# Hopf Bifurcation of Maglev System with Coupled Elastic Guideway

No. 10
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ABSTRACT: In this paper, the dynamical behavior of single magnet suspension system of maglev train considering the elastic guide-way is researched through HOPF bifurcation theory. With three states feedback variables (gap, current, velocity), the HOPF bifurcation conditions and corresponding vibration frequency of the coupled system is presented under different flexible guideway conditions, that is, the guideway parameters are changeable. These results can provide with beneficial help for improving the traditional design of flexible guideway and control algorithms.

## 1 INTRODUCTION

In the present, the TR08 in Shanghai often appears violently flexible vehicle-guideway-coupled vibration while running at the steel guideway in the maintenance depot or crossing the turnoff at low velocity. The CMS03 train developed by Changsha Institute of Technology also vibrates seriously while suspending on the floating guideway. But the TR08 train in Shanghai line doesn't vibrate on the cement girder whose density is 7 ton per meter.

In order to solve the problem of vibration phenomena of maglev train, we have to propose more strict requirements for the precision and dynamical performance of the system, and investigate the practical engineering problem via nonlinear system theory. Early in 1986, Nagai and Masao[1] had paid attention to the nonlinear self-oscillation owing to neglecting elastic deformation in the controller design, and designed a control algorithm which can make the vehicle suspend stably on the flexible guideway and be insensitive to the flexible conditions. Youhe Zhou and Xiaojing Zheng[2-3] established a constant coefficient differential equation of guideway dynamical equations with variable coefficients, and judged the dynamic stability of maglev system via Lyapunov characteristic index, and then studied the probability the appearing chaotic phenomenon under the nonlinear condition. Guidong Liu and Longhua She[4] given the HOPF bifurcation condition of nonlinear maglev controller, and discussed the bound condition and vibration frequency which can make the controller have self-oscillation. Xiaohong Shi and Longhua She[5-6] utilized the numerical value calculation method of HOPF bifurcation to study the corresponding relationship between guideway parameters and HOPF bifurcation, and gave the intrinsic frequency scope making the guideway suspending stably. They also qualitatively explained the variable relationship of system load, the controller frequency, the rigidity of second system, the inherent frequency of guideway and so on.

In this paper, the relationship of variable elastic parameters of guideway and the possibility of vehicle-guide-way-coupled vibration was investigated through three states feedback variables (gap, current, velocity) to determine the corresponding coupled frequency. Then the coupled vibration phenomenon was made clear by HOPF theory. Of course, there exist some systems not using the state feedback control methods. But the idea of this paper is also valuable, that is, using the method of fixed control parameters to study the influence to the system with variable guideway parameters.

## 2 ELASTIC COUPLED VIBRATION MODEL OF SINGLE MAGNET SYSTEM

### *2.1 physical model of suspension system*

The attractive electromagnetically levitated train is suspended generally by many suspension magnets.

The HSST-100L model in Japan has 32 suspension points. There are 16 suspension points in CMS03 train in China. The influence of every suspension points can be neglected owing to using the mechanical decoupling and essential control decoupling. Therefore, this paper will make single suspension point as research object to study the control problem of flexible coupled vibration between vehicle and guideway. Every suspension point has the principle structure as the Fig. 1. The guideway is a simple pivot, $m$ is the mass under the second system, $M$ is the mass above the second system, and the mass of electromagnet is included in the mass of $m$. If we select the air-spring as second suspension system, the dynamical rigid coefficient and damp coefficient between $m$ and $M$ will be very small, so the influence of the two parameters can be neglected while studying the elastic coupled vibration. In addition, suppose that the length of electromagnet is much less than the length of guideway, so the electromagnet force can be adopted as a centralized model.

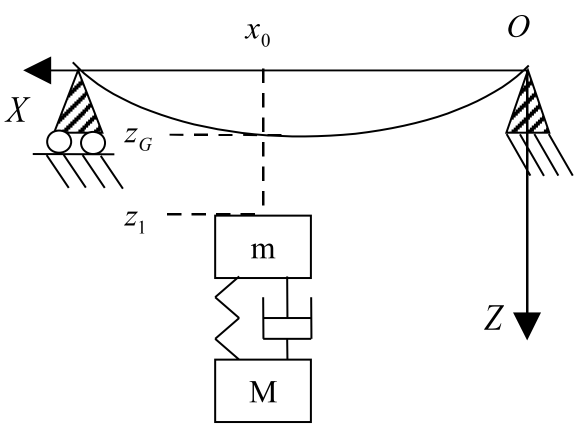


Figure 1. the principle structure of suspension point

In the Figure 1, $OZX$ is the coordinate connected with guideway pier, $x_0$ is the horizontal coordinate , $z_1$ is the vertical displacement of electromagnet in the place of $x_0$, $z_G$ is the vertical displacement of guideway in the place of $x_0$.

With respect to electromagnet, suppose the symbol parameters as following. Here $u$ represents the voltage of the electromagnet winding, $i$ is the current of the electromagnet, $R$ is the resistance of the electromagnet, $L_d$ is the inductance of the electromagnet, $F_e$ is the electromagnetical force, $C_1$ is the electromagnetical coefficient, $g$ is the acceleration of gravity and $t$ is the time variable.

With regard to guideway, suppose the symbol parameters as following. Here $E_G$ denotes YANG elastic variable, $I_G$ is the section inertia of guideway, $\rho_G$ is the line density of the mass, $l_G$ is the span of guideway, $m_G$ is the mass of guideway ($m_G = \rho_G l_G$), $\phi_n$ is the $n$ th vibrational function, $q_n$ is the $n$ the broad coordinate, $\eta_n$ is the damp ration of the $n$ th model, $\omega_n$ is the inherent frequency of the $n$ th model and $Q_n$ is the broad force.

Considering the simple girder, the Bernoulli-Euler vibrational partial differential equation is adopted to solve the system equations as follows.

$$z_G(x,t) = \sum \phi_n(x) q_n(t) \tag{1}$$

$$\phi_n(x) = \sqrt{\frac{2}{m_G}} \sin\left(\frac{n\pi}{l_G} x\right) \tag{2}$$

$$\ddot{q}_n + 2\eta_n \omega_n \dot{q}_n + \omega_n^2 q_n = Q_n \tag{3}$$

$$\omega_n = \left(\frac{n\pi}{l_G}\right)^2 \sqrt{\frac{E_G I_G}{\rho_G}} \tag{4}$$

$$Q_n = \sqrt{\frac{2}{m_G}} \sin\left(\frac{n\pi x_0}{l_G}\right) \bullet F_e \tag{5}$$

With regard to the electromagnet, we can get the equations as follows.

$$L_d = \frac{2C_1}{(z_1 - z_G)} \tag{6}$$

$$u = Ri + \frac{2C_1}{(z_1 - z_G)} \frac{di}{dt} - \frac{2C_1 i}{(z_1 - z_G)^2} \frac{d(z_1 - z_G)}{dt} \tag{7}$$

With regard to the dynamical system, the equations can be gotten as follows.

$$F_e = C_1 \left(\frac{i}{z_1 - z_G}\right)^2 \tag{8}$$

$$(M + m) g - F_e = m\ddot{z}_1 \tag{9}$$

Suppose that the $n$ th coupled vibration between vehicle and guideway occurs, the equations above can be concluded as follows

$$(M + m) g - C_1 \left(\frac{i}{z_1 - z_G}\right)^2 = m\ddot{z}_1 \tag{10}$$

$$u = Ri + \frac{2C_1}{(z_1 - z_G)} \dot{i} - \frac{2C_1 i}{(z_1 - z_G)^2} (\dot{z}_1 - \dot{z}_G) \tag{11}$$

$$\ddot{z}_G + 2\eta_n\omega_n\dot{z}_G + \omega_n^2 z_G = C_2 C_1\left(\frac{i}{z_1 - z_G}\right)^2 \quad (12)$$

The three equations above (10-12) make up of complex coupled nonlinear physical model, where $C_2$ is a new defined parameter. Here $C_2$ means the square of magnitude of the unitary vibration functional in this place.

$$C_2 = \left(\sqrt{\frac{2}{m_G}}\sin\left(\frac{n\pi}{l_G}x_0\right)\right)^2 \quad (13)$$

From the analysis above, we can find out that for different $n$, that is, there are different order coupled vibration between vehicle and guideway, and the forms of functions (10-12) are unchangeable but the value of $\eta_n, \omega_n, C_2$ in (12). Therefore, we can study the first order coupled vibration in this paper.

### *2.2 State space model of the system and three state feedback close-loop control*

The physical meaning indicates that $z_1, \dot{z}_1, z_G, \dot{z}_G, i$ are independent state variables of the system above. At present, the suspension gap, the vertical vibrational acceleration and the current of the electromagnet can be measured directly by some sensors, while $z_G, \dot{z}_G$ are difficult to be measured. Especially with the moving of vehicles at the guideway, the horizontal coordinates of $z_G, \dot{z}_G$ are changable ceaselessly. So the nonsingularity transformation is designed as follows.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ \dot{z}_1 \\ z_G \\ \dot{z}_G \\ i \end{bmatrix} \quad (14)$$

Then a new set of independent state variables which are easy to be measured can be gotten as $x_1 = z_1 - z_G$, $x_2 = \dot{z}_1 - \dot{z}_G$, $x_3 = z_1$, $x_4 = \dot{z}_1$, $x_5 = i$. We can prove that after the non-singularity linear transformation, the controllability of the equilibrium point is unchangeable. After the new state variables are selected, considering the first order coupled vibration, the equations (10-12) can be described as that

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{M+m}{m}g - (\dfrac{C_1}{m} + C_2C_1)\dfrac{x_5^2}{x_1^2} - 2\eta_1\omega_1 x_2 \\ \qquad + 2\eta_1\omega_1 x_4 - \omega_1^2 x_1 + \omega_1^2 x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{M+m}{m}g - \dfrac{C_1 x_5^2}{m x_1^2} \\ \dot{x}_5 = \dfrac{x_5}{x_1}x_2 - \dfrac{R}{2C_1}x_1x_5 + \dfrac{x_1}{2C_1}u \end{cases} \quad (15)$$

Here we take suspension gap, the displacement and the current of the electromagnet as the state feedback variables, that is,

$$u = u_{ec} + k_s(x_1 - s_e) + k_{bi}x_4 + k_c x_5 \quad (16)$$

Here $s_e$ is the expected gap of the guideway, $k_s$ is the feedback parameter of the suspension gap, $k_{bi}$ is the coefficient of vertical velocity of the electromagnet and $k_c$ is the feedback coefficient of current.

## 3 RELATED HOPF THEORY

In the traditional HOPF theory[4], for normal nonlinear differential equation

$$\dot{x} = f(x,\mu), x \in R^n, \mu \in R \quad (17)$$

the balance point of the system is $x = x_0(\mu)$, that is $f(x_0(\mu),\mu) = 0$. After suitable transformation, the balance point $x = x_0(\mu)$ can be switched to the origin. In general, suppose the balance point of the system is the origin. Suppose that $x$ and $\mu$ are resolved near to the origin, and $f(x,\mu) \equiv 0$ when $\mu$ is a part of region including zero. The Jacobian matrix [6] can be gotten as $A(\mu) = D_x(0,\mu)$, if

(*i*) $A(\mu) = D_x(0,\mu)$ has a pair of complex roots, $\lambda$ and $\bar{\lambda}$. Here $\lambda(\mu) = \alpha(\mu) + i\omega(\mu)$ where $\omega(\mu_0) = \omega_0 > 0$, $\alpha(\mu_0) = 0$, $\alpha'(\mu_0) \neq 0$;

(*ii*) the other enginvalues of $A(\mu_0)$ have negative real part.

Then the system will have HOPF bifurcation with the parameter $\mu = \mu_0$, that is, a periodical resolution occurs near to the point $\mu = \mu_0$.

HOPF theory is mature and perfect, but if the order of the mathematical model is very high, the calculation will be very complex and the result is difficult to conclude. Therefore, some researchers brought forward some simple and effective algebra criterion. Using the Hurwitz determinants, an algebraic criterion and corresponding computational method for determining the Hopf bifurcation point is proposed in the reference[7]. This method does not need to calculate all the eigenvalues of Jacobian matrix of the system for any parameter and saves computer time demand. Using the method, the critical speed of wheelset and the hunting are studied.

The algebra criterion is described as follows.

$$\lambda^n + a_1(\mu)\lambda^{n-1} + ... + a_{n-1}(\mu)\lambda + a_n(\mu) = 0 \qquad (18)$$

The characteristic equation $\det(A(\mu) - \lambda I) = 0$ is changed as

$$\lambda^n + a_1(\mu)\lambda^{n-1} + ... + a_{n-1}(\mu)\lambda + a_n(\mu) = 0 \qquad (19)$$

where $a_i(\mu)$, $\Delta_i(\mu)$ is denoted as $n$, $\Delta_i$ $(i = 1, 2, ...n)$. The Hurwitz determinant can be constructed as follows by the coefficients of equation (18).

$$\Delta_m = \begin{vmatrix} a_1 & 1 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 1 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{2m-1} & a_{2m-2} & a_{2m-3} & a_{2m-4} & \cdots & am \end{vmatrix}, (m = 1, 2, ..., n)$$

where if $i > n$, $a_i = 0$.

**Theory 1.** Real coefficient algera equation (18) has a pair of pure imaginary roots and other $(n-2)$ roots all have negative real part, if and only if $a_1 > 0, a_2 > 0, ..., a_n > 0$ and $(i = n-3, n-5, ...)$, where $\Delta_i > 0$ is the Hurwitz determinant of equation (18).

**Theory 2.** If real coefficient algebra equation (18) has a pair of pure imaginary roots $\pm\omega i$ and other $(n-2)$ roots all have negative real part, then $\omega^2 = \frac{\Delta_{n-3}}{\Delta_{n-2}} a_n$.

**Theory 3.** If the eigenpolynominal equation (18) of the Jacobian matrix of the system (17) has negative real part, and Hurwitz determinant is satisfied with the conditions below,

$(i)$ $\Delta_{n-3}(\mu_c) > 0$

Where $\mu_c = \min_{\mu}\{|\mu - \mu_0| : \Delta_{n-1}(\mu) = 0\}$, then equation (18) has a pair of pure imaginary root $\pm\omega i$ at the point of $\mu = \mu_c$ and other eigenvalues all have negative real part. Suppose that $U$ and $V$ are left and right eigenvector to the eigenvalue $i\omega_c$ of the matrix $A(\mu_c)$.

(ii) $\mathrm{Re}(UBV) \neq C$, where $B = \left.\frac{A(\mu)}{d\mu}\right|_{\mu=\mu_c}$, then there exists a HOPF bifurcation at the point of $\mu = \mu_c$ for the system (17). That is, near to the point of $\mu = \mu_c$, the system (17) has periodical movement.

## 4 HURWITZ CRITERION OF HOPF BIFURCATION IN THE APPLICATION OF MAGLEV COUPLED VIBRATIONAL SYSTEM

In this paper, based on the three state feedback control (gap, current, velocity), we discuss the HOPF bifurcation conditions and corresponding vibration frequency of the coupled system, which are gotten under different flexible guideway parameters. Here we take practical data in the maintaince depot of Shanghai high speed maglev line as parameters of equations above, which are summed up as follows.

Table 1.

| parameters | $m$ | $M$ | $g$ | $R$ | $C_1$ | $s_e$ |
|---|---|---|---|---|---|---|
| values | 500 | 1000 | 9.8 | 1 | 0.0025 | 0.01 |

By the HOPF theory and simulation, we can get that the system is stable if the three control parameters and three guideway parameters are set up as follows.

Table 2.

| parameters | $k_c$ | $k_{bi}$ | $k_s$ | $\omega_1$ | $\eta$ | $C_2$ |
|---|---|---|---|---|---|---|
| values | -50 | 500 | 140000 | $90\pi$ | 0.001 | 0.002 |

Then we will analyze the HOPF bifurcation and vibration cases when the parameters $k_s$、$k_{bi}$ $k_c$ are fixed and $\eta_1, \omega_1, C_2$ are changeable. That is, when the train is running at the guideway, the control parameters are unchangeable, but if the parameters of guideway are variable, the coupled vibration case may occur.

For the equations (16-17), the balance point can be calculated as

$$(x_1^0, x_2^0, x_3^0, x_4^0, x_5^0) = (s_e, 0, se + \frac{C_2(M+m)g}{\omega_1^2 C_1}, 0, \pm s_e\sqrt{\frac{(M+m)g}{C_1}})$$

And the state of the balance point are satisfied with $x_1 = s_e$, $u_{ec} = (R - kc)x_5$.

The system has two singular points whose characteristic are same, and they are only determined by the parameter $k_c$. Therefore, we will analyze the case only when $x_s^0$ is positive, and simply denote $x_s^0$ as $\mathbf{x}_0$. Here $C_3$ is expressed as $C_3 = \sqrt{\frac{(M+m)g}{C_1}}$, and then the balance point is denoted as

$(s_e, 0, s_e + \frac{C_2 C_3{}^2}{\omega_1^2}, 0, s_e C_3)$.

At the balance point $\mathbf{x}^0$, after the system is lineared, we can get the Jacobian determinant [8] as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2(\frac{C_1}{m} + C_2 C_1)\frac{C_3^2}{s_e} - \omega_1^2 & -2\eta_1\omega_1 & \omega_1^2 & 2\eta_1\omega_1 & -2(\frac{C_1}{m} + C_2 C_1)\frac{C_3}{s_e} \\ 0 & 0 & 0 & 1 & 0 \\ 2\frac{C_1 C_3^2}{m s_e} & 0 & 0 & 0 & -2\frac{C_1 C_3}{m s_e} \\ \frac{s_e k_s}{2C_1} & C_3 & 0 & \frac{s_e k_{bi}}{2C_1} & \frac{s_e(k_c - R)}{2C_1} \end{bmatrix}$$

Then the corresponding characteristic polynomial of the Jacobian determinant can be inferred as

$$J(\lambda) = \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5$$

where

$$a_1 = \frac{-m s_e k_c + m s_e R + 4m\eta\omega_1 C_1}{2mC_1}$$

$$a_2 = \frac{-2m\eta\omega_1 s_e k_c + 2m\eta\omega_1 s_e R + 2C_3 k_{bi} C_1 + 2\omega_1^2 m C_1}{2mC_1}$$

$$a_3 = (-2C_3^2 C_1 R + \omega_1^2 m s_e R + 2C_3^2 C_1 k_c + 2C_3^2 C_1 C_2 m k_c$$
$$-2C_3^2 C_1 C_2 m R - \omega_1^2 m s_e k_c + 2k_s C_3 C_1 + 4\eta\omega_1 C_3 k_{bi} C_1$$
$$+2k_s C_3 m C_1 C_2)\frac{1}{2mC_1}$$

$$a_4 = \frac{4C_3^2 C_1 \eta\omega_1 k_c + 2\omega_1^2 C_3 k_{bi} C_1 + 4k_s C_3 C_1 \eta\omega_1 - 4C_3^2 C_1 \eta\omega_1 R}{2mC_1}$$

$$a_5 = \frac{-2C_3^2 C_1 \omega_1^2 R + 2C_3^2 C_1 \omega_1^2 kc + 2k_s C_3 C_1 \omega_1^2}{2mC_1}$$

Then the Hurwitz determinant can be built as follows.

$$\Delta_5 = \begin{vmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & 0 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & 0 & a_5 \end{vmatrix}$$

where

$$a_1 = 102+2\eta_1\omega_1$$

$$a_2 = 204\eta_1\omega_1 + 2424.871132 + \omega_1^2$$

$$a_3 = 79203.91680 + 102\omega_1^2 + 39601958.40C_2$$
$$+ 4849.742264\eta_1\omega_1$$

$$a_4 = 158407.8332\eta_1\omega_1 + 2424.871132\omega_1^2$$

$$a_5 = 79203.91680\omega_1^2$$

In the following, we will discuss the HOPF bifurcation cases of the coupled vibration system. From the Theory 1, if the eigenpolynominal appears a pair of pure imaginary engenvalues and the other characteristic roots have negative real part, it should satisfy $a_1 > 0, a_2 > 0, ..., a_5 > 0$ and $\Delta_4 = 0, \Delta_2 > 0$. That is

$$102+2\eta_1\omega_1 > 0$$

$$204\eta_1\omega_1 + 2424.871132 + \omega_1^2 > 0$$

$$79203.91680 + 102\omega_1^2 + 39601958.40C_2 + 4849.742264\eta_1\omega_1 > 0$$

$$158407.8332\eta_1\omega_1 + 2424.871132\omega_1^2 > 0$$

$$79203.91680\omega_1^2 > 0$$

We can see that because of the three positive guide parameters, the inequities $a_1 > 0, a_2 > 0, ..., a_5 > 0$ are always satisfied.

Now we try to determine the three guideway parameters with some suitable values, which are useful to search the HOPF points. Here let $\eta_1 = 0.005$ , $\omega_1 = 45\pi$ be fixed constant values, and then we can get $C_2 = 0.001021609147$ and $\Delta_2 = 171647 > 0$ by through the calculation of $\Delta_4 = 0$.

By verification, the parameters above are all satisfied with every condition of the Theory 1.

Based on the Theory 2, when the system has two pure imaginary root and the other roots are all have negative real parts, we can get

$$\omega^2 = \frac{\Delta_2}{\Delta_3} a_5$$

$$\Delta_2 = 170791$$

$$\Delta_3 = 1.34214296*10^{10}$$

$$a_5 = 1581360850.$$

By the Theory 2, we can get $\omega^2 = \frac{\Delta_2}{\Delta_3} a_5$. Then the pair of pure imaginary roots are $\pm\omega i = \pm 141.86i$.

By the Theory 3, we can verify the results above as follows, where left eigenvectors are

[-755.9688281-338.2810799*I, -3.244227090+5.618705407*I, 790.8090697+456.6112686*I, 2.943399970-5.409647967*I, -0.1883176719e-1-0.4701036538e-1*I]

The right eigenvectors are

[0.5823415524e-3-0.5395037417e-3*I, 0.7653206094e-1+0.8260887800e-1*I, -0.1198986822e-5-0.1935163616e-4*I, 0.2745153460e-2-0.1700840350e-3*I, 1.402153190-1.468820170*I]$^T$

$$B = \frac{dA(k_s)}{dk_s} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2.94*10^6 & 0 & 0 & 0 & 1212.435566 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

UBV=-1133.149063-563.89685*I .

Because the real part of UBV is not zero, the system has theoretical HOPF bifurcation point under the condition $\eta_1 = 0.005$ , $\omega_1 = 45\pi$ , $C_2 = 0.001$ and the vibration frequency is $141.86Hz$ .

The table below shows the corresponding coupled vibrational frequencies calculated under different parameters of guideway. It should be pointed out that when the selected inherent frequency $\omega_1$ is increasing, the coupled vibrational frequency $\omega$ calculated by the theories above is near to $\omega_1$, especially when $\omega_1 > 15\pi$ .

Table 3.

| number | $\omega_1$ | $C_2$ | $\eta_1$ | $\omega$ |
|---|---|---|---|---|
| 1 | $15\pi$ | 0.001 | 0.00936 | 54.96 |
| 2 | $15\pi$ | 0.003 | 0.061 | 60.15 |
| 3 | $15\pi$ | 0.005 | 0.11786 | 65.83 |
| 4 | $30\pi$ | 0.0005 | 0.00568 | 95.01 |
| 5 | $30\pi$ | 0.001 | 0.0113 | 95.73 |
| 6 | $30\pi$ | 0.003 | 0.0332 | 98.25 |
| 7 | $30\pi$ | 0.005 | 0.0542 | 100.75 |
| 8 | $30\pi$ | 0.008 | 0.0842 | 104.06 |
| 9 | $45\pi$ | 0.001 | 0.005 | 141.86 |
| 10 | $60\pi$ | 0.001 | 0.0024 | 190.28 |
| 11 | $60\pi$ | 0.003 | 0.0072 | 189.31 |
| 12 | $90\pi$ | 0.001 | 0.0008 | 282.92 |
| 13 | $90\pi$ | 0.003 | 0.0024 | 294.4 |

## 5 CONCLUSIONS

In this paper, the vehicle-guideway-coupled vibration problem of single magnet suspension system of maglev train is considered using the HOPF bifurcation tool. We discuss the relationship of different guideway parameters of the coupled system, and then the coupled vibrational frequency is calculated with quantitative analysis. It should be pointed out that only three state variables are chosen as feedback parameters in this paper, but other states are also can be applied to feedback control in practice. Next plan is to combine with more state variables under different elastic guideway conditions. The expected results should improve the design of flexible guideway or control algorithms.

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