{H}_0 e^{-\underline{\xi}}\underline{\Omega}$$

$$B_z = j\frac{\pi}{\tau\underline{\xi}}\mu\hat{H}_0 e^{-\underline{\xi}}\underline{\Omega}$$

The force perpendicular to the separation surface is, according to Maxwell's stress tensor:

$$dF_z = \frac{\mu}{2}\left[H_z^2 - H_y^2\right]dS = \frac{1}{2\mu}\left[B_z^2 - B_y^2\right]dS$$

The average specific force (pressure) is :

$$\frac{dF_z}{dS} = F_z' = \frac{\mu_0\hat{H}_0^2}{4}\left[\left|\frac{\pi}{\tau\xi}\right|^2 - 1\right] = \frac{\mu_0\hat{H}_0^2}{4}\left[\frac{1}{1+\frac{\mu_0\tau v}{\pi\rho}} - 1\right] \qquad [5]$$

The expression in [5] is always negative, which means a repulsion force.

If τ = 0.5 m ; ρ = 32 nΩm (Al) we have the following values:

- already for a speed of 0.15 m/s the expression in [] = -0.5
- for a speed of 15 m/s [] =-0.99
- for a speed of 150 m/s [] = -0.999

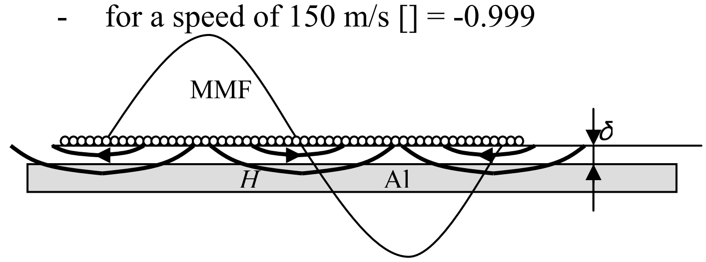

Figure 2 EDS structure and excitation

The repulsion force is proportional to the magnetic field square. If the distance d increases, the tangential magnetic field decreases. So for a given mass, it appears an equilibrium position corresponding to both repulsion and gravitation forces equilibrium.

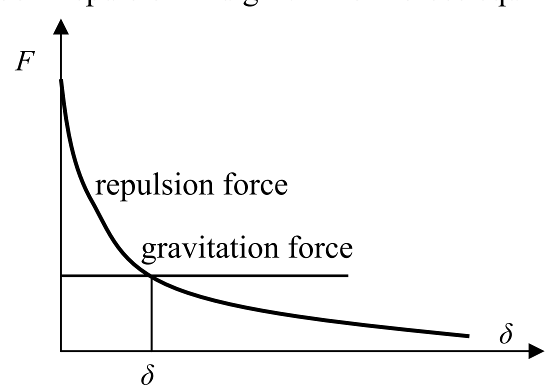

Figure 3. EDS force equilibrium.

This force has a maximum value of

$$F'_{z\max} = -\frac{\mu_0\hat{H}_0^2}{4}$$

The tangential field at the excitation layer level is given by:

$\vec{H} = -gr\vec{a}d\Theta$ with Θ = MMF :

$$\Theta = \hat{\Theta} \sin \frac{\pi y}{\tau}$$

$$H_y = -\frac{\partial \Theta}{\partial y}$$

$$\hat{H}_0 = \frac{\pi \hat{\Theta}}{\tau}$$

$$F'_{z\max} = -\frac{\mu_0 \hat{H}_0^2}{4} = -\frac{\mu_0 \pi^2 \hat{\Theta}^2}{4\tau^2} \qquad [6]$$

This force corresponds to a maximum with a distance δ = 0.

A more complete model is described in Section 3. Other solutions based on coils instead of conducting layer are possible, as an example such as used for the Japanese MLX.

1.2 Electromagnetic sustentation EMS

The EMS solution is based on an electromagnet (Fig 3) interacting with an iron surface and controlled in position, imposing a constant air gap δ.

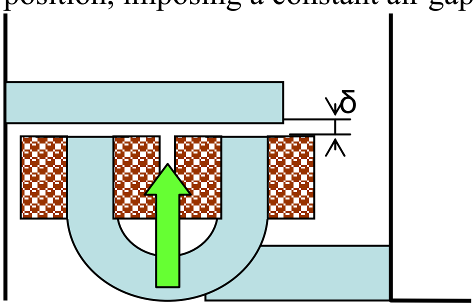

Figure 4. Electomagnetic system

The force per surface unit (pressure) is :

$$F'_\delta = \frac{B_\delta^2}{2\mu_0}$$

By Ampere's law:

$$\oint H dl = \Theta = Ni$$

$$\oint H dl = k_{sat} H_\delta 2\delta = \Theta$$

With Θ = Coil MMF

K_{sat} = saturation coefficient (1.05÷1.5)

$$H_\delta = \frac{\Theta}{2 k_{sat} \delta}$$

$$B_\delta = \frac{\mu_0 \Theta}{2 k_{sat} \delta}$$

$$F'_\delta = \frac{B_\delta^2}{2\mu_0} = \frac{\mu_0 \Theta^2}{8 k_{sat}^2 \delta^2} \qquad [7]$$

1.3 Comparison

It is possible to compare the respective performances of EMS and EDS systems. In this aim, the same MMF will be supposed in both cases.
So, using expressions [6] and [7]:

$$F'_{EDS} = -\frac{\mu_0 \pi^2 \hat{\Theta}^2}{4\tau^2} \qquad [8]$$

$$F'_{EMS} = \frac{\mu_0 \Theta^2}{8 k_{sat}^2 \delta^2} \qquad [9]$$

$$\frac{F'_{EDS}}{F'_{EMS}} = \frac{2\pi^2 k_{sat}^2 \delta^2}{\tau^2} \qquad [10]$$

Example:
k_{sat} = 1.2 , δ=0.02 m
τ = 0.5 m

$$\frac{F'_{EDS}}{F'_{EMS}} = 0.0455$$

With the same MMF, the EDS force per surface unit is about 1% of the EMS one. In other words, the necessary MMF for EDS is about 10 times higher than the one for EMS.
That is the reason for a superconducting coil for the EDS excitation.

2 2D EDS MODEL

2.1 Structure

The inductor defined in Figure 5 generates a magnetic field with a tangential component such as:

$$H_{\delta y} = \hat{H}_0 \sin(\frac{\pi y}{\tau})$$

According to the necessity to proceed to first and second order derivatives, it is more efficient to associate the complex calculus:

$$\underline{H}_{\delta y} = \hat{H}_0 e^{j\pi y/\tau} \qquad [11]$$

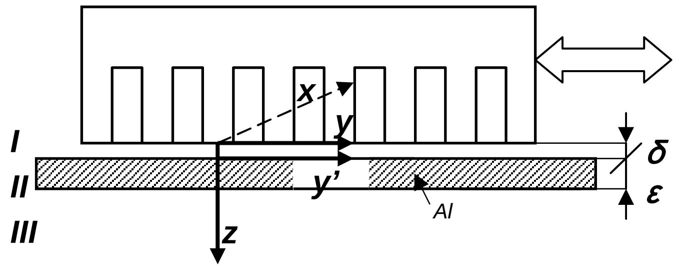

Figure 5. EDS 2D structure

Three domains are defined :

- I - The air gap domain
- II - The aluminium domain
- III - The air opposite to the air gap

2.2 Resolution

Domain I:

Laplace's law [1] can be applied. The corresponding solution is:

$$\underline{V}_{1\delta x} = (D_1 e^{-\lambda z} + D_2 e^{\lambda z}) \qquad [12]$$

With $\lambda = \dfrac{\pi}{\tau}$

Domain II:

Poisson's law [2] can be applied. The corresponding solution is:

$$\underline{V}_{2\delta x} = (R_1 e^{-\xi z} + R_2 e^{\xi z}) \qquad [13]$$

With $\xi = \sqrt{\left(\dfrac{\pi}{\tau}\right)^2 + j\dfrac{\mu_0 \pi}{\rho\tau} v} \qquad j = \sqrt{-1}$

Domain III:

Laplace's law [1] can also be applied. The corresponding solution has the same form:

$$\underline{V}_{3\delta x} = (D_3 e^{-\lambda z} + D_4 e^{\lambda z}) \qquad [14]$$

Between these different domains, the different following continuity relations can be applied:

- For z = 0:
  $\hat{H}_{1\delta y} = \hat{H}_0$
- For z = δ:
  $V_{1\delta x} = V_{2\delta x}$
  $H_{1\delta y} = H_{2\delta y}$
- For z = δ + ε:
  $V_{2\delta x} = V_{3\delta x}$
  $H_{2\delta y} = H_{3\delta y}$
- For z = ∞:
  $V_{3\delta x} = 0$

Out of these relations, it is possible to determine the integration constants: D_1, D_2, R_1, R_2, D_3, D_4 .

$$D_4 = 0$$

$$-D_1 + D_2 = \frac{\mu_0}{\lambda}\hat{H}_0$$

$$D_1 e^{-\lambda\delta} + D_2 e^{\lambda\delta} - R_1 e^{-\xi\delta} - R_2 e^{\xi\delta} = 0$$

$$\frac{\lambda}{\mu_0}\left(-D_1 e^{-\lambda\delta} + D_2 e^{\lambda\delta}\right) + \frac{\xi}{\mu_0}\left(R_1 e^{-\xi\delta} - R_2 e^{\xi\delta}\right) = 0 \qquad [15]$$

$$D_3 e^{-\lambda(\delta+\varepsilon)} - R_1 e^{-\xi(\delta+\varepsilon)} - R_2 e^{\xi(\delta+\varepsilon)} = 0$$

$$-\frac{\lambda}{\mu_0} D_3 e^{-\lambda(\delta+\varepsilon)} + \frac{\xi}{\mu_0}\left(R_1 e^{-\xi(\delta+\varepsilon)} - R_2 e^{\xi(\delta+\varepsilon)}\right) = 0$$

The different pressures interacting on the aluminium plate can be determined from Lapalce's tensor:

For the normal pressure p_n:

$$p_n = \frac{1}{2}\mu_0\left(H_n^2 - H_t^2\right) = \frac{1}{2}\Re\left(H_n H_n^* - H_t H_t^*\right) \qquad [16]$$

For the tangential pressure p_t:

$$p_t = \mu_0 H_n H_t = \mu_0 \Re(H_n H_t^*) \qquad [17]$$

2.3 Results

Relations [16] and [17] have been applied to an EDS system with the following data:

Pole pitch τ = 0.5 m, 2 poles

Inductor and plate width 1 m

Aluminum plate of 10 mm

MMF = 20'000 A

In Figure 6, the normal and tangential pressures are represented as a function of speed between 0.5 and 5 m/s.

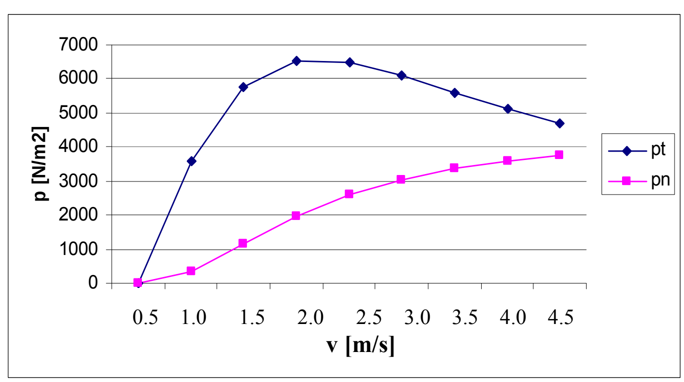

Figure 6. EDS 2D structure – Pressures as a function of speed (low speed)

In Figure 7, the same normal and tangential pressures are represented as a function of speed between 5 and 55 m/s.

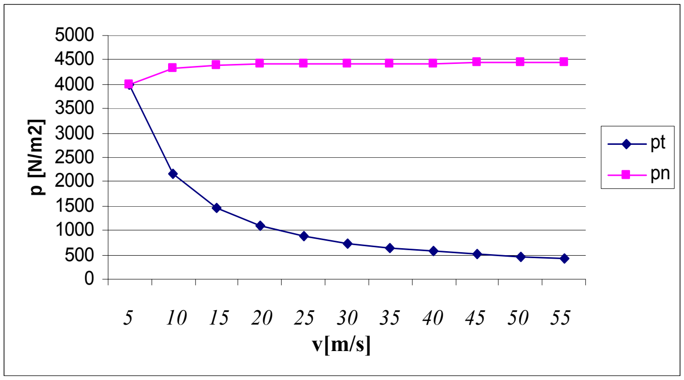

Figure 7. EDS 2D structure – Pressures as a function of speed (high speed)

On Figure 8, the normal and tangential pressures at a speed of 50 m/s are represented as a function of the gap δ, from 0 to 100 mm. Logically, both pressures are decreasing with the air gap. For a zero air gap, the value for the normal pressure is 4960 N/m^2, which is the same as given by relation [8].

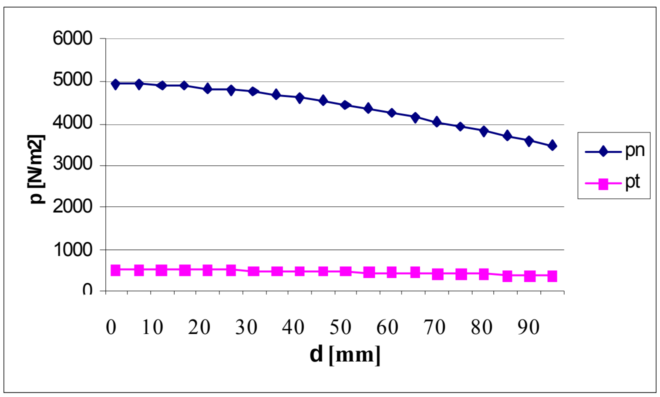

Figure 8. EDS 2D structure – Pressures as a function of air gap at 50 m/s

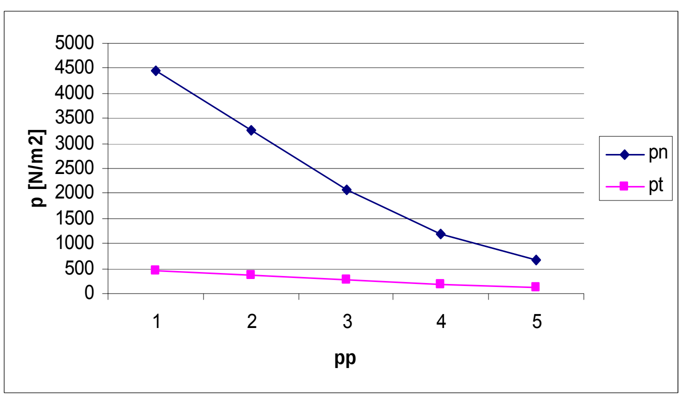

Figure 9. EDS 2D structure – Pressures as a function of pole pair number at 50 m/s and a gap of 50 mm.

On Figure 9, the pressures are represented at a speed of 50 m/s for an axial length of 1 m and different pole pair number with the relation:

$$\tau = \text{length}/(2pp)$$

The force decreases with the pole pair number or with the pole pitch decreasing.

On Figure 10, the normal and tangential pressures at a speed of 50 m/s are represented as a function of the aluminum plate thickness ε.

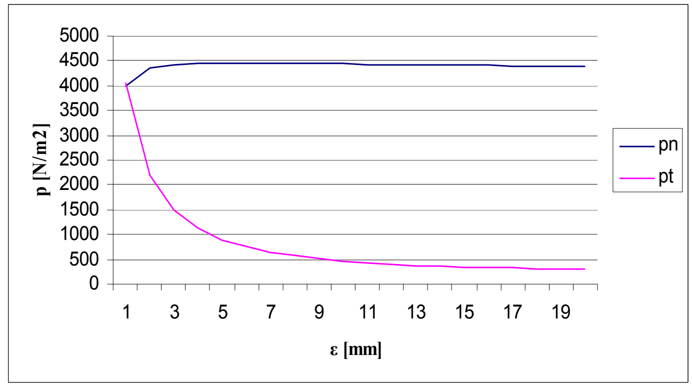

Figure 10. EDS 2D structure – Pressures as a function of aluminum plate thickness ε at 50 m/s

The plate thickness has a very low influence on the normal pressure or levitation force above 1 mm. On the contrary, the influence is important on the tangential pressure or on the drag force. A thickness of 8 mm or above is necessary to limit this effect.

3 CONCLUSION

The presented methodology to analyze the EDS system and to compare it to the EMS system is a simple but efficient way to proceed with an aluminum plate fixed to the track structure. It allows a direct parametric analysis. The same procedure can also be applied to coils fixed on the track.

4 SYMBOLS

All symbols in MKSA unit system

B	Flux density
$D_{1,2,3,4}$	Integration constants
E	Electric field
F	Force
F’	Force per surface unit
H	Magnetic field
j	Complex unit number
$K_{1,2}$	Integration constants
L	Length
p	Pressure
$R_{1,2}$	Integration constants

S	Surface
t	Time
v	Speed
x,y,z	Coordinates
δ	Air gap
ε	Aluminum plate thickness
λ	π/τ
ξ	[13]
τ	pole pitch
μ	permeability
μ_0	vacuum permeability
Θ	MMF
Ω	[4]

Indexes

n	normal
sat	saturated
t	tangential
x,y,z	in the direction x,y,z

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