

# Phase Set Shift Effect for Cogging Force Reduction in Linear Motor

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**ABSTRACT:** This paper discusses the reduction effect of cogging force in a linear motor using the phase set shift. Due to the cogging force has a periodic characteristic, the sum of cogging force can be decreased with proper phase set shift. The phase set shift has a drawback of thrust loss as in a skew effect. With multi-phase set stator structure, however, cogging reduction is much effective than loss in a thrust force. It is found that certain value of the phase set shift decreases the cogging force according to number of the phase set and it is effective for cogging force reduction. It is also verified that this method shows similar effect with stator skewing.

## 1 INTRODUCTION

Recently, permanent magnet linear motors have been very much in demand for industrial applications. They have a higher positioning precision and better dynamic performance than rotary motors, which implement the linear motion by means of a geared system. One of the major problems with the operation of linear motors is their large cogging force. The cogging force is caused by the force of attraction between the permanent magnets and stator structures. It also produces thrust ripples and has harmful effects on the positioning accuracy and dynamic performance. Thus, the cogging force should be minimized for precise and stable operation [1]-[3].

Generally, the cogging force can be divided into the core cogging force and the teeth cogging force. The core cogging force, whose period is the pole pitch, is caused by the interaction of the PM and back iron. The teeth cogging force originates from the force of attraction between the PM and teeth structure. The sum of these two cogging force components is related to the great-common-divisor(GCD) of the pole pitch and teeth pitch.

In linear planar machines, various cogging force reduction methods have been proposed, such as chamfering, bifurcating, slot opening and skewing, etc. In linear tubular machines, different techniques have been adopted, due to their curved structure.

These include varying the magnet width, using special shaped magnets, adjusting the pole pitch in the mover structure, etc. [1]-[5].

This paper examines the linear combination characteristics of the cogging force and proposes a method of reducing the cogging force using the phase set shift. The higher harmonics of the cogging force related to the pole pitch and teeth pitch are also presented. The length of the phase set shift can be determined by considering the cogging force characteristics and the similarity between the phase set shift and the skew effect is discussed.

## 2 COGGING FORCE ANALYSIS OF LINEAR MOTOR

### 2.1 Linear Combination Characteristics of the Teeth Cogging Force

Since the teeth cogging force is caused by the interaction between the teeth and magnets, the cogging force of one tooth and one magnet can be the basis function of the total cogging force. This means that the total cogging force of the entire set of teeth and magnets can be obtained by adding the basis component while taking into consideration the phase difference. If  $F(x)$  is the basis cogging force function of a single tooth and single magnet, the total cogging force is given by :

$$F_{Total}(x) = \sum_{j=0}^{N_p-1} \sum_{i=0}^{N_T-1} F(x - i\tau_T - j\tau_p) \quad (1)$$

where  $\tau_p$ ,  $\tau_T$ ,  $N_p$  and  $N_T$  are the pole pitch, teeth pitch, number of poles and number of teeth, respectively.  $F_{Total}(x)$  is a periodic function and its period is related to the pole pitch and teeth pitch. To prove this relation, basic models with various teeth and pole structures are shown in Figure 1. In order to excluding the effect of the core cogging force, the size of the back core is made sufficiently large. The results of this analysis are shown in Figure 2.

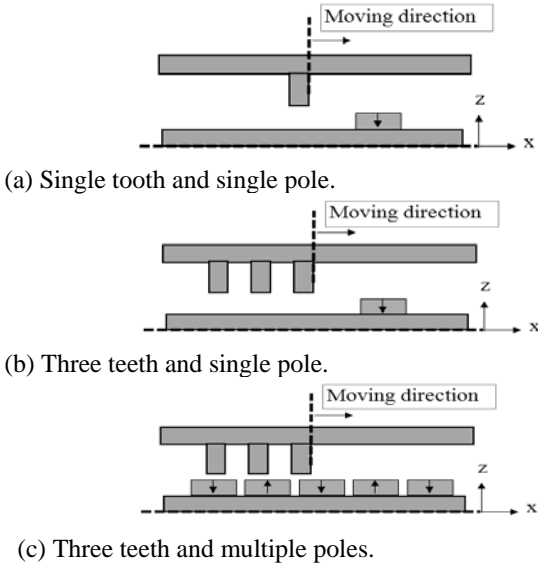
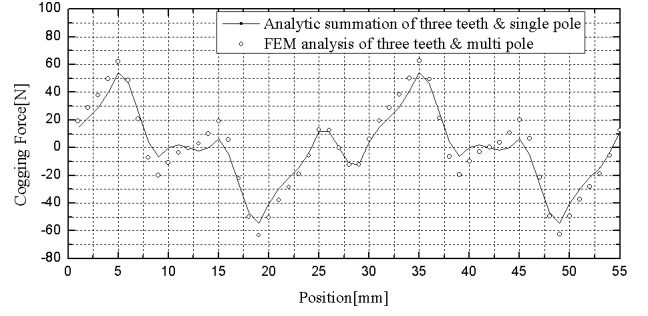
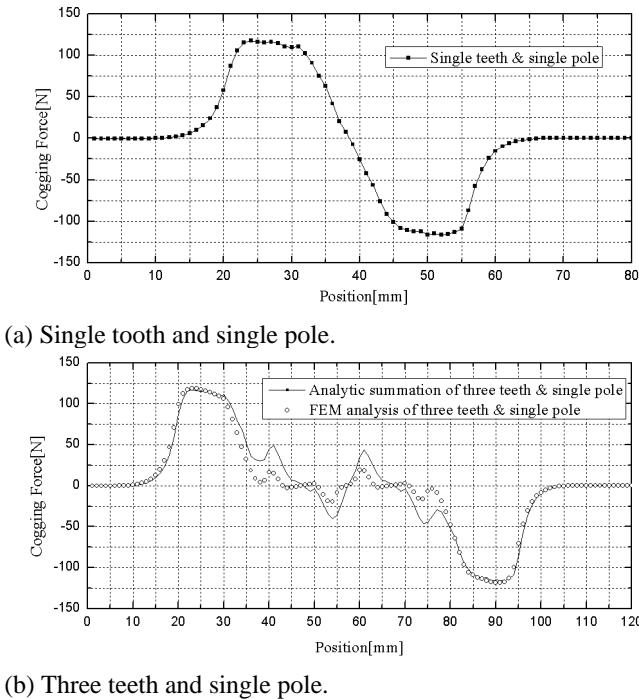


Figure 1. Basic model for cogging force analysis.



(c) Three teeth and multi pole.

Figure 2. Cogging force comparison of analytic summation and FEM analysis.

The cogging force caused by a single pole and single tooth arises at the entry region and exit area. The analytic summation of the basis cogging force, while considering the phase difference of three teeth is shown in Figure 2(b). The result of the comparison of the analytic summation result and FEM analysis result of three teeth with the multi-pole model is shown in Figure 2(c). As shown in these figures, the cogging force can be explained by the summation of the basis function considering the phase difference caused by the machine structure.

## 2.2 Fourier Series Expansion of Cogging Force

As mentioned earlier, the cogging force is described by a periodic function. Therefore, the Fourier series expansion of the cogging force can help to understand the relation between the period of the cogging force and the GCD of  $\tau_p$  and  $\tau_T$ . The Fourier series expansion equation for a periodic function which has a period of  $2L$  is shown below [6].

$$F(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right) \quad (2)$$

The Fourier series coefficient of the FEM analysis result of Figure 2(c) is shown in Figure 3. Only the sine coefficient is shown because the cosine coefficient is negligible. Among the sine coefficient,  $b_1$ ,  $b_3$  and  $b_6$  have meaningful values. With these coefficients, the relation between the cogging force period and the GCD of the pole pitch and teeth pitch can be explained. The wavelength of  $b_1$  is same as the pole pitch and the wavelength of  $b_3$  is the same as the GCD of the teeth pitch and pole pitch. The main component of the cogging force originates from the pole pitch and GCD. Therefore, to reduce the cogging force, we should concentrate on these periodic characteristics.

### 3 ANALYSIS OF LINEAR PLANAR MOTOR

#### 3.1 Cogging Force Analysis of Double Phase Set Motor

The basic model has a 2 pole/3 slot structure. This structure has a relatively larger cogging force than the multi-pole with multi-slot structure. Although the cogging force can be reduced effectively in this structure, the multi-pole and multi-slot structure is more effective. The FEM analysis result of the double phase set linear motor is shown in Figure 6. In the basic model,  $\tau_{PS}=30\text{mm}$  and  $\tau_{PS}=20\text{mm}$ . The period of the cogging force is 30mm without any phase set shift. The sine coefficients of the cogging force analysis result without any phase set shift are shown in Figure 7. The cogging force analysis result with varying  $\tau_{PS}$  is demonstrated in Figure 8. As shown in this figure, certain values of  $\tau_{PS}$  can reduce the maximum cogging force. This relation can be clarified by analyzing the Fourier series expansion of the cogging force. The main sine coefficients are listed in Table 1 according to  $\tau_{PS}$ .

As mentioned earlier, the sine coefficients  $b_1$ ,  $b_3$  and  $b_6$  have significant values. As  $\tau_{PS}$  becomes larger,  $b_3$  is reduced, while  $b_1$  changes slightly until  $\tau_{PS}=5\text{mm}$ . At this point, the third harmonic almost vanishes. This is because it has a period with a wavelength of 10mm and, consequently, a 5mm shift means that the cogging force induced by one phase set is shifted by 180 degrees which causes the two components to cancel each other out.

From the viewpoint of cancelling out the harmonics, minimizing all of the coefficients is the best way of reducing the cogging force. However, the value  $\tau_{PS}$  required to cancel out the  $b_1$  component is relatively large and this can cause a huge thrust loss. Cancelling out  $b_6$  is ineffective, because of the  $b_3$  component. The minimum cogging force appears at  $\tau_{PS}=6\text{mm}$ , due to the third harmonic reduces the peak value of the first harmonic wave. However, from the viewpoint of cancelling out the third harmonics, we concentrated on the  $\tau_{PS}=5\text{mm}$  point.

#### 3.2 Cogging Force Analysis of Multi-Phase Set Motor

The concept of the phase set shift can be extended to the multi-phase set model. The cogging force for the multi-phase set are analyzed and plotted in Figure 9. The x-axis is converted to the electrical angle and the y-axis is normalized with respect to the number of sets of phases. As shown in Figure 9, there is a certain angle required to minimize the cogging force for each number of sets of phases. In the 3 phase set

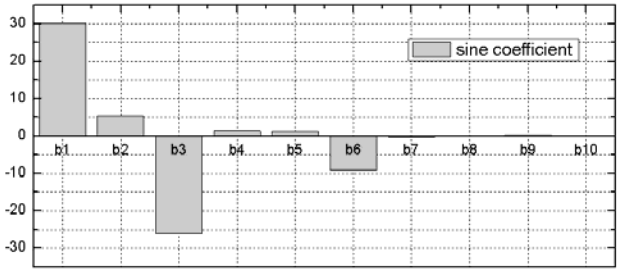


Figure 3. Sine coefficient of Fourier series expansion.

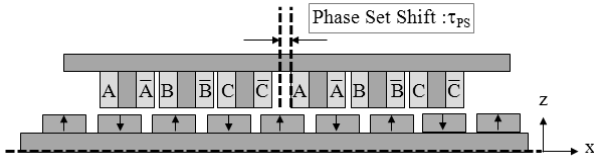
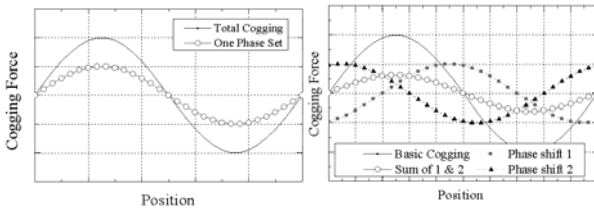


Figure 4. Concept of phase set shift.



(a) Without phase set shift. (b) With phase set shift.

Figure 5. Effect of phase set shift.

#### 2.3 Concept of Phase Set Shift

The phase set shift motor treated in this paper is shown in Figure 4. Considering the linear combination property of the cogging force, each phase set should have the same cogging force waveform and, ideally, the total cogging force should be equal to the sum of the individual components. The summation of the cogging force without any phase difference is shown in Figure 5(a). If the proper phase set shift is adopted, the summation of the individual cogging forces can be decreased, as shown in Figure 5(b). The total cogging force is related to the cogging force of each phase set,

$$F_{Total} = F_{PS1}(x) + F_{PS2}(x) \quad (3)$$

$$F_{PS1}(x) = F_{PS2}(x - \tau_{PS}) \quad (4)$$

where  $F_{PS1}$  and  $F_{PS2}$  are the cogging force components for the two phase sets and  $\tau_{PS}$  is the gap between the phase sets.

model, the local minimum value arises near the electrical angles of 120 and 240 degrees. In the 4 and, 5 phase set models, multiples of 90 and 72 degrees have the local minimum cogging force. These multiples can be candidates for  $\tau_{PS}$ , which minimizes the cogging force.

These values are related to the third harmonics of the Fourier series expansion and the number of phase sets. For each phase set, 180, 120, 90, and 72 degrees can cancel out the third harmonics. The optimum value of  $\tau_{PS}$  can be expressed as follows :

$$\tau_{PS} = \frac{GCD(\tau_T, \tau_P)}{N_{PS}} \quad (5)$$

The calculation result of the cogging force is treated with the Fourier series expansion, and the noticeable sine coefficients are listed in table 2. Similarly to table 1, not only the b3 component is cancelled out, but also the total cogging force is decreased with the above, proposed value of  $\tau_{PS}$ .

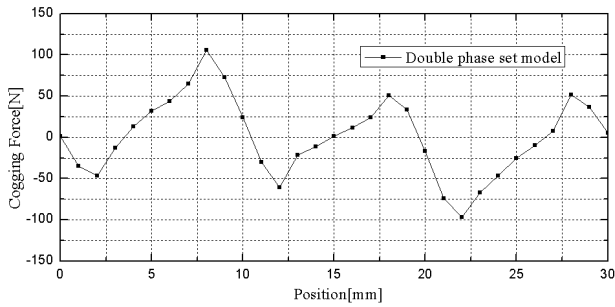


Figure 6. Cogging force of double phase set model.

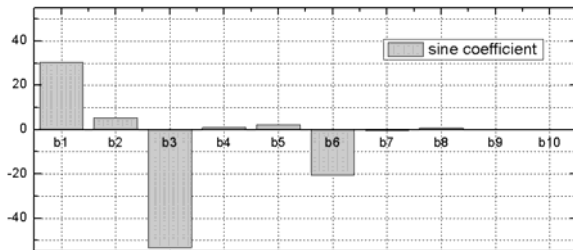


Figure 7. Sine coefficients of Fourier series expansion.

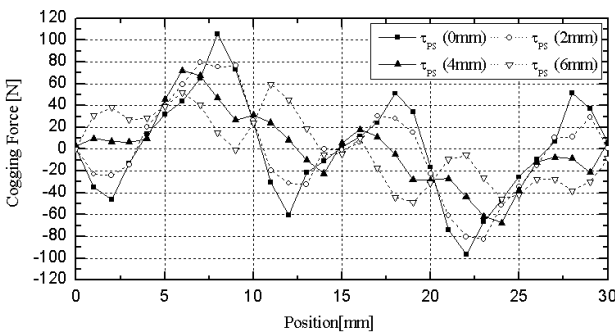


Figure 8. Cogging force according to  $\tau_{PS}$ .

Table 1. Fourier sine coefficients and maximum cogging force.

Item	b1	b3	b6	Max cogging
0mm shift	30.48	-53.31	-20.65	105.58 [N]
1mm shift	34.19	-51.12	-16.47	99.38 [N]
2mm shift	37.19	-43.75	-9.12	82.54 [N]
3mm shift	39.09	-31.86	3.92	77.97 [N]
4 mm shift	39.48	-17.63	14.86	72.22 [N]
5 mm shift	39.77	-0.52	19.10	66.09 [N]
6 mm shift	37.87	14.73	18.51	59.33 [N]
7 mm shift	36.70	30.75	8.38	66.25 [N]
8 mm shift	32.78	41.43	-2.40	68.93 [N]

Table 2. Comparison of cogging and thrust force for multi phase set.

Item	b1	b3	b6	Max cogging	
2 phase set	0 deg	30.48	-53.31	-20.65	105.58 [N]
	180 deg	39.77	-0.52	19.10	66.09 [N]
3 phase set	0 deg	29.97	-81.00	-29.60	138.10 [N]
	120 deg	45.26	-1.49	-3.85	60.78 [N]
4 phase set	0 deg	29.12	-109.9	-39.58	173.54 [N]
	90 deg	48.87	-2.02	-5.57	62.31 [N]
5 phase set	0 deg	30.16	-136.7	-47.05	198.69 [N]
	72 deg	48.87	-3.25	0.09	58.52 [N]

Table 3. Comparison of cogging and thrust force for multi phase set.

Item	2 set	3 set	4 set	5 set
Angle[deg]	180	120	90	72
Cogging force[%]	62.6	44.0	35.9	29.5
Thrust force[%]	97.8	93.9	94.2	94.6

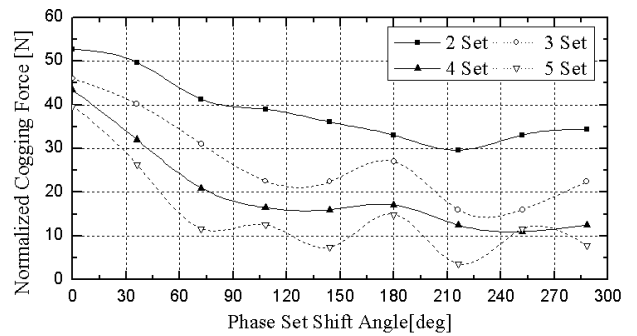


Figure 9. Normalized cogging force result for multi-phase set models.

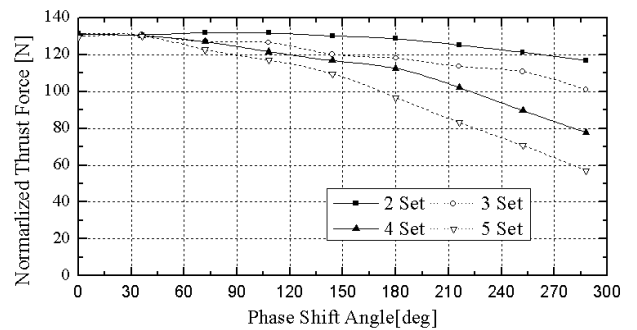


Figure 10. Normalized thrust force result for multi-phase set models.

### 3.3 Thrust Force Analysis of Linear Planar Motor

The phase set shift has the drawback of the thrust force loss, as shown in Figure 10. As the shift angle becomes bigger, the current phase for each phase set differs more and more from the exact angle and position. This causes the loss of the thrust force. Especially in the 5 phase set model, the thrust loss is significant, because of the accumulated current shifts in the side phase sets. Thus, selecting minimum value of  $\tau_{PS}$  is desirable among the candidates. A comparison of the cogging and thrust force results at the optimum phase set shift is listed in Table 3. As shown in the table 3, a small amount of thrust loss occurs while the cogging force is greatly decreased. The reduction portion of the cogging force is much greater than the loss of the thrust force. The greater the number of phase sets employed, the more effective cogging force reduction can be possible.

### 3.4 Relation of Phase Set Shift and Skew Effect

The skew effect provides an effective method of reducing the cogging force by twisting the permanent magnet or stator teeth structure. The effects of the phase set shift and skew are almost the same. The phase set shift can be described as skewing along the x-axis. The effects of the phase set shift on the cogging force and thrust force are similar. The cogging force is reduced, while some thrust force loss also occurs.

While skew involves some difficulties, such as difficulty of applying stator skew in a coil structure, the phase set shift has quite a simple structure. Accordingly, the manufacturing procedure is quite simple. This means that the phase set shift can be a substitute for stator skewing. Even in a linear tubular machine, which has a complicated curved structure, the skew effect can be achieved through the phase set shift. Another merit of the phase set shift is that it can coexist with the skew effect, because of the difference in the axis of twist in the two cases. The phase set shift is related to the x-axis, while the skew concerns the y-axis. Therefore, the coexistence of an armature with phase set shift and primary with magnet skewing is possible. The cogging force reduction can be maximized by using these two methods together.

## 4 CONCLUSION

A method of reducing the cogging force using the phase set shift is presented. It was found that certain values of the phase set shift decrease the cogging

force according to the number of sets of phases by eliminating the third harmonic of the cogging force. Though it has the drawback of inducing small loss of the thrust force, the cogging force reduction method is effective and it can be easily applied to the manufacturing process. It was also confirmed that this method has a similar effect to stator skewing.

## 5 REFERENCES

- Bianchi N., Bolognani S., Cappello A.D.F., "Reduction of Cogging Force in PM Linear Motors by Pole-Shifting," IEE Proceedings- Electric Power Applications, Vol. 152, Issue 3, May 2005.
- Choi H., Lim J., Jung H, Hong S., Cho D., Hwang S., and Oh S., "Design of flat-type linear generator for free-piston engine," ICEMS, 2004
- Hor P. J., Zhu Z. Q., Howe D., and Rees-Jones J., "Minimization of Cogging Force in a Linear Permanent Magnet Motor," IEEE Transactions on Magnetics, Vol. 34, No. 5, September 1998.
- Kreyszig E., "Advanced Engineering Mathematics," John wiley & sons, INC.
- Yoshimura T., Kim H.J., Watada M., Torii S., and Ebihara D., "Analysis of the Reduction of Detent Force in a Permanent Magnet Linear Synchronous Motor," IEEE Transactions on Magnetics, Vol. 31, Issue 6, Part 2, November 1995.
- Zhu Z. Q., Hor P. J., HoweD., and Rees-Jones J., "Calculation of Cogging Force in a Novel Slotted Linear Tubular Brushless Permanent Magnet Motor," IEEE Transactions on Magnetics, Vol. 33, No. 5, September 1997.