

# Optimum Shape Design of Single-Sided Linear Induction Motors Using Response Surface Methodology and Finite Element Method

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**ABSTRACT:** This paper deals with finding the optimal ratio of height and length of Single-Sided Linear Induction Motors (SLIM) using Finite Element Method (FEM) for magnetic field analysis coupled with optimal design methodology. For effective analysis, FEM is conducted in time harmonic field which provides steady state performance with the fundamental components of voltage and current. The ratio of height to length providing the required output power is obtained by Response Surface Methodology (RSM) and optimal values are presented by the variation in output power. When output power is small, the ratio is high and as the power increases, the ratio shows a converged value. Considering the general application of linear motors, using a small ratio can be limiting, however, the shape ratio for maximum thrust can be identified.

## 1 INTRODUCTION

Single-Sided Linear Induction Motor (SLIM) has been developed for use in the industry, transportations, OA, FA, because of the advantages of direct drive and simple structure.

SLIM is very useful in situations requiring linear motion since it produces thrust directly, so the opportunity for industry application is increased [1], [2]. It is possible to divide linear motors into the following categories by the application;

- (1) force machines
- (2) power machines
- (3) machines

Force machines are short duty machines operating at very low speed, and efficiency is not a major consideration with regard to overall performance. Power machines are often operated at medium or high speed and are continuous-duty machine. They must have high efficiency. Energy machines are short-duty machines and have found applications as accelerators. At present most linear machines are used in low speeds and standstill applications. As power machines, shape design of SLIM is dealt in this paper.

The optimum design of SLIMs is subject to

performance constraints as the design changes according to application. In this paper the variables of SLIMs for servo system are optimized using FEM and response surface methodology. In order to design SLIMs to have maximum force density, numerous design variables can be used and optimized. Among design variables, the shape ratio, which is the ratio between stator length and height, is chosen and optimal values for various outputs are determined.

The RSM is used for the experiments, which seeks for the relationship between design variables and response in interest area through statistical fitting methods based on the observed data from system. The response is generally obtained from real experiments or computer simulations. In the case of rotating machines, research on the ratio of stator and rotor diameter for maximum torque have been conducted [3], [4], however, in the case of linear motor, little work has been done.

Therefore, this paper deals with the mover shape ratio of linear motors. By using FEM and optimization methodology, the shape ratio providing maximum output power is presented. The optimization is conducted for various output powers. Maximum output power significantly depending on the shape ratio of mover's width and length is investigated.

## 2 ANALYSIS THEORY

### 2.1 Determination of yoke and tooth width

Determination of tooth and yoke thickness is important for maximum output. Therefore, it is essential to determine tooth width and yoke thickness since they are not considered as design variables in this paper. In order to assign reasonable values of tooth and yoke thickness, an equivalent magnetic circuit network is used [4].

In each model, tooth and yoke thickness are determined to have minimum reluctance. Assumptions required in the equivalent magnetic circuit network method are as follows

- Permeability of tooth and yoke are constant.
- End effects are not considered
- HT, AL, and slot area are determined in the initial design
- Leakage flux is not considered

Equation (1) shows the total reluctance of a simplified equivalent magnetic circuit. Instead of total magnetic circuit, a simplified model is used based on the symmetry and periodicity. Since the slot area is predetermined in the initial design, total reluctance is the function of tooth width  $x$ , and yoke thickness consequently determined by slot area and tooth

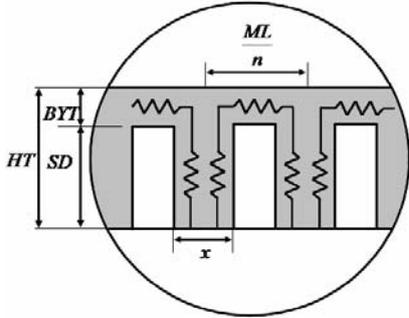


Figure 1. Equivalent magnetic circuit of mover

width. Total reluctance of simplified model is calculated as the variation of  $x$ , and the value providing minimum reluctance is chosen for each experimental model.

$$\begin{aligned} R_{total} &= R_{tooth} + R_{yoke} \\ &= 2 \times x \cdot L_{st} / \mu ((HT + SD) / 2) \\ &\quad + ((HT - SD) \cdot L_{st} / \mu ((ML / n) - x)) \end{aligned} \quad (1)$$

$$SD = \text{slot area} / ((ML / n) - x) \quad (2)$$

where,  $R_{total}$  is the total reluctance,  $R_{tooth}$  is the tooth

reluctance,  $R_{yoke}$  is the yoke reluctance,  $L_{st}$  is stack length,  $\mu$  is the permeability,  $x$  is the tooth width.

### 2.2 Optimization

For optimal design, sampling of experimental data should be conducted initially. In order to do that Design of Experiment (DOE) is necessary for effective experiments.

Among various DOEs, Optimal Latin-Hypercube Design (OLHD) is used and an approximation is constructed, then Progressive Quadratic Response Surface Methodology PQRS is used for to find an optimal value [5].

#### 2.2.1 Design of experiment

To achieve optimum design, sampling points are arranged using OLHD, which is one of many DOE methods [6]. All the axes of factors are arranged by identical numbers using OLHD.

In other words, all factors are arranged to be on same level and axis of factors are equal.

As sampling points are arranged in this way, they may be highly correlated to one another, therefore sampling points should be evenly distributed by optimum conditions as follows.

As it is not known beforehand which design will lead to the best performance in terms of  $\tilde{\phi}_p$  criterion ( $0 \leq \tilde{\phi}_p \leq 1$ ) referred to in equation (3), (4) and (5), one cannot know which design use.

Fig. 2 shows an example of design for two variables.

$$\phi_p = \left[ \sum_{i=1}^{n_p-1} \sum_{j=i+1}^{n_p} d_{ij}^{-p} \right]^{1/p} \quad (3)$$

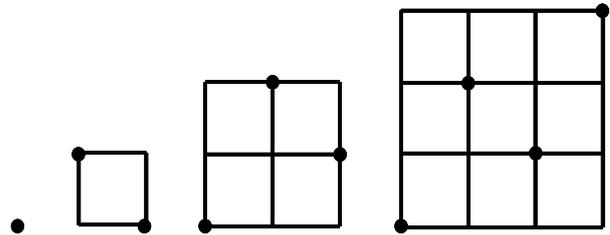


Figure 2. Example of design for two variables

$$d_{ij} = d(x_i, x_j) = \left[ \sum_{k=1}^{n_v} |x_{ik} - x_{jk}|^t \right]^{1/t}, \quad p = 50, \quad t = 1 \quad (4)$$

$$\tilde{\phi}_p = \min(\phi_p) / (\max(\phi_p) - \min(\phi_p)) \quad (5)$$

where,  $n_p$  is the number of points of the design, and  $d_{ij}$  is the inter-point distance between all point pairs in

the design,  $\max(\phi_p)$  and  $\min(\phi_p)$  are the maximum and minimum values of  $\phi_p$  found in the generated DOEs.

The OLHD has the advantages in the cases of many design variables. One sampling point being on one level is difficult due to a large number of factors, therefore sampling points should be spread out evenly by applying optimum conditions. The main effect is obtained by a small number of experiments and the number of sampling points can be set freely [7]. Consequently, time and the number of experiments can be modified and conducted.

However, the disadvantage of OLHD is the difficulty in recognizing correlations between design variables. This is because the level of all the design variables should be identical and interactions between design variables are difficult to figure out. Reappearance is impossible because sampling point is randomly decided. The number of factors is three, however the design of experiments is by application of OLHD because good results are produced by a smaller number of experiments compared to CCD (Central Composite Design) or FFD (Full Factorial Design). Data processing experiments are conducted ten times for each output.

### 2.2.2 Progressive quadratic response surface methodology

In this paper, the progressive quadratic response surface methodology is used, which is one method of optimum design. The reason is that the conjugated gradient method on the basis of a non-linear optimum can't be used.

All of the design parameters are independent. PQRS is a method to solve this problem. Just  $(2n+1)$  calculations are necessary by choosing  $(2n+1)$  sample points to make quadratic approximation functions. And progressive calculation rest quadratic terms about interaction by normalized Quasi-Newton method does not need additional calculation [8].

## 3 RESULT & DISCUSSION

### 3.1 Analysis model

A 2-pole/12-slot design model is presented in this study (paper). The dimension of an initial model is shown in Fig. 3, and its brief specifications are listed in Table I.

In order to identify the shape ratio according to output power, 5 SLIM models were designed, 250W, 375W, 500W, 625W, and 750W, respectively, with constrains of identical pole numbers, slot numbers,

winding fill factors, current densities, air gap lengths etc. Fig. 4 shows the winding configuration of designed model. The presented winding configuration provides more compact size of linear motor than the conventional winding presented in work [9], but causing undesired high harmonics of current and high thrust force ripples.

By using FEM and optimization methodology, optimal designs of 5 SLIMs with different output powers for maximum power density were obtained. So optimization of SLIMs is firstly performed by the objective function and constraint conditions which are defined as follows:

- Objective function:

Thrust force  $\geq 23.2\text{N}$ ,  $91.1\text{N}$ ,  $151.3\text{N}$ ,  $205.0\text{N}$ ,  $323.7\text{N}$

- Constraint conditions:

Mover volume = constant according to output power, Output power @250W, 375W, 500W, 625W, 750W = Constant

Design variables are given in Fig. 5. Since the tooth width and yoke thickness were determined by using equivalent magnetic circuit method firstly, they are excluded in this optimal design. In addition to ML and HT, thickness of conductor plate (CT) is added.

Fig. 6 and Fig. 7 show the equal potential distribution of SLIM under the starting and rated condition. It can be found, at starting, slip is 1 and main flux does not flow through conductor plate.

RSM is applied to make appropriate response models of the thrust force [10]. The polynomial models of the responses are given by (6) ~ (10), respectively.

$$\hat{Y}_{250W} = 0.518 - 0.846X_1 + 2.177X_2 - 10.644X_3 + 0.004X_1^2 + 0.050X_2^2 - 29.081X_3^2 - 0.030X_1X_2 + 1.237X_1X_3 - 1.606X_2X_3 \quad (6)$$

$$\hat{Y}_{375W} = -0.447 - 0.026X_1 - 2.898X_2 + 14.884X_3 + 0.066X_2^2 - 40.834X_3^2 - 0.011X_1X_2 - 0.072X_1X_3 - 2.84X_2X_3 \quad (7)$$

$$\hat{Y}_{500W} = -7.763 + 1.519X_1 + 0.395X_2 - 38.802X_3 - 0.008X_1^2 - 0.035X_2^2 - 63.008X_3^2 + 0.008X_1X_2 + 20.766X_1X_3 - 1.789X_2X_3 \quad (8)$$

$$\hat{Y}_{625W} = -0.207 + 2.154X_1 + 1.151X_2 - 2.297X_3 - 0.005X_1^2 - 0.031X_2^2 - 8.953X_3^2 + 0.007X_1X_2 - 0.270X_1X_3 + 0.812X_2X_3 \quad (9)$$

$$\hat{Y}_{750W} = 1.773 - 0.463X1 + 13.168X2 + 2.799X3 - 0.008X1^2 - 0.093X2^2 - 30.205X3^2 - 0.021X1X2 + 0.633X1X3 - 0.916X2X3 \quad (10)$$

where  $X1$  is AL,  $X2$  is HT, and  $X3$  is CT.

Fig. 8 shows the converged design variables of 500W SLIM.

The other SLIMs are optimized by identical process. Design solutions of 5 SLIMs are listed in Table II.

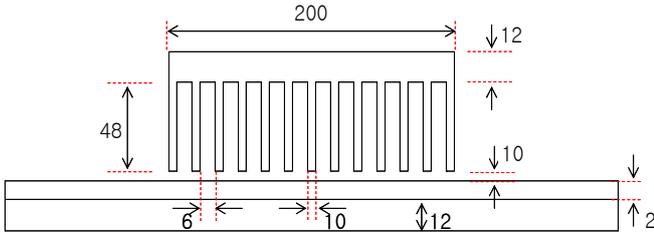


Figure 3. Dimensions of initial model

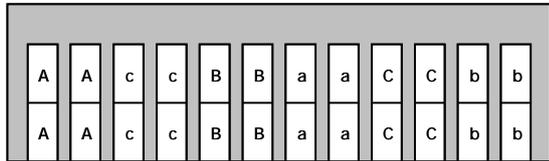


Figure 4. Winding configuration

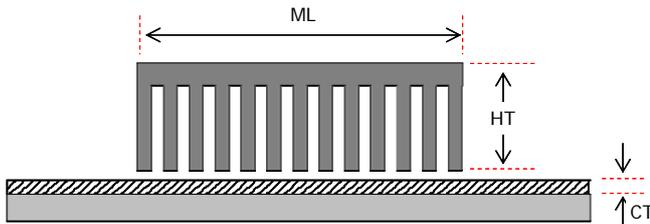


Figure 5. Variables for optimal design

TABLE I  
SPECIFICATION OF THE SLIM

|                        | Value                  |
|------------------------|------------------------|
| Number of poles        | 2                      |
| Number of phases       | 3                      |
| Number of slots        | 12                     |
| Input voltage          | 220 V                  |
| Frequency              | 60 Hz                  |
| Conductor conductivity | $3.12 \times 10^7$ S/m |
| Back iron permeability | $0.5 \times 10^7$ S/m  |
| Mover material         | S23                    |

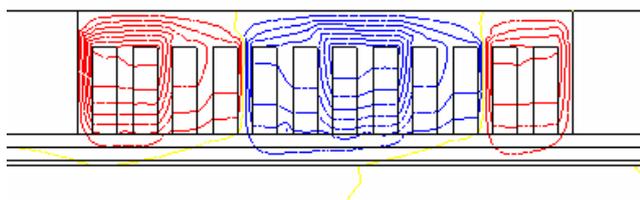


Figure 6. Equi-potentials of Model 3 at starting (slip:1)

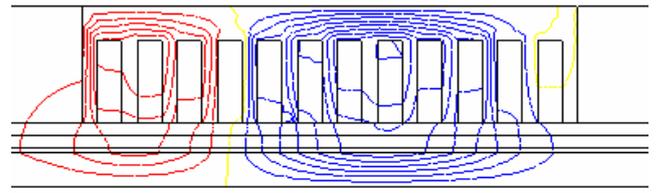


Figure 7. Equi-potentials of Model 3 at rated condition (slip:0.2)

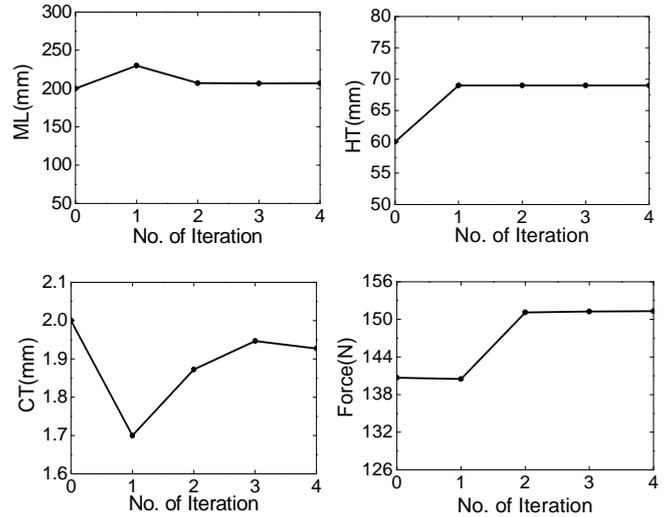


Figure 8. Converged design variables of 500W SLIM

TABLE II  
RESULT OF OPTIMUM DESIGN

|      | AL(mm) | HT(mm) | CT(mm) |
|------|--------|--------|--------|
| 250W | 115.0  | 25.5   | 1.15   |
| 375W | 172.5  | 51.8   | 1.28   |
| 500W | 207.1  | 69.0   | 1.90   |
| 625W | 227.1  | 78.3   | 2.13   |
| 750W | 255.0  | 87.5   | 2.55   |

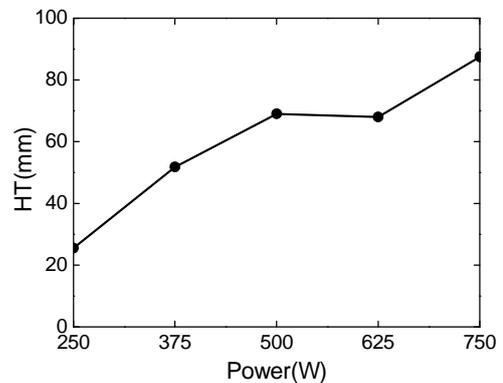


Figure 9. Determined HT for output power

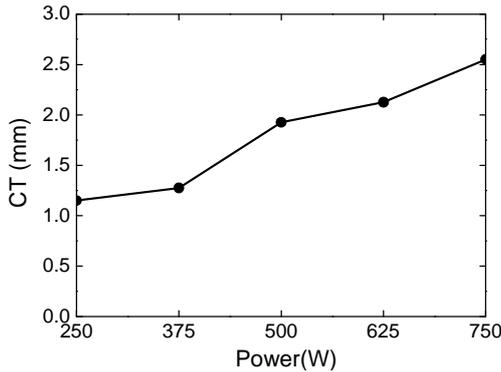


Figure 10. Determined CT for output power

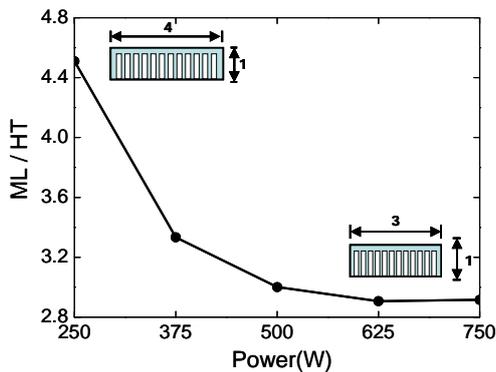


Figure 11. Shape ratio for output power

Fig. 9, Fig. 10, and Fig. 11 show the responses of ML, HT, and CT related to the output power increasing. Optimal values of design variables show the proportionality to the output power as shown in Fig. 9 to Fig. 11. For estimating the variation of ML and HT, the ratio of ML and HT is calculated. It is confirmed that as the output power increases, the shape ratio is decreases.

Therefore, in the design of SLIM, appropriate shape ratios should be chosen for maximizing output power.

#### 4 CONCLUSION

Optimal shape ratio of height and length of mover of SLIMs with respect to output power is determined in this paper. The results show that as the power becomes larger within a studied range, the shape ratio becomes smaller and converge on 2.9 (ML/HT).

The optimal design result can be restricted due to practical limitations of linear motors. However, the results can be a good reference for high power density design of SLIM.

#### ACKNOWLEDGEMENT

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