

# Levitation Controller Design for an Electromagnetic Suspension System Based on Backstepping Algorithm

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**ABSTRACT:** Active control is indispensable to electromagnetic suspension system in order to stabilize the open-loop system. For simplicity, linear control strategy is widely utilized in practical maglev systems. However, electromagnetic suspension system is inherently highly nonlinear, which makes linear control strategies sometimes fail in experiments. To conquer the nonlinearity of the suspension system, backstepping algorithm is introduced to the nonlinear model of a magnetic suspension control system in this work. The effectiveness of the proposed controller is verified by simulation, and its suspension performance is discussed under appointed disturbance. The results illustrate that the designed controller based on backstepping strategy can satisfy the performance requirements of the suspension system, and it has fine robustness to disturbance.

## 1 INTRODUCTION

Maglev suspension system can be used in various situations [1], such as maglev train, maglev bearing, maglev scale and maglev assistant launch system. Because the maglev suspension system is a nonlinear unstable system, and the values of its parameters are not fixed, the PID control method [2][3] or other nonlinear control methods [4][5] are often used to make the system stable. Backstepping method, which is put forward in the end of last century [6], is a nonlinear method to design controller especially for nonlinear system. A special method is adapted to construct the controller step by step, and it has the advantages to deal with unmatched uncertainty. The backstepping method is successfully applied in the controller design of planes and missiles. With nonlinear adaptive backstepping method, Chin-I Huang control velocity and position of linear induction motor, and the results of simulation and experiment are presented [7]. S.S.GE used adaptive backstepping method to do research on the control of a kind of chaos system, and the simulation result is presented [8].

Considering the nonlinear characteristic of maglev suspension system [9] and the advantages of backstepping method, a levitation controller using this method is discussed and designed, and the performance of this controller is validated by simulation, which shows that the backstepping method can be used to gain fine performance.

## 2 MODELING OF MAGLEV SYSTEM

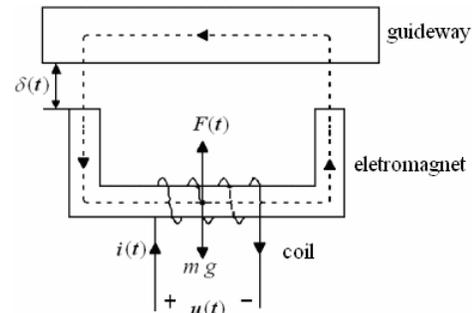


Fig1: The diagram of a maglev system

As shown in Fig.1, the mathematic model of the system can be obtained as [10]:

$$F = \frac{\mu_0 AN^2}{4} \cdot \left( \frac{i}{\delta} \right)^2 \quad (1)$$

$$m\ddot{\delta} = m \cdot g - F + f_d \quad (2)$$

$$u = Ri + \frac{\mu_0 AN^2}{2} \cdot \left( \frac{i}{\delta} - \frac{i}{\delta^2} \cdot \dot{\delta} \right) \quad (3)$$

Where  $F$  is suspension force provided by the electromagnet,  $\mu_0$  is permeability of vacuum,  $A$  is the area of magnetic pole,  $N$  is the number of turn of coil,  $i$  is the current of coil,  $\delta$  is the length of gap,  $g$  is acceleration of gravity,  $f_d$  is the force of disturbance;  $u$  is control voltage,  $R$  is the resistance of coil. Equation (1) is the formula of suspension force. Equation (2) is the dynamics of the electromagnet in vertical direction. Equation (3) shows the relationship between the control voltage and the current of the coil. Equations (1)~(3) are the mathematical model of maglev suspension system.

Choose  $X = [\delta \ \dot{\delta} \ i]^T$  as the state variable of the system, the system can be described as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - k_a \cdot x_3^2 / x_1^2 + f_d / m \\ \dot{x}_3 = k_r x_1 (u - R x_3) + x_2 x_3 / x_1 \end{cases} \quad (4)$$

Where  $k_a = \frac{\mu_0 AN^2}{4m}$  and  $k_r = \frac{2}{\mu_0 AN^2}$ .

Noting the gap is  $\delta_0$  and the current is  $i_0$  when the system is in balance. The equilibrium point of a suspension system is not equal to the origin. In order to transfer the equilibrium point to the origin, following coordinate transform is made use of, that is  $y_1 = x_1 - \delta_0$ ,  $y_2 = x_2$ ,  $y_3 = x_3 - i_0$ . Then the model is:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = g - k_a \cdot (y_3 + i_0)^2 / (y_1 + \delta_0)^2 + f_d / m \\ \dot{y}_3 = k_r (y_1 + \delta_0) (u - R i_0 - R y_3) + y_2 (y_3 + i_0) / (y_1 + \delta_0) \end{cases} \quad (5)$$

### 3 CONTROLLER DESIGN

Backstepping method is adapted to design controller of nonlinear system. The model in formula (4) is a three-order system, the controller can be obtained by the following three steps.

#### Step 1:

Define a virtual control for the first equation in model (5), say  $\alpha_1$ , and let  $z_1$  be a virtual state variable representing the difference between the actual and virtual controls, that is  $z_1 = y_2 - \alpha_1$ . So the first equation in model (5) can be rewritten as:

$$\dot{y}_1 = z_1 + \alpha_1 \quad (6)$$

To the above one-order system in (6), we construct a Lyapunov function to be  $V_1 = 0.5 y_1^2$ . Then the differential of the function is:

$$\dot{V}_1 = y_1 \dot{y}_1 = y_1 z_1 + y_1 \alpha_1 \quad (7)$$

Choosing the virtual control to be:

$$\alpha_1 = -k_1 y_1 \quad (8)$$

Where  $k_1 > 0$ . The influence of  $z_1$  in (7) is not considered in this step, and the other items of  $\dot{V}_1$  equals  $-k_1 y_1^2$ , which is negative definite.

According to formula (6) and (8), we can get:

$$\dot{\alpha}_1 = -k_1 \dot{y}_1 = -k_1 y_2 \quad (9)$$

#### Step 2:

Note  $u_a = g - k_a \cdot (y_3 + i_0)^2 / (y_1 + \delta_0)^2$  to make the system simple. A new virtual control  $\alpha_2$  is defined, and the virtual state variable is  $z_2 = u_a - \alpha_2$ . If the influence of disturbance force is neglected, the second equation in model (5) is:

$$\dot{y}_2 = z_2 + \alpha_2 \quad (10)$$

Then a new Lyapunov function based on the function  $V_1$  is constructed, and it is  $V_2 = V_1 + 0.5 z_1^2$ . The differential of the new function is:

$$\dot{V}_2 = y_1 \dot{y}_1 + z_1 \dot{z}_1 = -k_1 y_1^2 + y_1 z_1 + z_1 (z_2 + \alpha_2 + k_1 y_2) \quad (11)$$

Choosing the new virtual control:

$$\alpha_2 = -k_2 z_1 - k_1 y_2 - y_1 \quad (12)$$

Where  $k_2 > 0$ . Then the formula (11) is:

$$\dot{V}_2 = y_1 \dot{y}_1 + z_1 \dot{z}_1 = -k_1 y_1^2 - k_2 z_1^2 + z_1 z_2 \quad (13)$$

The influence of  $z_2$  in (7) is not considered in this step, and the other items in  $\dot{V}_2$  are  $-k_1 y_1^2 - k_2 z_1^2$ , which is negative definite. Furthermore, the differential of virtual control is:

$$\dot{\alpha}_2 = -(k_1 + k_2) u_a - (k_1 k_2 + 1) y_2 \quad (14)$$

#### Step 3:

We also note

$$u_b = k_r (y_1 + \delta_0) (u - R i_0 - R y_3) + y_2 (y_3 + i_0) / (y_1 + \delta_0)$$

to make the system stable, then the control voltage  $u$  can be calculated as:

$$u = \frac{u_b - y_2 (y_3 + i_0) / (y_1 + \delta_0)}{k_r (y_1 + \delta_0)} + R (y_3 + i_0) \quad (15)$$

A new Lyapunov function is constructed based on the formal function  $V_2$ , and it is shown as  $V_3 = V_2 + 0.5 z_2^2$ . The differential of new function is:

$$\begin{aligned} \dot{V}_3 &= y_1 \dot{y}_1 + z_1 \dot{z}_1 + z_2 \dot{z}_2 \\ &= -k_1 y_1^2 - k_2 z_1^2 + z_1 z_2 + z_2 (\dot{u}_a - \dot{\alpha}_2) \\ &= -k_1 y_1^2 - k_2 z_1^2 + z_1 z_2 + \\ &\quad z_2 [K_\delta y_2 - K_i u_b + (k_1 + k_2) u_a + (k_1 k_2 + 1) y_2] \end{aligned} \quad (16)$$

Where  $K_\delta = \frac{2k_a (y_3 + i_0)^2}{(y_1 + \delta_0)^3}$  and  $K_i = \frac{2k_a (y_3 + i_0)}{(y_1 + \delta_0)^2}$ .

In order to make  $\dot{V}_3 \leq 0$  satisfied for all value of state variables, we can choose the variable  $u_b$  to be:

$$u_b = \frac{1}{K_i} [z_1 + k_3 z_2 + K_\delta y_2 + (k_1 + k_2) u_a + (k_1 k_2 + 1) y_2] \quad (17)$$

Where  $k_3 > 0$ . Substituting formula (17) into (16), we have:

$$\dot{V}_3 = -k_1 y_1^2 - k_2 z_1^2 - k_3 z_2^2 \quad (18)$$

It is obviously that  $\dot{V}_3$  is negative definite. For  $V_3$  is positive definite at the same time, the close-loop system is asymptotically stable at the origin.

Substituting the expressions of  $z_1$  and  $z_2$  into formula (17), the variable is rewritten as:

$$u_b = \frac{1}{K_i} \begin{bmatrix} (k_1 + k_2 + k_3) u_a \\ + (k_1 k_2 + k_1 k_3 + k_2 k_3 + K_\delta + 1) y_2 \\ + (k_1 + k_3 + k_1 k_2 k_3) y_1 \end{bmatrix} \quad (19)$$

Substitute the expression (19) into formula (15), the formula of control voltage can be obtained.

### 4 SIMULATION RESULTS AND ANALYSIS

The the values of parameters are:  $N = 320$ ,  $A = 0.84 \times 0.028 \text{ m}^2$ ,  $m = 750 \text{ kg}$ ,  $R = 0.5 \ \Omega$ ,  $\delta_0 = 10 \text{ mm}$ ,  $i_0 = 31 \text{ A}$ . The controller is designed based on backstepping method by the three steps above, and

the corresponding values of controller parameters are:  $k_1 = 30$ ,  $k_2 = 20$ ,  $k_3 = 10$ .

Supposing the initial gap  $\delta(0) = 20$  mm, the curves of gap, current and control voltage when the system is arising have been simulated, and the result is shown in fig.2. The following conclusions are drawn from the simulation result: the gap keeps to be 20mm while the control voltage changes up to be about 350V at beginning, and the current arises at the same time. After 0.17s, the current equals to 80A, which makes the system begins to leave the guideway, while the gap is smaller than 20mm. After the adjusting time for 0.5s long, the gap and current achieve their respective steady value, and the control voltage needed is only 15.5V.

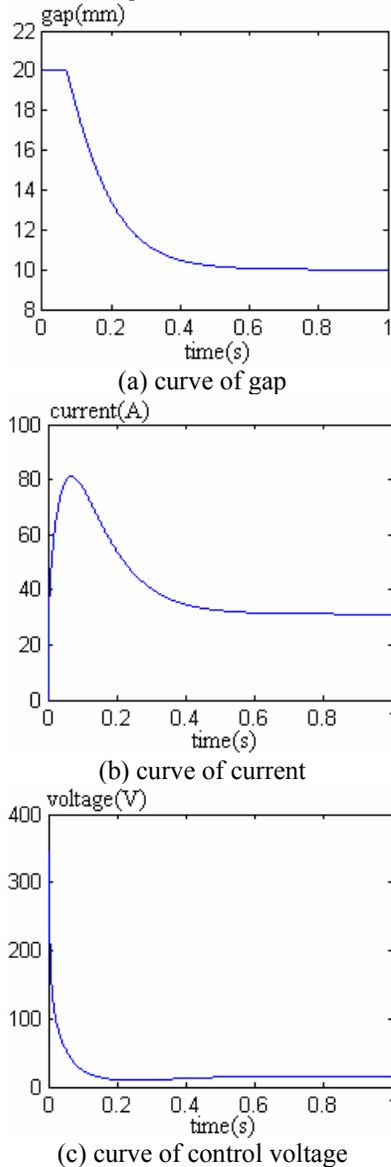


Fig.2: Simulation result when arising

In order to analyze the influence of disturbance to the suspension system, a disturbance force which equals to 20% of the suspension weight is added to the system, and the simulation result is shown in fig.3. We can draw the conclusion from fig.3 that:

when the disturbance is added into the system, the variation of gap is smaller than 1.5mm, while the variation of current is smaller than 7A, and the variation of control voltage is little too. So the controller designed by backstepping method can stabilize the suspension system, and achieve a satisfying performance even the disturbance exists.

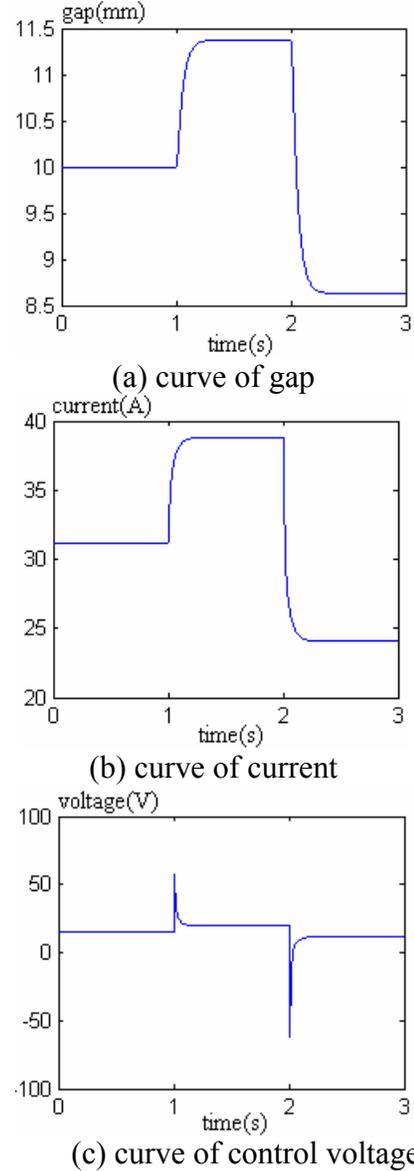


Fig.3: Simulation result when disturbance exists

## 5 CONCLUSIONS

The nonlinear model of magnetic suspension system is established in this paper, and a controller for levitation is designed based on the backstepping method. Then the control effect is simulated in order to validate the feasibility of this method. The simulation results show that, by using the controller designed before, the maglev system can arise smoothly. In this process, the maximum current needed is 80A, and the control voltage needed is

about 350V, which accord with the practical value in experiment. When disturbance is added into the system, the variation of gap is very small, and the variation of current and control voltage are in reasonable range. So we can draw the conclusion that the backstepping method can be used to design controller for suspension maglev system to gain fine performance.

## 6 REFERENCES

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