

# Development of the Integrated Railway Vehicle Model considering Catenary, Bogie and Wheel-Rail Contact

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**ABSTRACT:** Currently widely used railway vehicle is operated by supplied electric power from the catenary and the wheel of the vehicle is driven on the rail. The behaviors of the bogie system and the contact between wheel and rail are related with vehicle safety and ride quality. Also, dynamic interaction between catenary system and pantograph is related with the current collection performance of the railway vehicle. Because these consist of a railway vehicle, these behaviors have an effect on the each other. So, pre-evaluation of the whole system or each system is very important in the development of the railway vehicles. In this paper, using flexible multibody dynamic analysis method, simple integrated railway vehicle model was developed. And the driving simulation in high speed was carried out. Bogie, pantograph and wheel set are modeled as rigid body and catenary system is modeled as flexible body using the absolute nodal coordinate formulation which can effectively express behavior of the elastic large deformable parts. The contact force between wheel and rail is calculated using the Kalker's FASTSIM algorithm for straight line including irregularity of the rail. Using developed model, the situation that the vehicle drives on the rail with irregularity and interacts with a certain number of spans was simulated. Through the driving simulation, driving behaviors such as contact force of the wheel-rail and displacement of the wheel set and current collection performance such as contact force of the pantograph and loss of contact are calculated and evaluated. Developed simple integrated railway vehicle model is expected to be the foundation for developing more detail vehicle model.

## 1 INTRODUCTION

Railway vehicle largely consist of wheel sets, bogie, car body and pantograph. And each component has their role to drive the railway vehicle. The wheel sets in contact with the rail support and drive the vehicle. And the bogie plays a role for improving vehicle stability by absorbing vibrations which are transferred from the rail. The electric energy for operating vehicle is supplied from overhead contact line through pantograph. In order to estimate dynamic characteristics of such railway vehicle, lots of analysis program are developed form the past. In recent years, commercial analysis programs such as VAMPIRE, SIMPACK and VI-Rail are commonplace. Such commercial analysis programs have a lot of advantages however these programs can not consider dynamic interaction

between pantograph and overhead contact line. In this paper, using a flexible multibody dynamic analysis technique, the integrated railway vehicle model which includes pantograph-catenary system, wheel-rail contact etc. is developed. Using developed model, driving simulation on irregular rail is carried out in high speed. And each component is verified by international standard and comparing with commercial program. Through simulation, possibility of the integrated railway vehicle is presented.

## 2 CATENARY-PANTOGRAPH INTERACTION MODULE

### 2.1 Absolute Nodal Coordinate Formulation

In this paper, in order to make the difference between other analysis programs, the analysis model, which can consider dynamic interaction between catenary

system and pantograph, have been added. The catenary system can be considered as the large deformable flexible body, so it is modeled using absolute nodal coordinate formulation. Figure 1 represents absolute nodal coordinates for the inertial coordinates system of the beam element  $j$  of the large deformable elastic body  $i$ . An ANCF beam element consists of two nodes at both ends, and each node has the information of the position and its gradients. Therefore, absolute nodal coordinate vector can be represented as

$$\mathbf{e}^{ij} = [\mathbf{e}_A^{ijT} \ \mathbf{e}_B^{ijT}]^T = [e_1^{ij} \ e_2^{ij} \ e_3^{ij} \ e_4^{ij} \ e_5^{ij} \ e_6^{ij} \ e_7^{ij} \ e_8^{ij}]^T \quad (1)$$

The global position vector of an arbitrary point on the beam is represented using the global shape function and absolute nodal coordinates.

$$\mathbf{r}^{ij} = \begin{bmatrix} r_X^{ij} \\ r_Y^{ij} \end{bmatrix} = \mathbf{S}^{ij}(x) \ \mathbf{e}^{ij}(t) \quad (2)$$

Here,  $\mathbf{r}^{ij}$  is an arbitrary point in the beam element for the global reference frame and  $\mathbf{S}^{ij}$  is the shape function of the element. The shape function can be expressed using the cubic polynomial equation on the deformation in longitudinal and transversal direction.

$$\mathbf{S}^{ij}(x) = \begin{bmatrix} S_X^{ij} \\ S_Y^{ij} \end{bmatrix} = \begin{bmatrix} S_1 & 0 & S_2 & 0 & S_3 & 0 & S_4 & 0 \\ 0 & S_1 & 0 & S_2 & 0 & S_3 & 0 & S_4 \end{bmatrix} \quad (3)$$

$$S_1 = 1 - 3(\xi)^2 + 2(\xi)^3, \quad S_2 = l(\xi - 2(\xi)^2 + (\xi)^3)$$

$$S_3 = 3(\xi)^2 - 2(\xi)^3, \quad S_4 = l(-(\xi)^2 + (\xi)^3)$$

Where,  $\xi = x/l$ ,  $l$  represents the length of the beam elements. The kinetic energy of the elastic body  $i$  of the beam element  $j$  can be obtained by using displacement relationship represented in Equation 4.

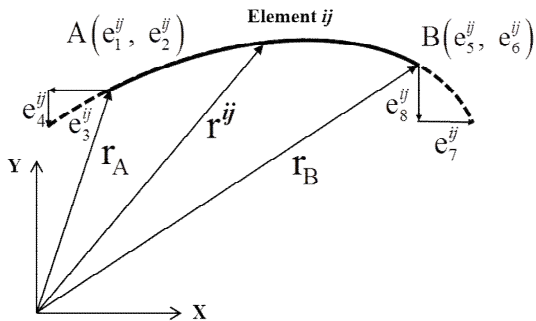


Figure 1. The absolute nodal coordinate on a beam element  $j$  on the large deformable body  $i$

$$T^{ij} = \frac{1}{2} \int_{V^{ij}} \rho^{ij} \dot{\mathbf{r}}^{ijT} \dot{\mathbf{r}}^{ij} dV^{ij} \quad (4)$$

$$= \frac{1}{2} \dot{\mathbf{e}}^{ijT} \left( \int_{V^{ij}} \rho^{ij} \mathbf{S}^{ijT} \mathbf{S}^{ij} dV^{ij} \right) \dot{\mathbf{e}}^{ij}$$

$$= \frac{1}{2} \dot{\mathbf{e}}^{ijT} \mathbf{M}^{ij} \dot{\mathbf{e}}^{ij}$$

In Equation 4,  $\rho^{ij}$  and  $V^{ij}$  means density and volume of a beam element, respectively. And  $\mathbf{M}^{ij}$  is the mass matrix of a beam element. Mass matrix is a function of shape function with time invariant, length and mass of the beam elements. Therefore it can be defined as Equation 5.

$$\mathbf{M}^{ij} = \int_{V^{ij}} \rho^{ij} \mathbf{S}^{ijT} \mathbf{S}^{ij} dV^{ij} = m^{ij} \int_0^l \mathbf{S}^{ijT} \mathbf{S}^{ij} dx \quad (5)$$

Finally, elastic force for a beam element can be defined as the sum of the axial deformation and the bending deformation. The strain energy of the axial deformation and the bending deformation can be defined as Equation 6 and Equation 7, respectively.

$$U_l = \frac{1}{2} \int_0^l EA \varepsilon^2 dx \quad (6)$$

$$U_t = \frac{1}{2} \int_0^l EI \kappa^2 dx \quad (7)$$

Where,  $E$  is the modulus of elasticity,  $\varepsilon$  is the strain for the axial deformation,  $I$  is the second moment of area and  $\kappa$  is the curvature of the deformed beam. Using the vector of the element generalized elastic forces for the axial deformation and bending deformation, the elastic force can be obtained as

$$\mathbf{F}_e = \mathbf{F}_l + \mathbf{F}_t \quad (8)$$

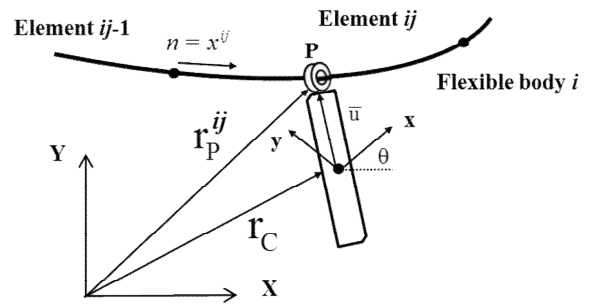


Figure 2. Sliding joint moving on a flexible body.

## 2.2 Sliding Joint

In this paper, in order to express dynamic interaction between catenary and pantograph, a sliding joint constraint is applied such as Figure 2. It is adequate to implement dynamic interaction between rigid body and large deformable flexible in multibody analysis. This constraint makes the point P defined as rigid body coordinate same as the point P defined as the beam element which consists of contact wire. To define the constraint equation, a non-generalized coordinate  $n$  is introduced. The constraint equation of the sliding joint can be defined as Equation 9.

$$\begin{aligned} \Phi(\mathbf{q}_r, \mathbf{q}_a, n, t) &= \mathbf{r}_p^{ij} - \mathbf{r}^R = \mathbf{0} \\ &= \mathbf{S}^{ij}(x^{ij} = n) \mathbf{e}^{ij}(t) - \mathbf{r}^R = \mathbf{0} \end{aligned} \quad (9)$$

Here,  $\mathbf{r}_p^{ij}$  is the global displacement vector defined from point P, and  $\mathbf{r}^R$  is the position vector defined from rigid body reference frame. Non-generalized coordinate  $n$  defines the position that sliding joint is constrained on a beam element. So,  $n$  must be less than the length of the beam element which belongs to a sliding joint.

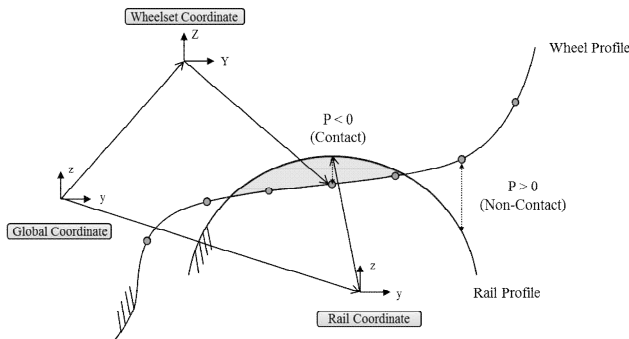


Figure 3. Contact point searching algorithm.

## 3 WHEEL-RAIL CONTACT MODULE

### 3.1 Contact Point Searching Algorithm

One of the most important features of the railway vehicles is that wheel and rail contact each other. The wheel-rail contact is difference from the common mechanical contact formulation. And to calculate contact force, various algorithms have been developed. In general, calculating contact force is accomplished through the following process. First, contact points are found by using searching algorithms and geometric information. And second, contact forces depending on the amount of

penetration are determined. Algorithms for searching contact point used in this paper are as follows. First, inputted wheel and rail profile data are represented using spline curve. And then initial position of the wheel and rail are determined by considering radius of the wheel and distance between each wheel, so on. And then variation of the contact points, which occur from relative motion of the wheel and rail, are transformed to global coordinate and calculated to distance between two points. At this point, when penetration depth is greater than 0.001 mm, penetration is assumed to occur. In the next step, wheel and rail displacements, which are changed by the contact force, are updated. And the above step is repeated.

### 3.2 Contact Force Calculation Algorithm

According to Hertz contact theory, shape of the contact point between wheel and rail is elliptical. Because the elliptical contact surface is divided as adhesion and slip zones, wheel speed and actual vehicle speed is different. This phenomenon is called creep. By creep phenomenon, creepage is occurred and it is defined by relative velocity between track and wheel. Equation 10 is the definition of the creepage.

$$\begin{aligned} \zeta_x &= \frac{(\mathbf{v}^w - \mathbf{v}^r)^T \mathbf{t}_1^r}{V} \\ \zeta_y &= \frac{(\mathbf{v}^w - \mathbf{v}^r)^T \mathbf{t}_2^r}{V} \\ \zeta_s &= \frac{(\mathbf{v}^w - \mathbf{v}^r)^T \mathbf{n}^r}{V} \end{aligned} \quad (10)$$

In Equation 10, subscript  $w$  and  $r$  define wheel and rail, respectively.  $\mathbf{t}_1^r$  and  $\mathbf{t}_2^r$  in Figure 4 is the unit vector of the movement and lateral direction.  $V$  is the speed of the direction of the railway vehicle and it can be defined as Equation 11.

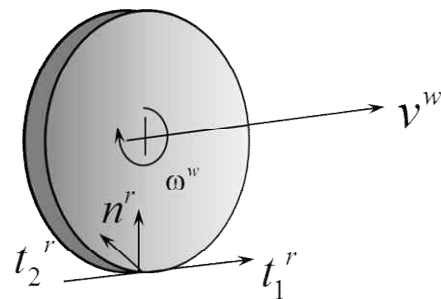


Figure 4. The definition of the each vector on contact point.

$$V = v^w t_1^r \quad (11)$$

In this paper, to calculate contact force, Kalker's FASTSIM algorithm is employed. The FASTSIM is an algorithm for increasing computing time and it is modified from simplified creep theory. The total algorithm for calculating contact force is represented in Figure 5.

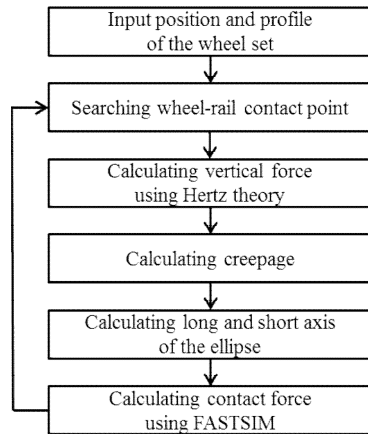


Figure 5. The process of the calculating contact force.

#### 4 INTEGRATED RAILWAY VEHICLE MODEL

Figure 8 shows a simple integrated railway vehicle model developed in this study. As mentioned earlier, railway vehicle model consist of wheel-rail contact module, bogie, car body and pantograph-catenary system. The wheel includes its shape and the rail includes shape and lateral and vertical irregularities as shown in Figure 6. And Figure 7 shows the profile of wheel and rail used in this paper. The bogie consists of primary and secondary suspension. The actual suspension system of the railway vehicle is complicated and it includes non-linear coil spring, yaw damper, anti-roll bar, z-link, air spring and so on. However, suspension model in this paper is applied to simplify. The catenary system with single contact line consists of the contact wire, messenger wire and dropper. As dropper is important component which connects contact and messenger wire, it distributes load of the contact wire to messenger wire. In analysis model, it was modeled as spring force element which has only tensile stiffness. The steady arm, which supports contact wire, is modeled using spring force element, too. Tension is applied to the end of each wire. The pantograph is assumed as 3 degree of freedom of mass-spring-damper system,

and sliding joint is used as a constraint to express the dynamic interaction with flexible body of the catenary system. In the model, catenary system is modeled as ANCF beam elements, and other elements are modeled as a rigid body. The main parameters of the vehicle model are as shown in Table 1.

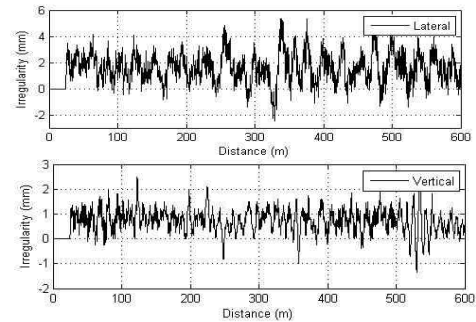


Figure 6. Lateral and vertical irregularities of the rail.

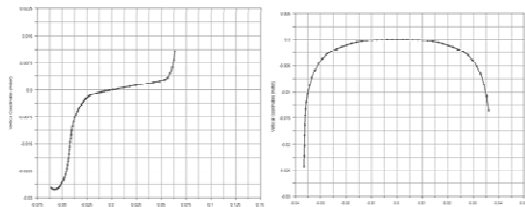


Figure 7. The profile of the wheel and rail.

Table 1 Main parameters of the vehicle model

| Parameter                           | Unit            | Value    |
|-------------------------------------|-----------------|----------|
| Distance between two bogies         | m               | 17.25    |
| Distance between two wheel-sets     | m               | 2.5      |
| Mass of car body                    | t               | 37.5     |
| Mass of bogie                       | kg              | 3825     |
| Mass of wheel-set                   | kg              | 1836     |
| Mass of collector header            | kg              | 7.2      |
| Mass of articulation frame          | kg              | 15       |
| Primary vertical spring stiffness   | N/m             | 50,000   |
| Primary vertical dashpot            | Ns/m            | 10,000   |
| Secondary vertical spring stiffness | N/m             | 86,000   |
| Secondary vertical dashpot          | Ns/m            | 30,000   |
| Collector header spring stiffness   | N/m             | 4,200    |
| Collector header dashpot            | Ns/m            | 10       |
| Articulation frame spring stiffness | N/m             | 50       |
| Articulation frame dashpot          | Ns/m            | 90       |
| Tension of contact wire             | N               | 20,000   |
| Tension of messenger wire           | N               | 16,000   |
| Dropper stiffness                   | N/m             | 100,000  |
| Mass/unit length of contact wire    | kg/m            | 1.35     |
| Mass/unit length of messenger wire  | kg/m            | 1.07     |
| Young's modulus of contact wire     | GPa             | 118      |
| Young's modulus of messenger wire   | GPa             | 110      |
| Area of contact wire                | mm <sup>2</sup> | 150.0E-6 |
| Area of messenger wire              | mm <sup>2</sup> | 65.49E-6 |

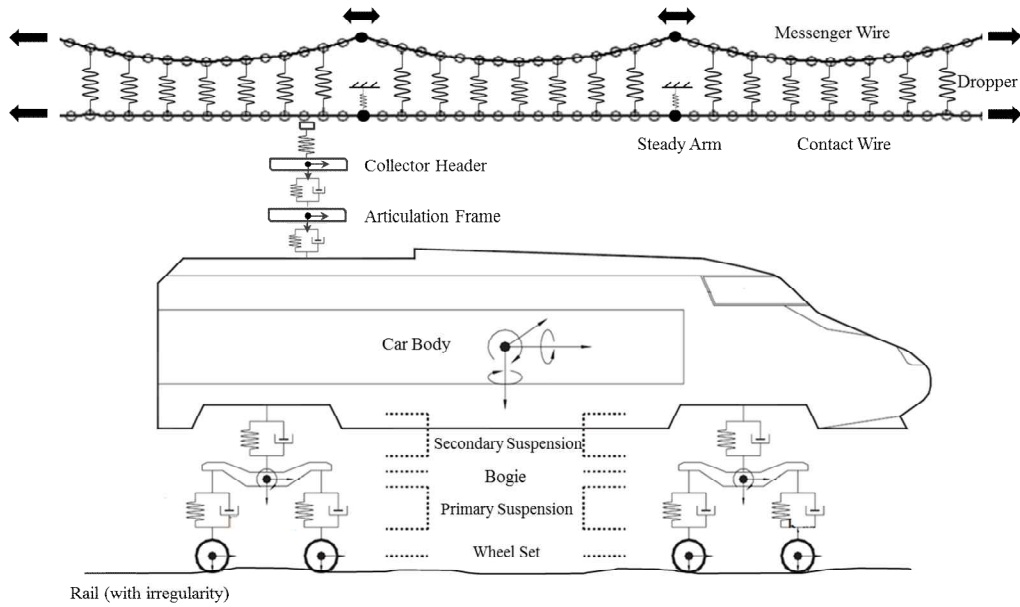


Figure 8. The integrated railway vehicle model.

## 5 DRIVING SIMULATION AND ANALYSIS RESULTS

Using generated railway vehicle model, driving simulation in high speed is carried out. The distance of the simulation track is total 600m for straight line with 10 spans of catenary wire. Through simulation, behavior of the each part of the train can be analyzed but, the main concern of the simulation is the dynamic interaction between catenary and pantograph considering behavior of the wheel and the rail. Before the driving simulation, stable conditions of the wire and each part for tension and gravity are obtained by static analysis. Next, static force of 120 N is applied to the articulation frame of the pantograph. Figure 9 shows the stable condition of the contact force after applying force. The speed of the train is 250km/h and while the vehicle drives on the irregular rail, pantograph interacts with contact wire. Figure 10 is the vertical displacement of the wheel-set, bogie and collector header of the pantograph. Wheel-sets vibrate up and down due to

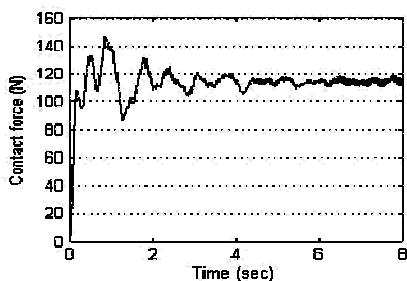


Figure 9. Contact force after applying static force.

the irregular rail, and vibration passed from the bogie is reduced by primary suspension. The displacement of the pantograph collector header is increased during driving one span and decreased when the pantograph pass through steady arm. Figure 11 shows the contact force between contact wire and pantograph. The behavior of the contact force is similar with the behavior of the collector header. Table 1 represents the simulation results such as mean value and max/min contact force, etc. about current collection performance. Through the simulation and results, the current collection performance of the vehicle model can be estimated.

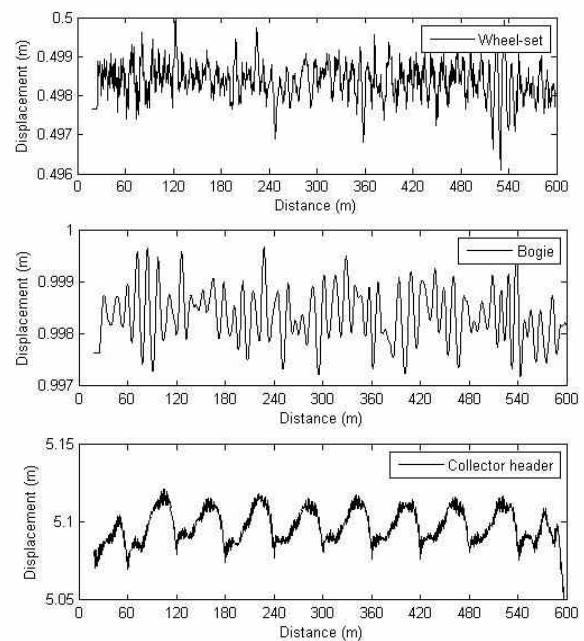


Figure 10. Displace of the wheel-set, bogie and collector header.

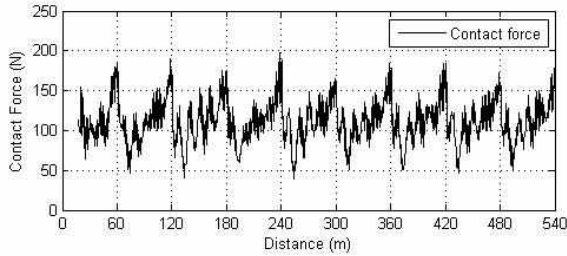


Figure 11. Contact force between contact wire and pantograph.

Table 2 Results about current collection performance

| Results                         | Unit | Value  |
|---------------------------------|------|--------|
| Speed                           | km/h | 250    |
| $F_M$                           | N    | 113.54 |
| $\sigma$                        | N    | 27.32  |
| Actual maximum of contact force | N    | 197.73 |
| Actual minimum of contact force | N    | 47.36  |

## 6 CONCLUSION

In this study, simplified railway vehicle model including catenary-pantograph, bogie carbody and wheel-rail contact module is developed using multibody dynamic analysis method. In order to express flexibility of the catenary wire, catenary is model by using absolute nodal coordinate formulation and other parts are modeled by rigid body. In the wheel rail contact module, contact point searching algorithm and contact force calculation algorithm are developed and employed. Through simulation, behavior of each body is estimated, especially, dynamic interaction between catenary and pantograph is focused. Using the integrated railway vehicle model, analysis of the current collection performance is carried out considering locomotive behavior such as wheel-rail, bogies and carbody.

## ACKNOWLEDGEMENT

### 7 ACKNOWLEDGEMENT

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