Optimization of a Magnetic Rail for the HTS-Superconducting Vehicle Maglev-Cobra

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ABSTRACT: One of the most promising applications of superconducting magnetic levitation is urban trains. In the Maglev-Cobra proposal, the conventional track is substituted by a rail of magnets interacting with superconductors installed in the vehicle. The main cost of this levitation system is the magnetic rail. Therefore, any improvement in the shape and configuration of magnets and superconductors has a significant budgetary impact. In this paper, the optimization of a magnetic rail is presented. The design variables are the geometrical parameters of the permanent magnets, iron magnetic circuit and superconductor. The objective is to find the geometry that minimizes the total cost, for given levitation and lateral forces, considering the mechanical properties and also assemblage restrictions. A Finite Element model is employed to compute the forces. A formulation for sensitivity analysis is obtained by differentiation of the equilibrium equation. The optimization is carried out with the Feasible Direction Interior Point Algorithm (FDIPA). For an initial feasible point, FDIPA produces a sequence of feasible points such that the cost function is reduced at each iteration. We present several numerical results with significant reduction of magnetic material. The construction of laboratory prototype to validate the results will be the next step of this research work.

1 INTRODUCTION

One of the most promising commercial applications of superconducting magnetic levitation is urban trains. This technology has many advantages when compared with other types of vehicles and its main cost is the magnetic rail [1]. This vehicle has significant reductions of noise, losses and operation costs compared with other usual transportation system. Since the load of a magnetic levitated (Maglev) vehicle is distributed along the line and not concentrated at the point of contact wheel-rail, the infrastructure costs can be reduced. Moreover, since the Maglev vehicle does not need trucks, wheels and axis it is lighter than Light Rail Vehicles. Another advantage is that the train can be made of small modules (~1m long), allowing the vehicle to do curves with a small radio following avenues, streets, rivers and channels. The Laboratory of Applied Superconductivity (LASUP/UFRJ) is developing a vehicle, called Maglev-Cobra, as shown in Figure 1.

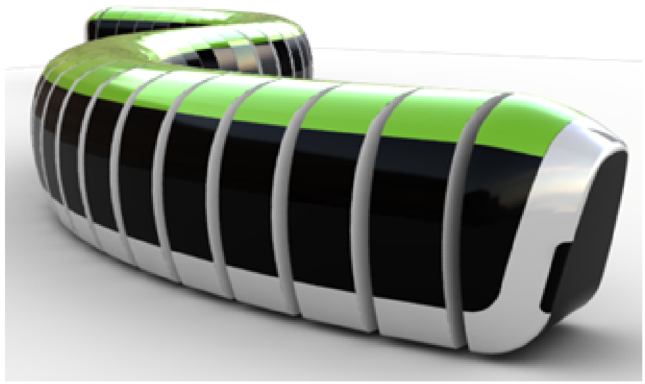

Figure 1. Maglev-Cobra.

As already mentioned the main cost of the levitation system is the magnetic rail. Therefore, any improvement in the shape and configuration of magnets and superconductors has a significant budgetary impact. To determine a design with minimal cost, ensuring a specific levitation force, an optimization is carried out with the Feasible Direction Interior Point Algorithm (FDIPA). The evaluation of the levitation force is performed with a Finite Element Model (FEM), considering the superconductor as a perfect diamagnetic material. Initially, the lateral diamagnetic material. Initially, the lateral force, that ensures the vehicle to perform curves, would be also a design restriction considered in the optimization process. However, in all simulations, this force was greater than the minimal necessary. Therefore, this restriction is only verified in the end of the optimization process.

The design variables are parameters that describe the geometry of the permanent magnets, iron magnetic circuit, aluminum and superconductor. The main objective is to find the design variables that minimize the total cost.

Numerical results, with significant reductions of magnetic material, are presented. Improvements in the superconductor model and construction of laboratory prototypes will be the next step of this research work.

2 DEVELOPMENT

The magnetic rail optimization is performed using FDIPA, a magnetic rail finite element model and an interface between them.

2.1 *Feasible Direction Interior Point Algorithm (FDIPA)*

The Feasible Direction Interior Point Algorithm (FDIPA) [2] is used to minimize an objective function $f(x)$, considering inequality constrains $g(x) \leq 0$. Starting at an initial feasible point x^1 defined by these inequalities constrains, this algorithm defines a sequence of feasible points, always in a descent direction of the objective function.

This is equivalent to solve Karush-Kuhn-Tucker (KKT) optimality conditions, presented in the system of equation 1.

$$\begin{aligned} &\nabla f(x) + \nabla g(x)\lambda = 0 \\ &G(x)\lambda = 0 \\ &\lambda \geq 0 \\ &g(x) \leq 0 \end{aligned} \tag{1}$$

where $G(x)$ is a diagonal matrix with $G_{ii}(x) \equiv g_i(x)$. A Newton's iteration to solve the nonlinear system of equations (1) in (x,λ) is stated as

$$\begin{bmatrix} H(x^k,\lambda^k) & \nabla g(x^k) \\ \Lambda^k \nabla g(x^k) & G(x^k) \end{bmatrix} \begin{bmatrix} x^{k+1} - x^k \\ \lambda_0^{k+1} - \lambda^k \end{bmatrix} = - \begin{bmatrix} \nabla f(x^k) + \nabla g(x^k)\lambda^k \\ G(x^k)\lambda^k \end{bmatrix}, \tag{2}$$

where $H(x^k,\lambda^k)$ is the Hessian of the Lagrangian and Λ is a diagonal matrix with $\Lambda_{ii} \equiv \lambda_i$. Instead of the Hessian of the Lagrangian, a matrix S^k symmetric and positive definite can be used (a quasi-Newton approximation of $H(x^k,\lambda^k)$ or the identity).

Defining the primal direction by $d_0^k = x^{k+1} - x^k$, it can be proved that it is a descent direction, but not necessarily feasible. To avoid this effect, the used direction d is obtained adding a ρd_1^k term to primal direction d_0^k, as presented in equation 3. This addition produces the effect of deflecting the primal direction d_0^k into a feasible region as presented in Figure 2

$$d^k = d_0^k + \rho d_1^k . \tag{3}$$

Solving the non linear system similar to (2), as presented in system of equation (4), the direction $d_1^k = x_1^{k+1} - x_1^k$ can be defined.

$$\begin{bmatrix} H(x^k,\lambda^k) & \nabla g(x^k) \\ \Lambda^k \nabla g(x^k) & G(x^k) \end{bmatrix} \begin{bmatrix} x_1^{k+1} - x_1^k \\ \lambda_1^{k+1} - \lambda^k \end{bmatrix} = - \begin{bmatrix} \nabla g(x^k)\lambda^k \\ G(x^k)\lambda^k + \lambda^k \end{bmatrix} \tag{4}$$

Using ρ as presented in equation 5, a new feasible point with a lower value of the objective function will be determined by a linear search method

$$0 \leq \rho \leq \frac{(\alpha - 1) d_0^{k^t} \nabla f(x^k)}{d_1^{k^t} \nabla f(x^k)} , \tag{5}$$

where α is a positive number lower than the unit.

The search direction of a two dimensional problem with one inequality constrain is presented in Figure 2.

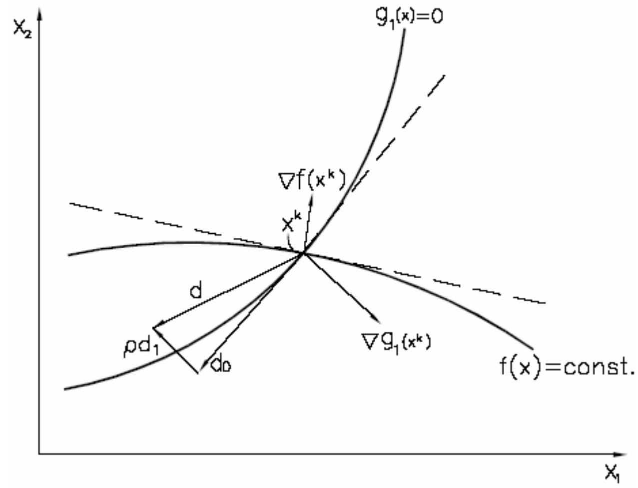

Figure 2. Search direction of FDIPA [2].

2.2 *Magnetic Rail Finite Element Model*

FEM software is used to evaluate the forces between the superconductor and the magnetic rail. The FEM method [3] can be divided in four steps:

- Division of the problem in elements;
- Definition of the elements properties;
- Interconnection of the elements and;
- Solution of the systems equation.

To determine the levitation force, the magnetic static field is evaluated using the magnetic vector potential $\boldsymbol{A}$ defined by equation 6, in the Coulomb gauge, i.e. $\nabla.\boldsymbol{A}=0$.

$$\nabla \times B = A \,. \tag{6}$$

So, defining $\boldsymbol{A}$, the magnetic field $\boldsymbol{B}$ is determined. The magnetic field intensity $\boldsymbol{H}$ is related to the magnetic flux density $\boldsymbol{B}$ as presented in equation 7.

$$B = \mu H + B_r \tag{7}$$

where μ is the magnetic permeability of the material and $\boldsymbol{B_r}$ is its resilient magnetization. In the air, aluminum and magnets, the vacuum magnetic permeability μ_0 is considered. Only in the magnets, $\boldsymbol{B_r}$ is different of zero. In the ferromagnetic material the used relation between them is presented in Figure 3.

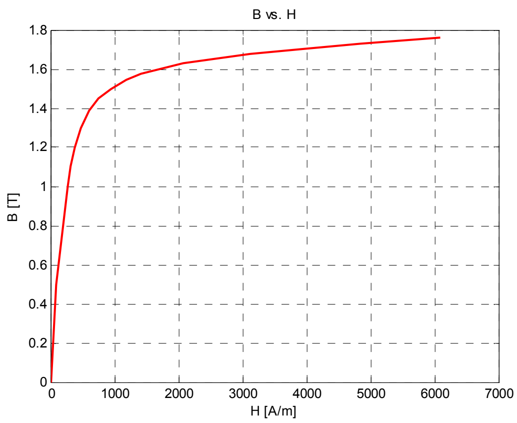

Figure 3. Relation between $\boldsymbol{B}$ and $\boldsymbol{H}$ in the ferromagnetic material.

The magnetic permeability of the superconductor varies with the evaluated force. The lateral and vertical forces are evaluated with the virtual work method.

The superconductor is modeled as a perfect diamagnetic material to determine the levitation force, as presented in Figure 4. However, the null μ implies in a division per zero in equation 7, so a numerical zero of 10^{-9} is used. This consideration implies in an evaluated force greater than the real levitation force. With a 10.2 mm gap between the magnets and the superconductor, the evaluated force is about twice the real one [4], as presented in Figure 5. However, the minimal levitation force considered in the optimization is also twice of the required in the design.

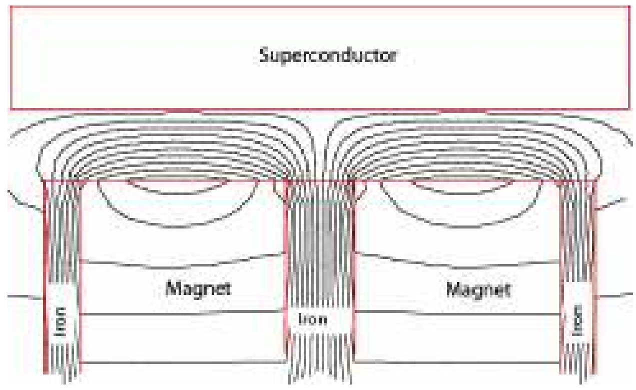

Figure 4. Simulation of levitation force for the initial geometry showing the magnetic flux lines, using the superconductor modeled as a perfect diamagnetic material.

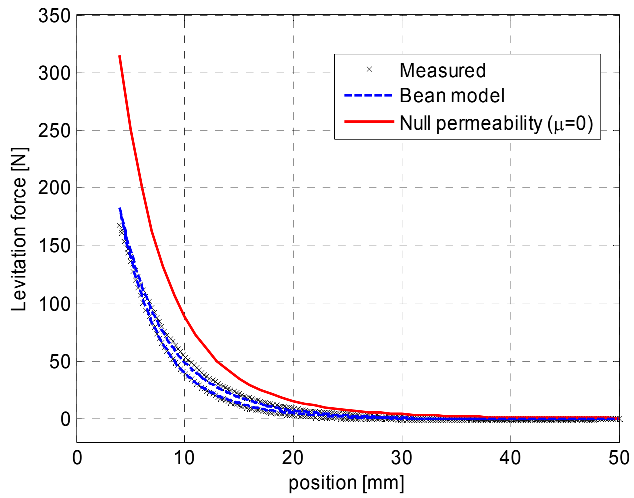

Figure 5. Simulated and measured levitation force versus superconductor bulk and magnetic rail gap, for an Up-down magnets configuration.

To evaluate the lateral force, the magnetic permeability μ_0 is imposed in the superconductor material. Therefore, the magnetic field of the permanent magnets in the superconductor area is determined, as presented in Figure 6. In this case, the superconductor is centered in the middle of the rail. The magnetic vector potential $\boldsymbol{A}$ on the boundary of the superconductor area is saved in a vector. Then, the superconductor is dislocated laterally, imposing the stored $\boldsymbol{A}$, as presented in Figure 7. This method represents an ideal field cooling [5].

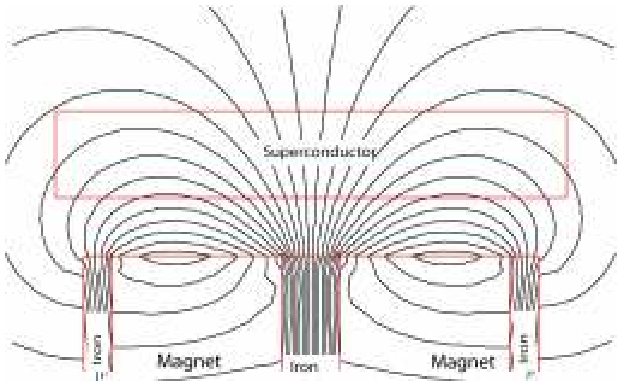

Figure 6. Ideal field cooling.

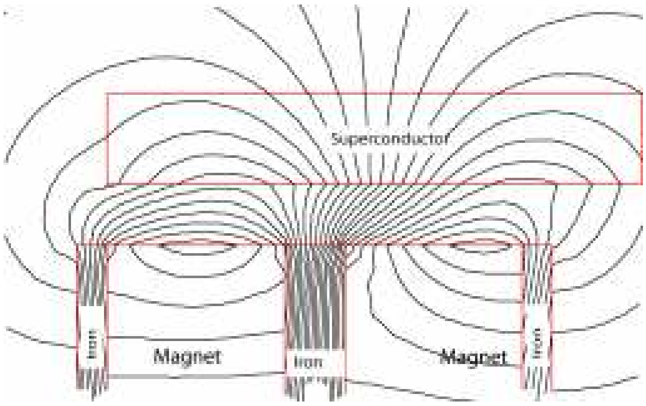

Figure 7. Simulation of the lateral force generated by the displacement of the superconductor with the magnetic field of its centered position cooled.

The levitation force of 12,300 N/m is the unique design restriction considered in the optimization process. Other restrictions were only implemented to ensure the correct evaluation of the FEM model. The levitation system design is known as half Hallbach (Figure 8). It is composed by a rectangular superconductor over two magnets in opposition concentrating the magnetic flux in the iron between them and also on their lateral faces, with aluminum filling the empty space to ensure mechanical resistance.

The initial geometry presented in Figure 8 is used because LASUP has already built this rail. A section of this configuration is presented in Figure 8. This rail shape has a simulated levitation force of 12,749 N/m with the null permeability model. Although having aluminum in the lateral parts of the rail presented in Figure 9, this material volume is practically constant and is not considered in the optimization process, equation 8.

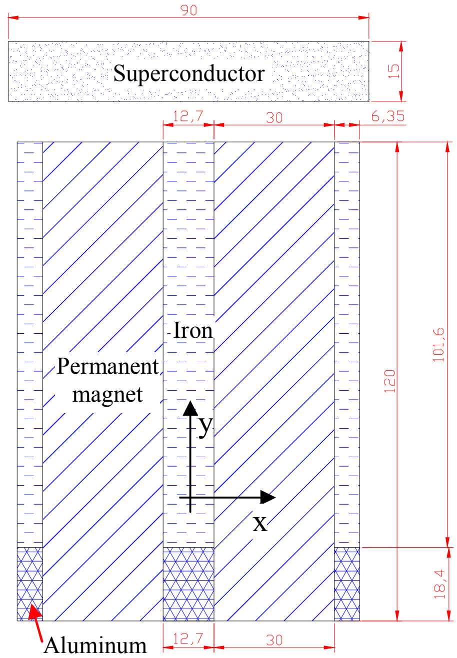

Figure 8. Representation of a two dimensional view of superconductor and magnetic rail, with bilateral symmetry by the "y" axis and geometry variables highlighted and in mm.

Figure 9. Section of the initial rail considered in the optimization process.

2.3 *Interface between FDIPA and FEM Software*

The interaction between the software is made using two ASCII text files, one generated by FDIPA and the other by FEM software. Initially, FDIPA generates one file containing the interaction indices and the geometry parameters. After that, FDIPA program starts reading the text file generated by FEM software, until the indices informed by the FEM software is the same of FDIPA's one. When this condition is reached, the levitation force is read and the optimization process continues. An analogous methodology is applied on the FEM software. The flowchart of this process is presented in Figure 10.

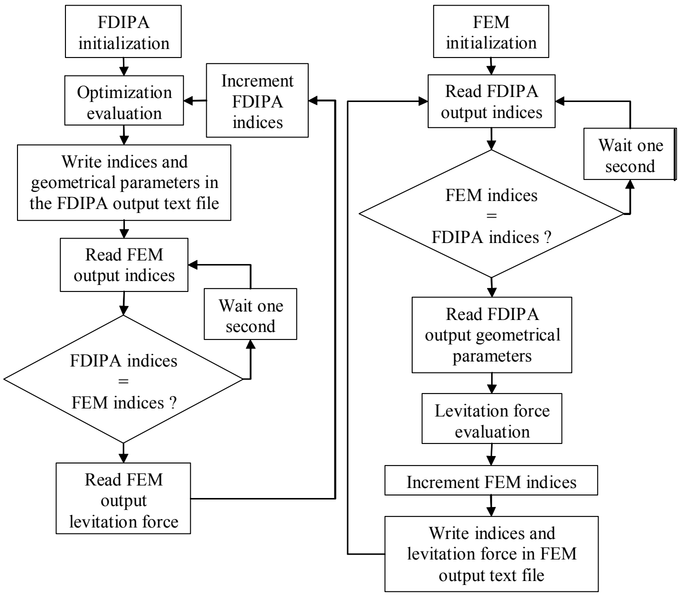

Figure 10. Flowchart of FDIPA and FEM software interaction.

2.4 *Objective Function*

The objective function to be minimized is the total cost of the levitation system. This cost function considers the materials volumes and their prices, except for the superconductor. In the superconductor case the price is given by its area. Equation 8 represents the objective function.

$$f = Rail\_length \cdot [(A2 \cdot P2) + (A3 \cdot P3) + (A4 \cdot P4)] + Train\_length \cdot .L_{SUP}.P1 \tag{8}$$

where:

A2 is the magnet area;
P2 is the magnet price per volume;
A3 is the iron area;
P3 is the iron price per volume;
A4 is the aluminum area;
P4 is the aluminum price per volume;
L_{sup} is the superconductor diameter;
P1 is the superconductor price per area.

3 RESULTS

The initial shape of the levitation system is presented in Figure 8. Appling the optimization process, after 21 iterations, the final configuration was reached, as presented in Figure 11. For this first optimization process, the final shape cost reduction is 37.1% (as presented in Figure 12) with a decrease on the levitation force of 3.2%. The volume of the permanent magnet material is reduced in 27,7 %.

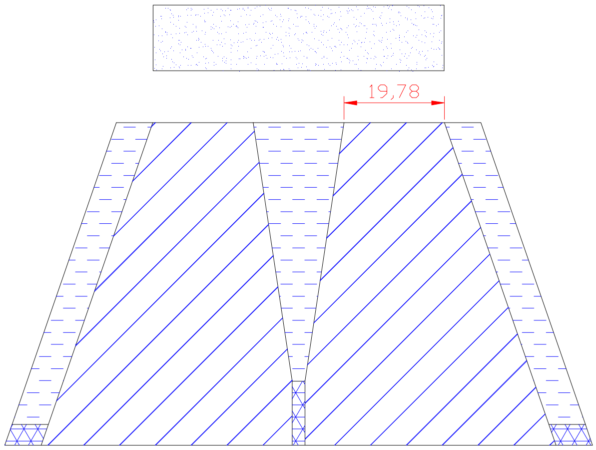

Figure 11. Representation of a two dimensional view of superconductor and magnetic rail final shape, after the first optimization process.

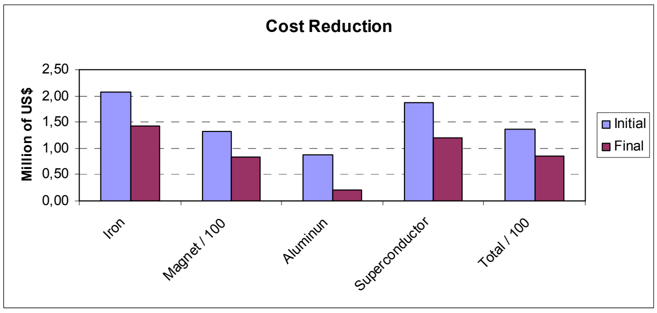

Figure 12. Cost reduction per material, for a simple rail of 100 km, after the first optimization process.

Was verified that the iron components presented in Figure 11 were not in a stable position. During the assemblage of the rail presented in Figure 9, the lateral components showed its instability. For the central iron component, a displacement of 0.1 mm, as presented in Figure 13, resulted in an expulsion force of 5,959.07 N/m. Considering the elastic deformation limit of the iron 18 kg/mm^2, the minimal width of this component was 0.033 mm. To ensure a non elastic deformation of this component, the minimal iron width of 2 mm was fixed.

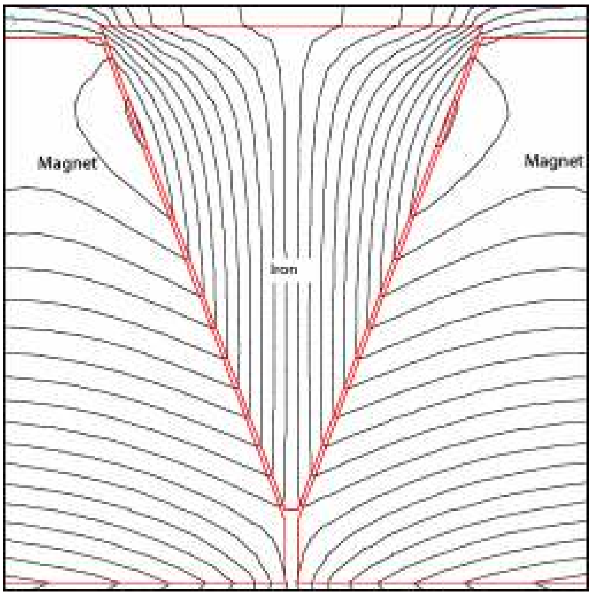

Figure 13. Simulation of the 0.1 mm displacement of the central iron component.

Therefore, an optimization process was performed considering noses in these iron components, assuring theirs correct position without the necessity of screws. In this new optimization process, the thickness of the superconductor was not considered as design variable. With the null permeability model, was verified that there is no considerable variation in levitation force with the thickness of the superconductor. Moreover, different widths, top and bottom, of the lateral iron components were considered.

At this second optimization process, the considered minimal levitation force was 12,000 N/m, the real design minimal levitation force. Appling the optimization process with the initial shape similar to the presented in Figure 11, after 9 iterations, the final configuration was reached, as presented in Figure 14. The final shape cost reduction is 25.28% compared with the initial shape (as presented in Figure 15) with a decrease on the levitation force of 5.87%. Although this shape has a smaller cost reduction comparing to the presented in Figure 11, there is no necessity of screws, reducing the cutting cost and also reducing the assemblage time.

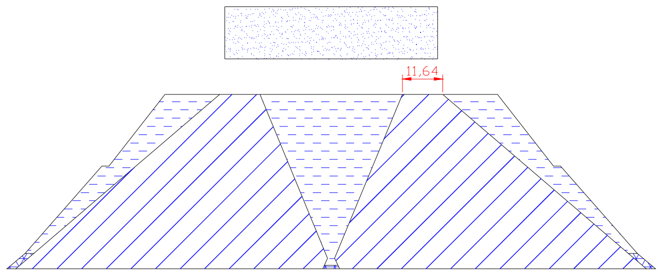

Figure 14. Representation of a two dimensional view of superconductor and magnetic rail final shape, after the second optimization process.

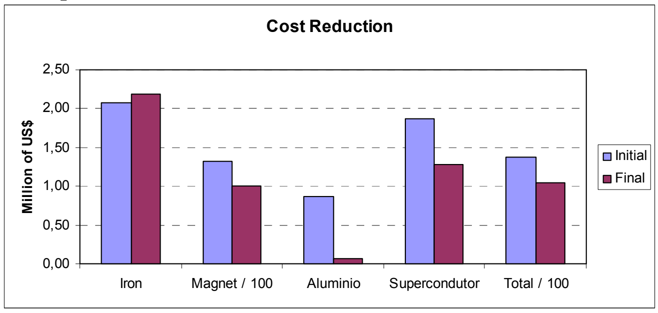

Figure 15. Cost reduction per material in the second optimization process, for a simple rail of 100 *km*.

The lateral stability of the rails configurations, Figures 8, 11 and 14, were analyzed. The highest lateral acceleration is 3,6 m/s^2[6], to ensure the travelers comfort. The vehicle weight is 1200 kg/m, implying in a minimal force of 4320 N/m. The maximum lateral displacement allowed is 10 mm. So, as shown in Figure 16, all configurations respect this restriction.

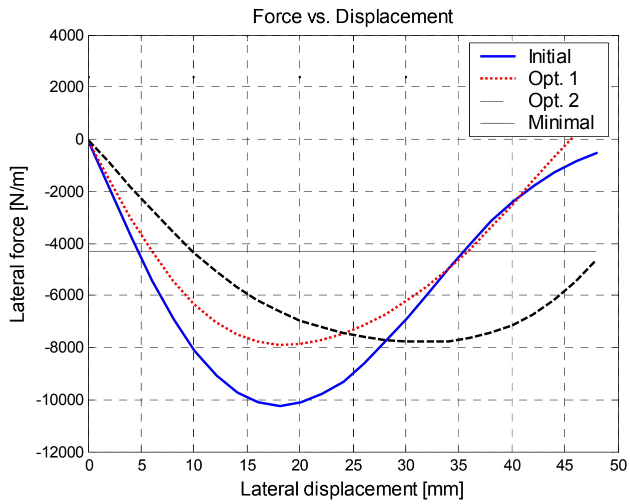

Figure 16. Lateral force versus lateral displacement for all configurations.

As presented in Figure 5, the levitation force can be simulated properly with the Bean Model [7]. However, it is impracticable to use this model in the optimization process because of its evaluation time.

The magnetic levitation force is simulated for all configurations with Bean Model, as shown in Figure 17. The maximum levitation force evaluated with this model are 10570, 9770 and 8176 N/m, to the initial, after the first step of optimization and after the second step, respectivly.

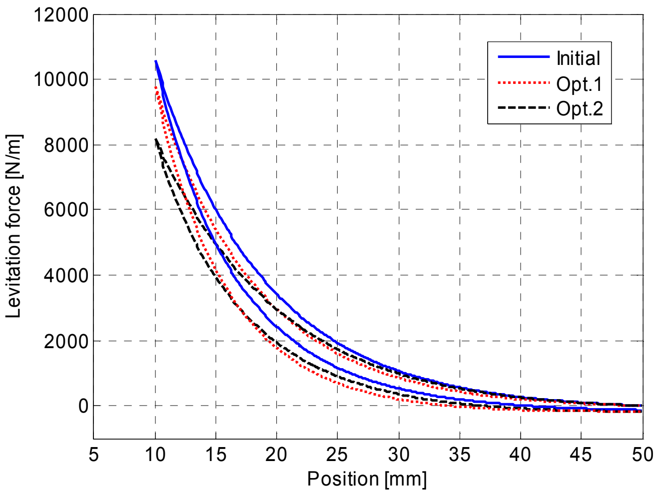

Figure 17. Simulation using Bean Model of the levitation force for all configurations.

4 CONCLUSIONS

The FDIPA was applied with success to the minimization cost of a magnetic device composed of superconductors and permanent magnets.

In future woks, the optimization process will be performed initially with Genetic Algorithm to prevent the optimization convergence in a local minimum far from the global minimal. After, the optimization will be performed with FDIPA, that has a higher accuracy. The optimization will also be performed with a better superconducting model (meta model [8] of Bean Model, for example), other design restrictions will be considered and experimental tests with laboratory prototypes will be performed to validate the simulation results.

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