Structure of Linear PM Halbach Array for EDS Maglev

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ABSTRACT: The paper is mainly on the structure of linear Halbach array in maglev. Study on the structure includes number of magnets per spatial wavelength, magnitude of each module and magnetization vectors. By analysis, 8-piece Halbach arrays are determined for maglev project. New approaches are also presented to optimize structure of 8-piece Halbach array. Higher magnetic fields are obtained with less magnet by changing the length, width and magnetization directions. Optimization indexes are maximized to get the rational length of the module which is in fact utilization ratio of magnetic materials; The rational width is calculated to get higher horizontal component of magnetic fields; and offset-Halbach structure is made to get even higher magnetic fields. All the above optimizations and designs are tested in laboratory with practical Halbach arrays.The study can be used in designs of magnets for further maglev systems.

1 INTRODUCTION

Ideal Halbach array is of one-sided flux structure, one side having enhanced magnetic field and the other side cancelled to zero[1][2]. Ideal linear Halbach array is a structure whose magnetization direction smoothly changes sinusoidally along the array, whereas it is impossible to be made in practice. Practical Halbach array has been improved since it was invented by K. Halbach in 1979[3]. Many optimizations and designs have been made to improve Halbach arrays to fit for engineering[4][5][6]. Halbach structure is used in General Atomics (GA) Urban Maglev[4] and Magplane[5]. In 1999,GA urban maglev adopted a dual Halbach array. The array is essentially an optimized structure, which can enhance the horizontal component of magnetic fields and cancel the vertical in order to maximize index $\eta = \frac{lift-to-drag\ \ ratio}{weight}$ [4]. In 2000, K.Davey optimized linear Halbach array for maglev, he put forward the index ($\eta = \frac{weight}{flux^2}$). By means of minimizing the index, he got an optimization structure with steel and air gap inside it [7]. In 2004, P. Suominen et al optimized a 24-piece hexapolar Halbach array, they put forward an offset-Halbach structure, in which all permanent magnets are oriented in such a way that none of the magnets has a magnetization vector perpendicular to the radial vector, they all have an offset angle. The offset structure has about 4 percent higher field than typical one[8]. In 2005, Q. Han et al optimized 4-piece and 8-piece Halbach arrays for maglev. They got the optimization length of modules for 4- piece and 8-piece array. Based on simulation of the harmonics and fundamental, they drew the conclusion that 8-piece structure is better for maglev [9].

In the paper, we firstly analyze magnetic fields of 8- and 12- piece linear Halbach array, then put forward optimization index and optimize the structure to get the length of each module; we also put forward rational width of module to get higher horizontal field; Offset-Halbach structure is made to get even higher magnetic fields. In the end, all the approaches above are tested in laboratory.

2 NUMBER OF MAGNETS PER SPATIAL WAVELENGTH

2.1 *Linear Halbach Array*

In linear Halbach array (as in Fig 1),when z>0, magnetic fields of the enhanced side can be written as the follows:

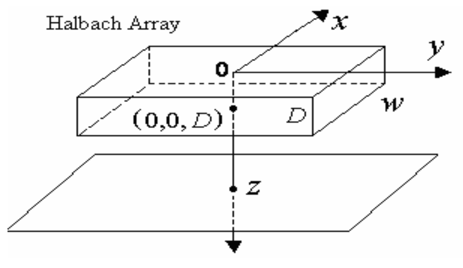

Fig 1 Magnet and the coordinate

$$B_m = B_r(1-e^{-kD})e^{-kz}\frac{\sin(\pi/m)}{\pi/m} \quad (1)$$

$$B_y = B_m \sin(ky) \quad (2)$$

$$B_z = B_m \cos(ky) \tag{3}$$

where k is the wave number, B_r is remanence, m is the number of magnets per spatial wavelength,and D is the thickness of magnets.

2.2 Determine the number of magnets per spatial wave- length

As shown in equation (1), the number of magnets per spatial wavelength influences magnetic fields of Halbach array. In 2005, Q.Han et al analyzed 4-piece and 8-piece respectively, and they concluded that magnetic fields of 8-piece Halbach array is suitable for maglev project[9]. Now, 8- and 12-piece are analyzed in order to determine whose structure is suitable for maglev.

On the assumption that magnitude of each module is $D\times D\times D$, let's analyze 8-piece and 12-piece Halbach arrays, where 8-piece has 3 wavelength and 12-piece has 2 wavelength, so they have the same weight.

As for 8-piece, the magnetic field:

$$B_8 = B_r\left[1-e^{-k_8 D}\right]\frac{\sin(\pi/8)}{\pi/8}e^{-k_8 z} \tag{4}$$

As for 12-piece, the magnetic field:

$$B_{12} = B_r\left[1-e^{-k_{12} D}\right]\frac{\sin(\pi/12)}{\pi/12}e^{-k_{12} z} \tag{5}$$

When $z>1.05D$, $B_{12}<B_8$.

As for linear Halbach array, we call $\xi=\dfrac{B^2}{W_{mag}}$ utilization ratio of magnetic materials, where B is magnitude of magnetic field and W_{mag} is the weight of PM. Comparisons between 8- and 12- piece are shown in Table I.

Table I Utilization ratio of materials

Gap (z)	0.10D	0.25D	0.50D	0.75D
$\dfrac{\xi_{8-piece}}{\xi_{12-piece}}$(%)	164.3	151.9	133.2	116.9
Gap (z)	1.00D	1.05D	1.25D	1.50D
$\dfrac{\xi_{8-piece}}{\xi_{12-piece}}$(%)	102.5	99.9	90.0	78.9

When $z<1.05D$, $\dfrac{\xi_{8-piece}}{\xi_{12-piece}}>1.0$ .In the project, the gap is less than the thickness of magnet, so 8-piece has larger utilization ratio, 8-piece is suitable for the project.

3 OPTIMIZATION OF MODULE LENGTH

Linear Halbach array has a relatively simple structure whose module is just box-shaped, which is helpful for us to optimize the structure. The optimization is to use least magnets to yield strongest magnetic fields. We take L as the levitation height, D as the thickness of the magnet, ρ as the specific gravity, W_{mag} as the weight of magnet per unit length along the array,D_8 as the module length of 8-piece Halbach array and φ as flux, so we get:

$$W_{mag} = \rho D \tag{6}$$

1) Maximize the index ξ_1:

$$\xi_1 = \frac{B^2}{W_{mag}} \tag{7}$$

ξ_1 is an index which reflects the same weight of linear Halbach arrays yield magnetic fields, i.e., it is utilization ratio of magnetic materials which Halbach array has. To make the ratio biggest means strongest field with least magnets.

2) Minimize the index ξ_2

$$\xi_2 = \frac{W_{mag}}{\varphi^2} \tag{8}$$

3) When only the fundamental wave is considered, suppose no harmonic, index ξ_3 is constructed with the amplitude of the fundamental component as the follow:

$$\xi_3 = \frac{B_m}{W_{mag}} \tag{9}$$

Based on equation (1) and (6), all the above indexes can be optimized from the equation:

$$\frac{\partial \xi_i}{\partial D_8} = 0,\ i = 1,2,3 \tag{10}$$

we get:

$$D_8 = -\frac{\pi D}{4\ln[L/(L+D)]} \tag{11}$$

This is the length of each module of Halbach array, i.e., our optimization result.

4 RATIONAL WIDTH OF MAGNETIC MODULE

4.1 Magnetic fields of linear Halbach array based on magnetization charges

Theory of magnetization charge is on magnetic field by means of superimposing fields of each magnetization charge , which is especially convenient and effectual for PM field . J.F. Hoburg analysed the static magnetic field of dual Halbach array for GA urban maglev on theory of magnetization charge [10]. One magnetization charge yields magnetic field as the follow:

$$\boldsymbol{H} = \frac{q_m}{4\pi\mu_0 r^2}\boldsymbol{r}^{\mathbf{0}} \tag{12}$$

As each module consists of a uniform magnetization surface charge density $\sigma_{sm} = \mu_0\|M\|$, where $\|M\|$ is the discontinuity in normal component magnetization across the magnet face. The charge density is a source of the magnetic field in the free space as the follow:

$$\boldsymbol{B} = \frac{\sigma_{sm}}{4\pi}\iint_{S'}\frac{\boldsymbol{a}_{rr'}dS'}{|\boldsymbol{r}-\boldsymbol{r}'|^2} \tag{13}$$

where $\boldsymbol{r}$ is vector from the origin of coordinate to the observation point P(x,y,z), $\boldsymbol{r}'$ is that to source $P'(x',y',z')$, $\boldsymbol{a}_{rr'}$ is unit vector from $P'(x',y',z')$ to P(x,y,z) .

In Fig 2, suppose one module space is $x=a\sim b$, $y=c\sim d$, $z=m\sim n$, when the plane of magnetization charge is parallel to the *xoy* plane, by equation(13) we get the fields as the follow:

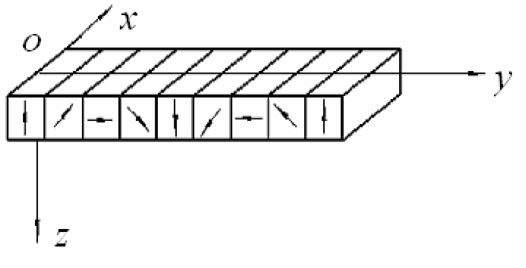

Fig 2 Halbach array and the coordinate system

$$B_x = \frac{\sigma}{4\pi}\left\{\ln\left[\frac{(y-d)-r_{bdz_0}}{(y-d)-r_{adz_0}}\cdot\frac{(y-c)-r_{acz_0}}{(y-c)-r_{bcz_0}}\right]\right\} \tag{14}$$

$$B_y = \frac{\sigma}{4\pi}\left\{\ln\left[\frac{(x-b)-r_{bdz_0}}{(x-b)-r_{bcz_0}}\cdot\frac{(x-a)-r_{acz_0}}{(x-a)-r_{adz_0}}\right]\right\} \tag{15}$$

$$B_z = \frac{\sigma}{4\pi}\begin{bmatrix}-\operatorname{arctg}\frac{(x-b)(y-c)}{(z-z_0)r_{bcz_0}}+\operatorname{arctg}\frac{(x-b)(y-d)}{(z-z_0)r_{bdz_0}}\\+\operatorname{arctg}\frac{(x-a)(y-c)}{(z-z_0)r_{acz_0}}-\operatorname{arctg}\frac{(x-a)(y-d)}{(z-z_0)r_{adz_0}}\end{bmatrix} \tag{16}$$

where:

$r_{acz_0}=\sqrt{(x-a)^2+(y-c)^2+(z-z_0)^2}$, $r_{adz_0}=\sqrt{(x-a)^2+(y-d)^2+(z-z_0)^2}$

$r_{bcz_0}=\sqrt{(x-b)^2+(y-c)^2+(z-z_0)^2}$, $r_{bdz_0}=\sqrt{(x-b)^2+(y-b)^2+(z-z_0)^2}$

In Fig 3, B_x=0, horizontal and vertical components are all sinusoidally changing along the array. B is fluctuating for B_y is lower than B_z in amplitude. Because it is finite for the width of magnet, B is lower than that based on equation (1).The volume of each module is (*width*×*length*×*height*) 75*mm*×12*mm*×25*mm*, levitation height is 6*mm*, B_r=1.38T.

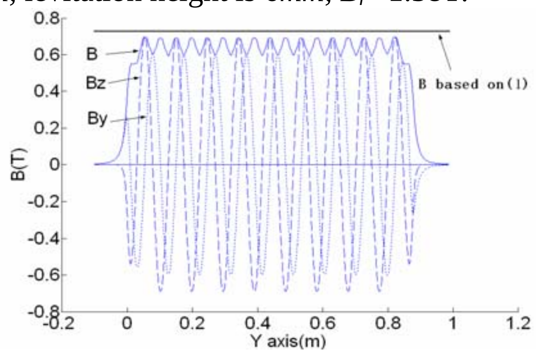

Fig 3 Fields of linear Halbach array based on magnetization charges

4.2 Rational width of module

4.2.1 Degree of field balance

Ignoring end effect of magnetic field, we only consider the stable part of fields along linear Halbach array. By electromagnetic induction, lift and drag force are induced. The horizontal component is just what we want, for it is necessary for the train to yield lift. We should enhance the horizontal and cancel the vertical, i.e., we should promote index as the follow:

$$\beta = \frac{\text{amplitude of horizontal component}}{\text{amplitude of vertical component}} = \frac{(B_m)_{horizontal}}{(B_m)_{vertical}} \tag{17}$$

β should also be optimized for us to get higher ratio of lift-to-drag. Let’s focus on geometrical structure of Halbach array. In part III of the paper, assume the width is large enough, and get horizontal and vertical components with the same amplitude and characteristics, thus β=100%. In fact, width is finite, so it is another case. Consider how β is affected by width and length of each module. As shown in Fig 4 and Fig 5, while the width increases, β also increases; while the length increases, β contra decreases. Width and length have great influences on the index β.

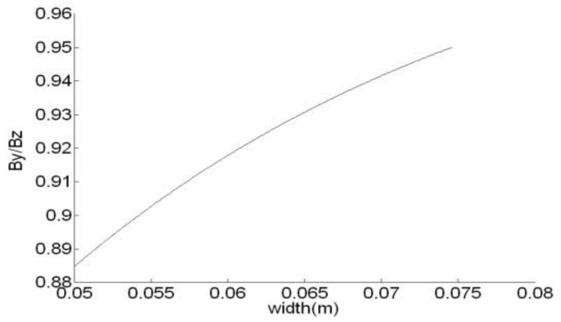

Fig 4 Curve of Width-(By/Bz)

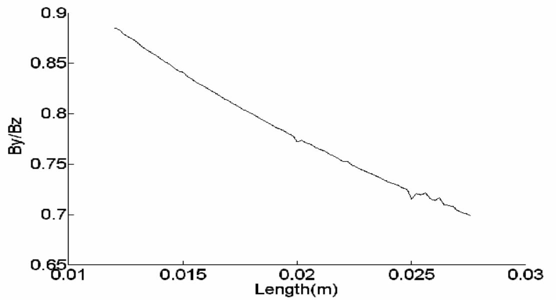

Fig 5 Curve of length-(By/Bz)

In Section 3, we get the length by means of optimization (see equation 11), now we should increase the width to get higher horizontal component. But as width increases the weight is also increasing sharply, so it is necessary for us to find a rational width, which makes β higher and weight also acceptable.

Remark: The index β has special meaning for design of Halbach array, which we call degree of field balance. In the optimized structure of Halbach array (see equation 11), width of module to make degree of balance up to 95% is called rational width of linear Halbach array. By a lot of simulation, when β=95%, horizontal component is enough higher and weight of magnets is also acceptable, i.e., β=95% is rational. By the way, it is by means of simulation on dynamic characteristics of maglev system that we decide whether weight is acceptable or not.

4.2.2 How to get rational width of module.

Rational width can be estimated by the nonlinear curve of width-index(β),as shown in Fig 6. Degree of field balance tends to 1.0, as the width is infinite, the amplitude of the two components are both equal to the value by equation (1) ;when width is zero, degree of balance is zero; as width increases, the balance also increases, before it gets to 95%, curve goes up sharply, designedly shown in Fig 7. For the magnetic field is non-linear and the balance curve is just ascending, we can only estimate the width of module by calculation.

As for one 8-piece Halbach array, which is optimized to get the length from equation (11), fields of each wavelength on the observation (x,y,z) can be calculated as the follow:

$$B_y^0 = \sum_{j=1}^{8}[(A_{j1} - A_{j2})\sin\theta_j + (A_{j3} - A_{j4})\cos\theta_j] \quad (18)$$

$$B_z^0 = \sum_{j=1}^{8}[(C_{j1} - C_{j2})\sin\theta_j + (C_{j3} - C_{j4})\cos\theta_j] \quad (19)$$

where θ is the angle between magnetization vector and the horizontal axis. By means of many simulations, for an optimized structure (by equation 11), rational width should be about 6 times of module length, see Fig7. Fig 8 is the flow chart to get the rational width by equation (18) and (19).

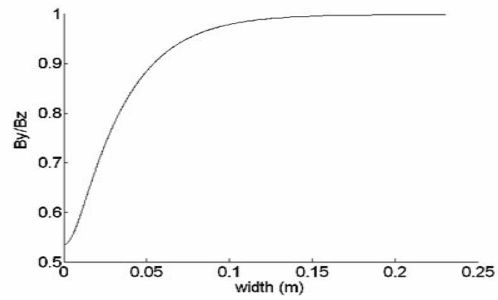

Fig 6 Field balance trend

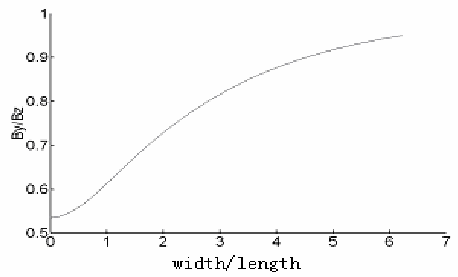

Fig 7 Curve of balance and width/length

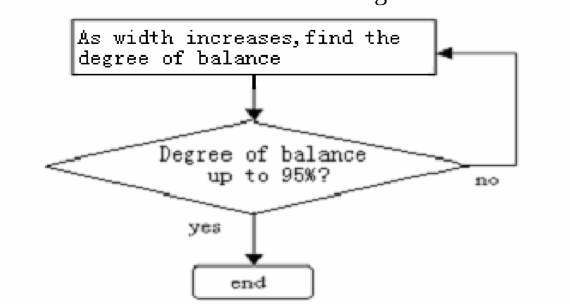

Fig 8 Flow chart for finding rational width

5 OFFSET-HALBACH ARRAY

Practical Halbach array is just an assembly with a set of modules whose magnetization vectors can be described as a sampling of a certain continuum. So the structure can be optimized in vector angles to enhance the fundamental, meanwhile to cancel the harmonics [6][8]. An offset-Halbach structure is made in this paper, whose structure is not arranged in the ordinary way, but with offset angles from the vector directions of typical Halbach array. Difference of two structures is shown in Table II, angles are defined between magnetization vectors and the horizontal axis. By simulation, we can get the offset angles that make magnetic induction largest. Offset-Halbach array can promote horizontal component of field about 4%, just changing the directions of magnetization without changing the geometrical structure, Field curves of offset-Halbach is shown in Fig 9.

Table II Typical and offset Halbach arrays

Module	Typical structure	Offset structure
Module1	90	θ_1
Module2	45	$180-\theta_1$
Module3	0	$180-\theta_2$
Module4	−45	θ_2-180
Module5	−90	θ_1-180
Module6	−135	$-\theta_1$
Module7	180	$-\theta_2$
Module8	135	θ_2

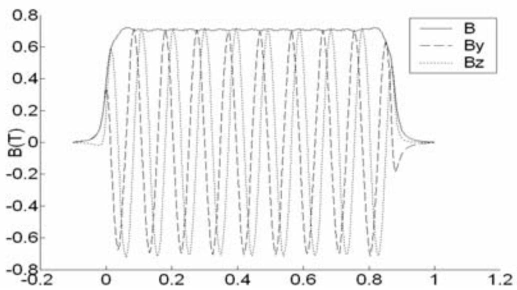

Fig 9 Magnetic field of offset-Halbach array

6 EXPERIMENT

On similarity principle, we try to make mini Halbach arrays to test the optimization of magnets' structure in order to make magnets for project of maglev. Four Halbach arrays are made, see Fig10.

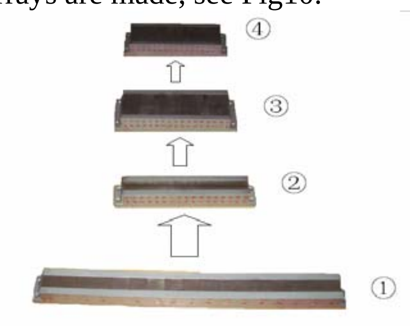

Fig10 Four Halbach arrays made in the lab

The last is offset-Halbach array, others are typical structure, all arrays are two-wave long and each block is as the follow (width×length×height):

①:25mm×25mm×25mm, typical Halbach array;

②:25 mm×12 mm×25mm, typical Halbach array;

③:75 mm×12 mm×25mm, typical Halbach array;

④: 75mm×12mm×25mm,offset-Halbach array, $\theta_1=109.5^0$, $\theta_2=154.5^0$

Remark: Array① is the ordinary structure directly made of normal magnet modules, which is adopted in many projects now[11];By means of equation (11),we get Array②;Based on Array②, by finding rational width, get Array③; At last, make offset-Halbach array (Array④) on base of Array③.

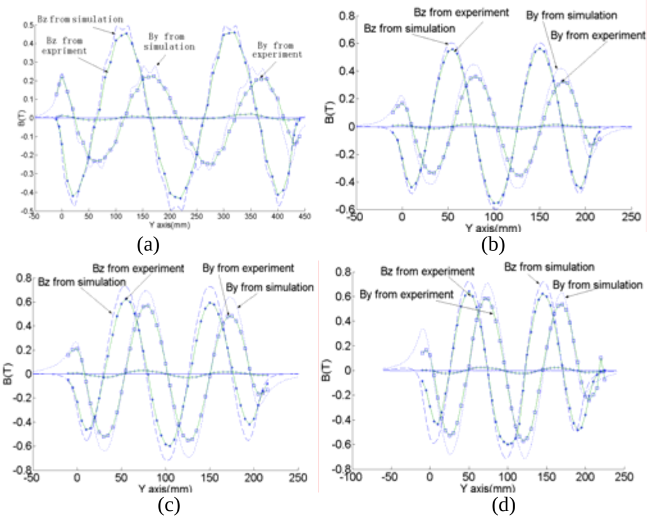

Fig 11 Fields from simulation and experiment.

(a):Array①,(b):Array②,(c):Array③,(d):Array④.*6mm* upon the enhanced side, along the centre axis of the surface, we measure the magnetic field and get the field curves in Fig 11.As is shown in Fig 11, horizontal and vertical components are sinusoidal, taking no account of end effects. Curves from experiments are consistent with those from simulation. From insert (a), structure without optimization has fields of much more distortion than others.

By means of least-squares fit (sinusoid fit), we get the data in Table III. From Table III, we get the follow Table IV.

Table III Data from experiment

Amplitude	*Bx*	*By*	*Bz*
array①	0.015	0.235	0.464
array②	0.015	0.363	0.562
array③	0.020	0.572	0.607
array④	0.021	0.590	0.621

Table IV Several results for comparison

	ξ	*By/Bz*(%)	Increment of horizontal component(%)
array①	0.0120	50.9	
array②	0.0600	64.6	
array③		94.2	④ vs ③: 3.2
array④		95.0	

From Array① to Array②: The index ξ_1 of Array ② is about 5 times of Array①, this means that Array② is better in utilization ratio, the optimization (equation 11) gives us not only good quality and less distortion of magnetic fields but also better utilization of magnetic materials.

From Array② to Array③: As for Array③, the degree of balance is much greater than that of module②, Horizontal component is sharply promoted to enhance the lift of maglev.

From Array③ to Array④: They have the same geometrical structures, however the increment of horizontal component is up to 3.2%. Although the number is lower than that of simulation (4%), on such a large base number of horizontal component, the number 3.2% is of good consideration on material and manufacturing of the arrays.

7 CONCLUSION

This paper provides a good tool for the design of magnets for further PM maglev systems.By analysis, 8-piece linear Halbach array is suitable for maglev project.To improve the maglev, we should do best to promote the profitable component, i.e. the horizontal one. Mainly on geometrical structure of Halbach array, we change the length, width and the directions of magnetization vectors to get much higher magnetic field to promote lift, especially horizontal component of field. 1). To get the strongest field with the least magnets, we provide an optimization index $\xi = \frac{B^2}{W_{mag}}$ and thenoptimize the length of magnetic module, we get the length $D_8 = -\frac{\pi D}{4\ln[L/(L+D)]}$. 2) We put forward the degree of field balance and the rational width of module, by finding the rational width of module, we promote the horizontal component sharply. This is useful for enhancing lift.3) Based on the above analysis, we make the offset-Halbach array and find offset angles of magnetization vectors to promote the field even higher.

Finally, all the above designs are tested by practical models of Halbach arrays in laboratory. Experiments verify those approaches and obtain satisfactory results.

Halbach array has a wide range of applications, the first step of engineering application is that Halbach array should be optimized and designed to yield the strongest fields. The techniques described in the paper are appropriate for further engineering designs of maglev and other interrelated projects.

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