

# Tracking Control of magnetic Levitation Systems Using Fast Gain Scheduling

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**ABSTRACT:** In this paper, we design a tracking controller for magnetic levitation systems using fast gain scheduling. The fast gain scheduling methodology to fast varying signals. Throughout the implemental test-bed, we verify the performance of the applied tracking control law. The tracking controller with  $K_2$  asymptotically derives the controlled output to track the reference air-gap and its performance also shown to be effective in case of large variation of the reference air-gap. By experimenting with electromagnetic levitation system, we conclude that the controller with  $K_2$  shows better tracking performance on fast tracking problems than one without  $K_2$ .

## 1 INTRODUCTION

Magnetically levitated systems are receiving increasing attention around the world and are being successfully implemented for many applications such as magnetic bearing, vibration isolation, and magnetic levitated train. Magnetic levitation systems which are highly nonlinear and have unstable dynamics, offer a number of practical advantages such as no mechanical contact, less component wear and vibration, smaller maintenance costs, etc. Classical feedback control with pole placement has been applied to the linearized models corresponding to a specific operating conditions. However, since the operating condition changes according to the mass variation and the force disturbance, its local controller could not achieve satisfactory performance with global operating points. In order to overcome such a limitation, proposed self-tuning controller using gain scheduling against unknown mass variation. On the other hand, proposed to use the feedback linearization approach to control the ball position to use the ball position in levitation over long ranges of movement. For tracking control of magnetic levitation systems formulated Hamiltonian modes and developed stabilizing controller for ball suspension systems. Moreover, the input-state

feedback linearization technique for tracking control of magnetic levitation system.

In this paper, we design a tracking controller for magnetic levitation systems using fast gain scheduling. The fast gain scheduling methodology to fast varying signals. Throughout the implemental test-bed, we verify the performance of the applied tracking control law.

## 2 SYSTEM DESCRIPTION

The single-axis model of the magnetic levitation system composed of two controlled magnetic is described by the following nonlinear dynamic equation.

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned}\tag{1}$$

with

$$f(x) = \begin{bmatrix} x_2 \\ -\frac{\mu N^2 A}{4m} \left( \frac{x_3 + x_4}{x_1} \right)^2 + G \\ \frac{x_2 x_4}{x_1} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 & 0 & \frac{2x_1}{\mu N^2 A} & 0 \\ 0 & 0 & 0 & \frac{2x_1}{\mu N^2 A} \end{bmatrix}$$

$$h(x) = x_1$$

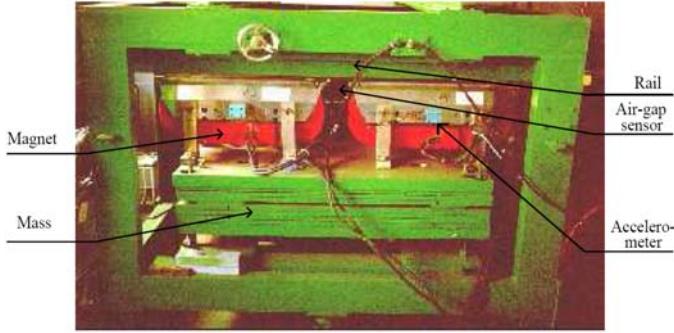


Fig. 1 Magnetic levitation system component of two controlled magnets.

Where,  $x = [x_1, x_2, x_3, x_4]^T \in R^4$  is the state vector and  $u = [u_1, u_2]^T \in R^2$  is the control input vector, and  $y \in R$  is the controlled output vector. Particularly,  $x_1$  is the vertical air-gap,  $x_2$  is the vertical velocity,  $x_3(x_4)$   $u_1(u_2)$  is the applied voltage of the magnet 1(2),  $m = 300\text{kg}$  is the total mass,  $N = 600$  is the number of turns of the coil wrapped around the magnet,  $A = 0.04\text{m}^2$  is the pole area,  $\mu = 4\pi \times 10^{-7} \text{H/m}$  is the permeability of free space,  $R = 1\Omega$  is the coil resistance,  $G = 9.8\text{m/sec}^2$  is the gravity constant, and  $r = 11\text{mm}$  is the reference air-gap.

### 3. TRACKING CONTROLLER DESIGN

Defining the reference air-gap as the scheduling parameter, the smooth function pair  $(\mathbf{x}(r), \mathbf{u}(r))$  satisfying;

$0 = f(\mathbf{x}(r)) + g(\mathbf{x}(r))\mathbf{u}(r)$  and  $r = h(\mathbf{x}(r))$   
for  $r \in \Gamma = \{r \in R \mid r > 0\}$  is computed by

$$\mathbf{x}(r) = \begin{bmatrix} r \\ 0 \\ \frac{r}{N} \sqrt{\frac{mG}{\mu A}} \\ \frac{r}{N} \sqrt{\frac{mG}{\mu A}} \end{bmatrix} \quad \text{and} \quad \mathbf{u}(r) = \begin{bmatrix} \frac{rR}{N} \sqrt{\frac{mG}{\mu A}} \\ \frac{rR}{N} \sqrt{\frac{mG}{\mu A}} \end{bmatrix} \quad (2)$$

Letting  $e = x - \mathbf{x}(r)$  the control objective is to minimize  $\lim_{t \rightarrow \infty} \|e(t)\|$  for the varying reference air-

gap  $r \in \Gamma$ . Then, for each fixed  $r$  the corresponding linearized closed loop system (1) with fast tracking control law  $u = k(x, r, \dot{r})$  written in the form of

$$\begin{aligned} \dot{x}_\delta &= A(r)x_\delta + B(r)u_\delta \\ u_\delta &= K_1(r)x_\delta \end{aligned} \quad (3)$$

Where the deviation variable are given by  $x_\delta = x - \mathbf{x}(r)$  and  $u_\delta = u - \mathbf{u}(r)$ . The linearized system coefficients are given by

$$A(r) = \partial\{f(\mathbf{x}(r)) + g(\mathbf{x}(r))\mathbf{u}(r)\} / \partial x$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{2G}{r_d} & 0 & -\frac{N}{r} \sqrt{\frac{\mu A G}{m}} & -\frac{N}{r} \sqrt{\frac{\mu A G}{m}} \\ 0 & \frac{1}{N} \sqrt{\frac{mG}{uA}} & -\frac{2rR}{\mu A N^2} & 0 \\ 0 & \frac{1}{N} \sqrt{\frac{mG}{uA}} & 0 & -\frac{2rR}{\mu A N^2} \end{bmatrix}$$

$$B(r) = \partial\{f(\mathbf{x}(r)) + g(\mathbf{x}(r))\mathbf{u}(r)\} / \partial u$$

$$= \begin{bmatrix} 0 & 0 & \frac{2r}{\mu A N^2} & 0 \\ 0 & 0 & 0 & \frac{2r}{\mu A N^2} \end{bmatrix}$$

and the linearized control law coefficients by

$$k_1(r) = \frac{\partial k(\mathbf{x}(r), r, 0)}{\partial x}.$$

In order to achieve the desire pole-placement in the linearized closed-loop system,  $k_1(r)$  is determined so that the eigenvalues of  $A(r) + B(r)k_1(r)$  have specified values with negative real parts for each  $r \in \Gamma$  with the controllable pair of  $A(r)$  and  $B(r)$ . Then, the tracking controller is defined by

$$k(x, r, \dot{r}) = \mathbf{u}(r) + K_1(r)(x - \mathbf{x}(r)) + K_2(r)\dot{r} \quad (4)$$

where  $K_1(r)$  is the state feedback gain and  $K_2(r)$  is the derivative gain.

$$\text{Determine } T(r, e) = g(x(r) + e)K_2(r) - \frac{\partial x(r)}{\partial r}.$$

Thus, we obtain the following theorem so as to construct tracking controller based on fast gain scheduling.

## Theorem 1;

Suppose that  $\partial\{f(\mathbf{x}(r)) + g(\mathbf{x}(r))k(\mathbf{x}(r)), r, 0\} / \partial x$  is Hurwitz for each fixed  $r \in \Gamma$ . Then, there exist finite constants  $\delta$  and  $\gamma$  such that for  $\|\dot{r}(t)\| \leq \delta$ ,  $t \geq 0$  and  $\|e(0)\| < \gamma$ ,  $\|e(t)\|$  goes to zero as  $t \rightarrow \infty$ , if there exist a control gain  $K_2(r)$  such that

$$\|T(r, 0)\| = 0, r \in \Gamma \quad (5)$$

Moreover, if there exist no  $K_2(r)$  satisfying (5) then we choose  $K_2(r)$  which provides a minimum constant  $T^* > 0$  such that

$$\|T(r, 0)\| = T^*, r \in \Gamma \quad (6)$$

In order to minimize  $\lim_{t \rightarrow \infty} \|e(t)\|$  [6].  
Therefore, we obtain

$$K_1(r) = \begin{bmatrix} -4378.5 & 4910.4 & 6.97 & 6.97 \\ -4378.5 & 4910.4 & 6.97 & 6.97 \end{bmatrix}$$

Where  $\lambda = 2$  is the multiple eigenvalues of the linearized closed loop system. Moreover, from theorem 1,  $K_2(r)$  is obtain by

$$K_2(r) = [4.01 \ 4.01]^T$$

## 4 EXPERIMENTAL VALIDATION

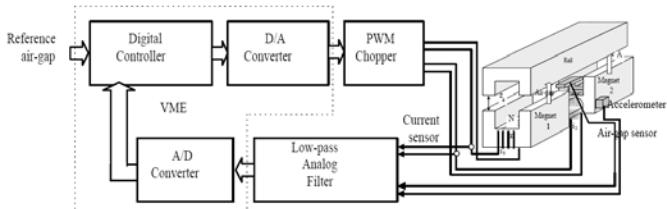
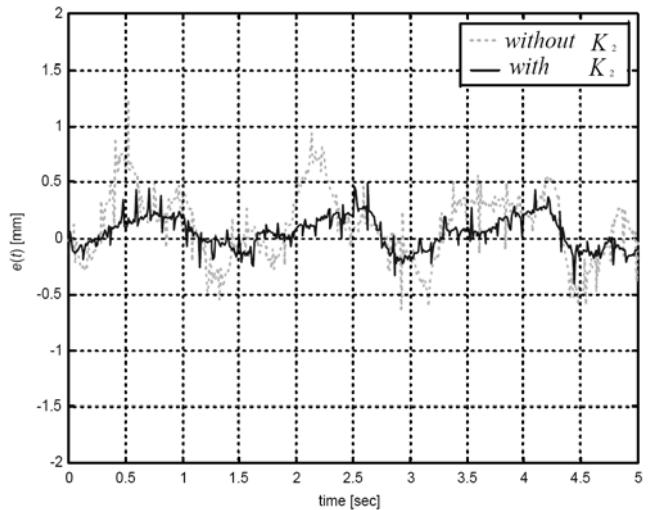


Fig. 2 Block diagram of the overall control system.

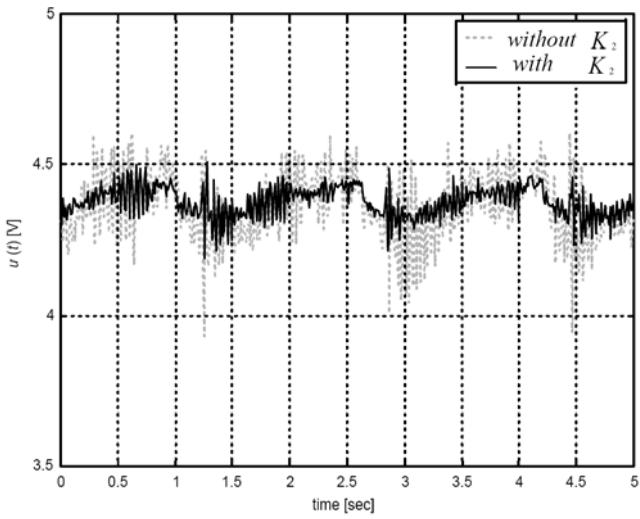
To verify effectiveness of the proposed control algorithm, the experimental hardware is constructed with two groups of controlled magnets and choppers, an air-gap sensor, an accelerometer shown in Fig.1 and Fig.2 shows the block diagram of overall experimental configuration. For the experimental evaluation of the fast tracking control, a digital control system based on Power PC (PPCIA 604, 225MHz) is constructed. The vertical air-gap sensor made by AEC is measured by an inductive-type air-gap sensor with an effective range of  $0 \sim 25\text{mm}$  and the acceleration of the magnet is measured by a servo-type accelerometer with an effective range of  $0 \sim \pm 3g$  which is made by Q-Flex. Furthermore the

magnet current is measured by a shunt-type current sensor with an effective range of  $0 \sim 100\text{A}$  which is made by Woo-Jin. The power with a dc link voltage of 300V consists of a 10 kHz PWM voltage-type chopper (actuator) made by K2E which has the input of energy supply and the output side of inductive load. The Power PC using VxWorks Tonado performs the control algorithm, as well as data acquisition and signal conditioning. Using this computer, the tracking controller is implemented at the sampling rate of 4 kHz. The vertical air-gap, acceleration and magnet current, which are measured by the voltage unit, are conditioned by a low-pass analog filter and then are sampled by 12-bit analog to digital (A/D) converters. Since the velocity sensor is not available, the velocity of magnet is converted from the integrated acceleration value with the high-speed sampling through the Power PC.

Experiment is conducted for both cases using conventional state feedback law(i.e.,(4) without  $K_2$ ) and using proposed state feedback law(i.e.,(4) with  $K_2$ ) as shown in Fig.3. Tracking controller with/without  $K_2$  is experimentally investigated on the electromagnetic levitation system with sinusoidal variation of the reference air-gap. The tracking controller with  $K_2$  shows better tracking performance than one without  $K_2$  for variation of the reference air-gap. Furthermore, Fig. 4 represents the worst case performance of tracking control using fast gain scheduling in this experiment. Concerning the control input, since the tracking controller with  $K_2$  is smaller than variation of the control input for input for one without  $K_2$  the tracking controller with  $K_2$  does not have any problem of saturation contrary to one without  $K_2$  in case of large variation of the reference air-gap.

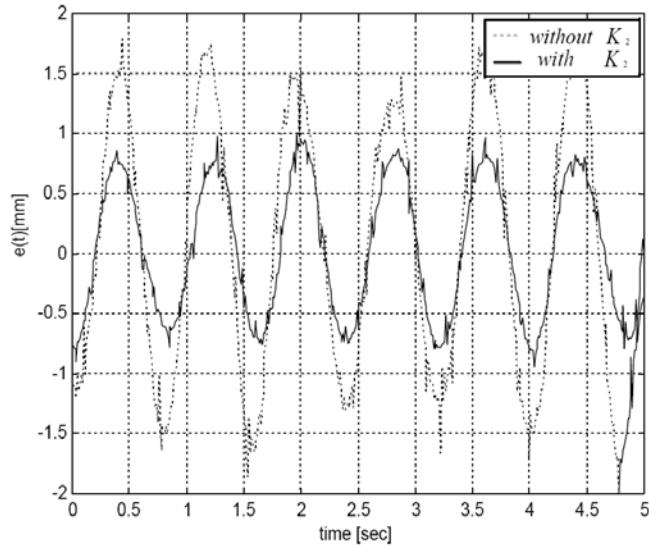


(a) Comparison of control error with/without  $K_2$

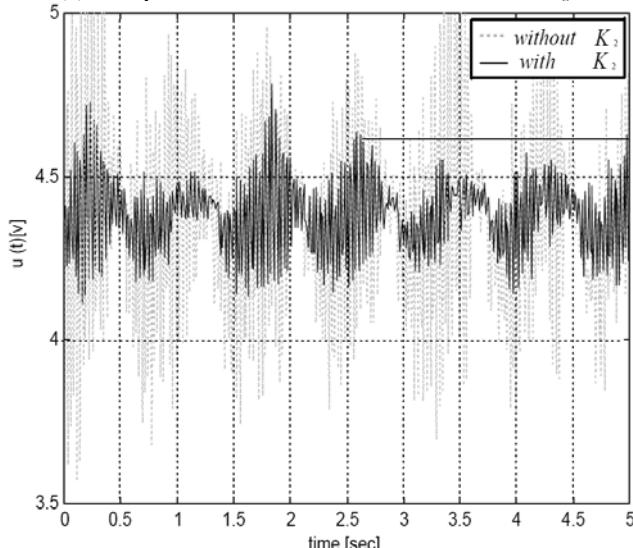


(b) Comparison of control input with/without  $K_2$

**Fig. 3** Experimental result for  $r(t) = 3\sin 4t$  [mm]



(a) Comparison of control error with/without  $K_2$



(b) Comparison of control error with/without  $K_2$

**Fig. 3** Experimental result for  $r(t) = 1.5\sin 8t$  [mm]

## 5 CONCLUSIONS

The tracking controller with  $K_2$  asymptotically derives the controlled output to track the reference air-gap and its performance also shown to be effective in case of large variation of the reference air-gap. By experimenting with electromagnetic levitation system, we conclude that the controller with  $K_2$  shows better tracking performance on fast tracking problems than one without  $K_2$ .

## 6 REFERENCES

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