Effects of the System Parameters to the Behavior of a Magnetic Top

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ABSTRACT: A magnetic top can be levitating without any control as far as it rotates at a certain speed. This fact suggests that the entirely passive magnetic bearing may exist. We had proposed a simple analytical method to describe the dynamic behaviour of a magnetic top. This analysis showed that the precession of a magnetic top is the essential to maintain levitation. In this paper, the effects of the key parameters such as weight or rotating speed of a top, to the dynamic behaviour of a rotating magnetic top, are investigated. A magnetic top is hopeful as a demonstration model or a relaxation item. The results of this paper provide the design criteria of a magnetic suspension system for such applications.

1 INTRODUCTION

The magnetic bearings have the distinct features such as clean, noiseless, vibration free and maintenance free. However, high cost due to control apparatus for 5 degrees of freedom of the rotor, prevent their wide applications at present. It is expected to develop a low cost magnetic bearing system.

The magnetic top, consist of a couple of ring shaped permanent magnets, PMs, can be levitating without any control while it maintains rotating by itself. This fact suggests that a magnetic top may be a hopeful candidate of the passive magnetic bearing. Several efforts had been made to explain the levitation mechanism of a magnetic top. San Miguel had reported that a magnetic top can be levitating by its rotating motion with precession based on a noble but complicated formulae.

We had proposed a simple analytical method to describe the dynamic behavior of a magnetic top, based on the equivalent side currents model for a permanent magnet. We had also proposed a design method based on the levitating area chart obtained from three dimensional, 3D, static analysis with the 3D dynamic motion analysis of a rotating magnetic top.

The dynamic behaviors of a magnetic top are dominated by the key parameters such as sizes of the both ground and rotating PMs, mass of a levitating top, tilt angle of a rotating PM, rotation speed and initial position related to the restoring center.

We had already investigated about the relationships between the shape and sizes of the levitating area based on the static 3D analysis, and the conditions to realize magnetic levitation are also studied based on the dynamic 3D analysis.

In this paper, the effects of the mass and the rotating speed of a magnetic top, to the levitating characteristics are studied. The dynamic behaviors of a levitating magnetic top are also investigated.

2 STATIC ANALYSIS OF A MAGNETIC TOP

2.1 *Analytical model*

A magnetic top is composed of a larger ring shaped stator PM on the ground and a smaller ring shaped PM in the top, as shown in Figure 1. In the analytical model, the axially magnetized ring shaped PMs are assumed as the equivalent coil current model. That is,

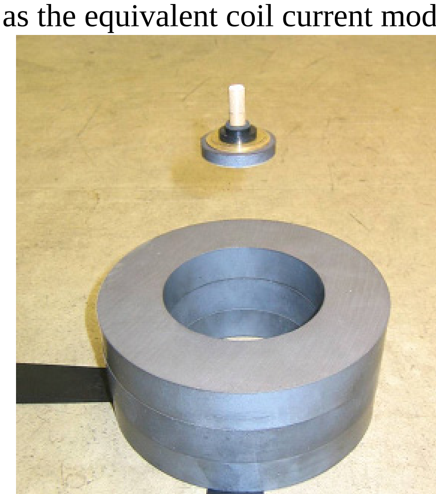


Figure 1. Levitating magnetic top.

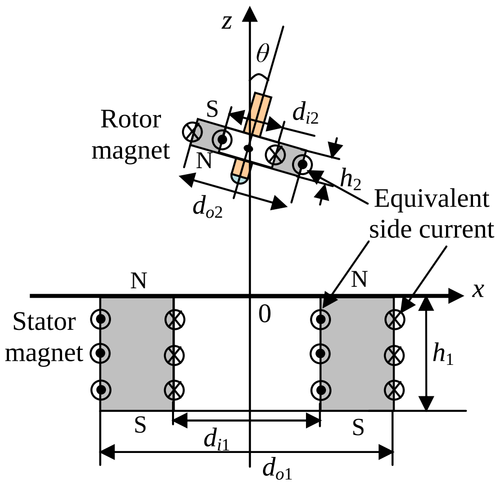


Figure 2. Analytical Model of The Magnetic Top

a ring shaped PM is replaced by the set of the circular coils located at the outer and inner surfaces of a ring shaped PM. The directions of currents in the outer and inner equivalent coils are inversed with each other, describing the axial magnetization of a PM, as shown in Figure 2.

The magnetic forces acting on the levitating PM are calculated as the interaction between the equivalent coil currents of the levitating PM and the magnetic field generated by the equivalent coil currents of the ground PM based on the Biot-Savart' law.

2.2 Levitating area in the magnetic force map

We had proposed the magnetic force map that shows the magnetic force acting on a PM placed at each designated points above the ground PM. This map shows the direction and the amplitude of the magnetic force in vector formula, where the vertical components of the magnetic forces are deducted by the weight of the top. Figure 3 shows the magnetic force map calculated for the parameters shown in the Table 1. The magnitude of the current in the equivalent coil is decided as the magnetic field density at a surface of the PM coincide with the measured value for the ferrite PM used in the experiments. Considering the thickness of the PMs, numbers of the equivalent coils are set as 2 for the rotor PM and 24 for the stator PM, in the calculation.

Figure 3 shows that a tilted magnetic top, located within the dotted rectangular area, named as the "levitating area", will be guided by the magnetic force along the direction of vectors towards the point A, named as the "restoring center" and its coordinates are (x_r, y_r, z_r)=(0, 0, 99.5). Although the levitating area is shown as two dimensional rectangle in Figure 3, the real shape of the levitating area is cylindrical.

Table 1 Parameters used in simulation

	Rotor PM	Stator PM
Outer diameter d_o [mm]	30	134
Inner diameter d_i [mm]	12	75
Thickness h [mm]	5	60
Eq. current I_{eq} [A/mm]	286	286
Mass m [g]	20.4	-
Tilt angle θ [deg]	-	1

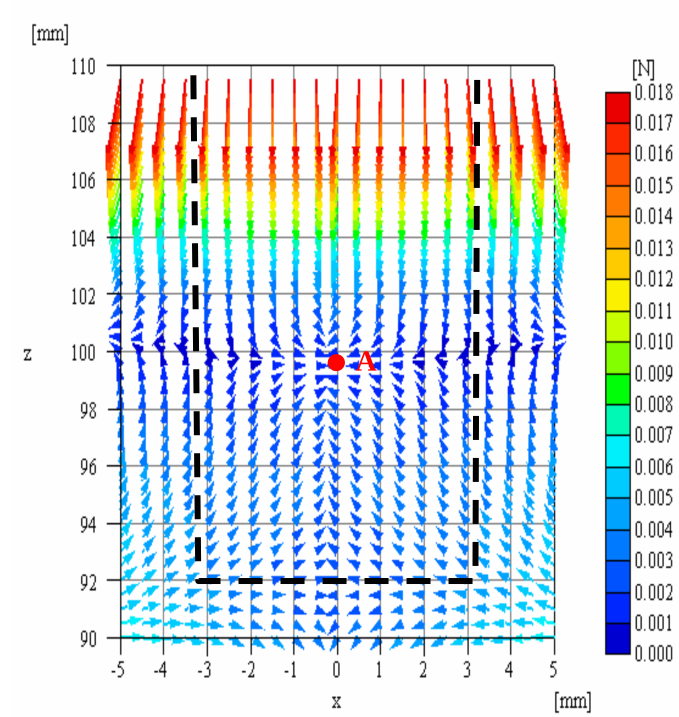


Figure 3. Levitating area and restoring center in the force map.

Calculations for the case of θ=0 showed that there is no levitating area nor restoring center for a vertical magnetic top. This results shows that the tilting is essential to maintain levitation of a magnetic top.

Although the static analysis gives the "levitating area", a magnetic top in this area can not always continue to levitate, considering dynamic motion of a top. However we can obtain rough guidelines for the size and shape of the levitating area.

2.3 Levitation hight and system parameters

We had already reported the effects of the PM sizes to the shape of the levitating area. Here, the effects of the thickness of the ground PM and the mass of a levitating top to the hight of the restoring center are discussed.

Figure 4 shows the relationships between the hight of the restoring center z_r (mm) and the mass of a levitating top m (g) when the thickness of the ground PM is 60, 40 and 20 mm. Calculated results show that the hight of the restoring center z_r becomes lower according to increase of the mass of a top, and to

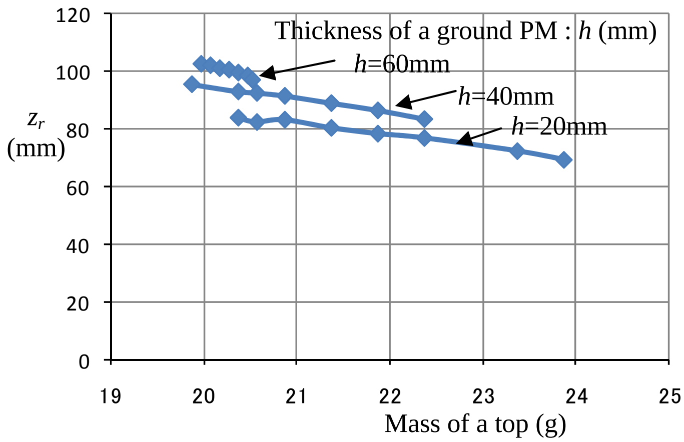


Figure 4. Hight of the restoring center v.s. mass of levitating top.

decrease of the thickness of a ground PM.

Calculations were made for various values for the mass of a top. However, there is no restoring center nor levitation area for a heavier or a lighter top shown in Figure 4. According to the results, a thin ground PM may realize successful levitation for wider mass variations of the rotating top.

3 DYNAMIC ANALYSIS OF A MAGNETIC TOP

3.1 Analytical model

After a rough design of a magnetic top system based of the static analysis in the previous chapter, a dynamic analysis is inevitable to confirm whether a rotating magnetic top can keep levitating or not. The behaviour of a magnetic top can be estimated by the equations of motion about the angular moment of a magnetic top, considering three dimensional layout of PMs with the tilt angle and the mechanical inertia of the rotating PM. Here, to simplify the analysis, aerodynamic damping effects are neglected.

The equations of motion for a magnetic top can be expressed as follows;

$$I_{Top} = m\frac{r_o^{\ 2} + r_i^{\ 2}}{2} \tag{1}$$

$$L_n = I_{Top} \times \omega \tag{2}$$

$$d\vec{L} = dt \times \vec{N} \tag{3}$$

$$\vec{L}_{n+1} = \vec{L}_n + d\vec{L} \tag{4}$$

where, I_{Top} is the moment of inertia of a magnetic top, m is the mass of a top, r_o and r_i are the outer and inner radius of the ring shaped PM in the top, L_n is the angular momentum vector around the axis of the rotating top at a time t_n , ω is the angular velocity of a top, $d\vec{L}$ is the incremental angular moment in a minute period dt, $\vec{N}$ is the torque, $\vec{L}_{n+1}$ is the angular momentum vector at a time $t_{n+1}=t_n+dt$.

The torque $\vec{N}$ in Equation 3 is estimated by the magnetic force acting on the levitating PM based on the equivalent coil current model. Some numerical simulations for the model shown in Table 1, were made based on the 4th order Runge-Kutta algorithm.

3.2 Effects of the initial position

Some experiments showed that the initial position related to the restoring center is one of the very important parameters. To realize successful rotation of a magnetic top, the initial position should be, at least, inside of the levitation area defined in the previous section. A magnetic top shows a variety of behaviours according to its initial point related with the restoring center.

Figure 5 shows the simulated trajectory of the center of a rotating PM in a top, in 60 seconds. The coordinates of the initial point is (1, 0, 98.5), where 1 mm apart in both x and z directions, respectively, from the restoring center. The mass of a top is 20.37g and rotation speed is 23rps, i.e., 1380rpm. The results shows that a rotating top is levitated in the area of +1.6mm/-1.6mm in both x and y directions, +1mm/-1mm in z direction, from the restoring center. The tilt angle varies in the range of 0 to 1.26 degrees related with z axis.

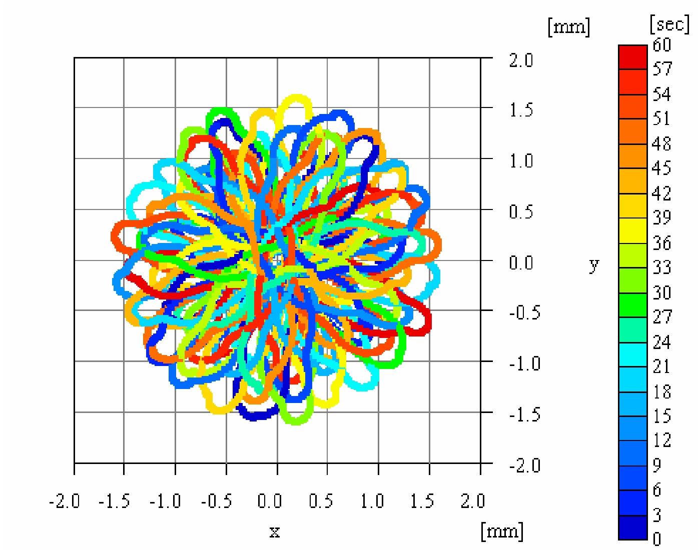
To investigate the effects of the initial point to the behaviours of a magnetic top, simulations were made for the case of the typical initial point of (1, 0, 99.5), that is, 1 mm apart in x direction from the restoring center, and (0, 0, 98.5), i.e., 1 mm apart in z direction from the restoring center.

Figure 6(a) and 6(b) show the simulated trajectory of the center of a PM of a top, rotating in 23rps in 60 seconds, for the initial points of (1, 0, 99.5) and (0, 0, 98.5), respectively.

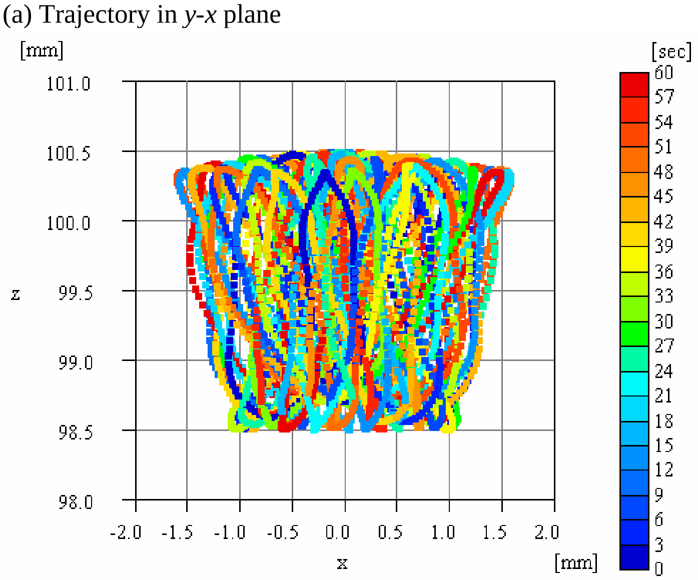
The magnetic top, started at the point 1 mm apart in x direction from the restoring center, is levitating in the range of +1.07mm/-1.07mm in both x and y directions, and +0.12mm/-0.06mm in z direction around the restoring center, as shown in Figure 6(a). The maximum tilt angle is 1.018 degree.

On the contrary, the magnetic top, started at the point 1 mm apart in z direction from the restoring center, is levitating in the range of +0.002mm/-0.002mm in both x and y directions, and +1mm/-1mm in z direction around the restoring center, as shown in Figure 6(b). The maximum tilt angle is 0.001165 degree.

These results show the ability of a magnetic top as an entirely passive magnetic bearings

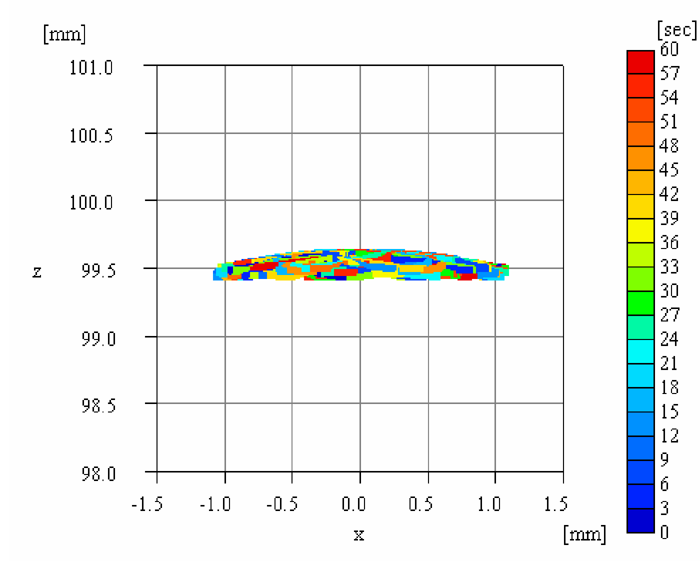


(a) Trajectory in *y*-*x* plane

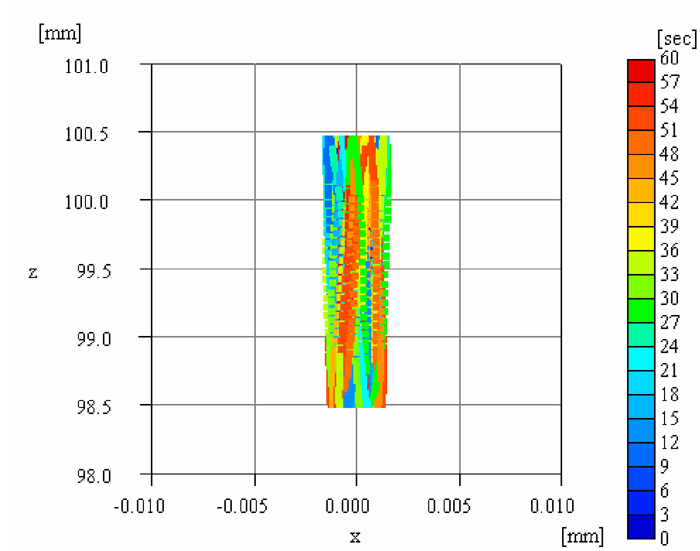


(b) Trajectory in *z*-*x* plane

Figure 5. Trajectory of a rotating magnet center of a top started at (1, 0, 98.5), 23rps(1380rpm)



(a) Trajectory of the PM center of a top started at (1, 0, 99.5)



(b) Trajectory of the PM center of a top started at (0, 0, 98.5)

Figure 6. Trajectory of the center of PM in a top in *z*-*x* plane.

3.3 Effects of the rotation speed

To realize successful rotation of a magnetic top, the rotation speed is also one of the most important parameters.

Figure 7 shows the simulated relationships between the maximum tilt angle and the rotation speed of a magnetic top. A magnetic top, whose parameters are listed in Table 1, can keep levitation while it rotates in the range of 18 to 50rps, i.e., 1080 to 3000rpm. When the rotation speed of a magnetic top becomes too low, it cannot keep its attitude with small tilt angle. On the other hand, when the rotation speed of a magnetic top becomes too high, a tilt angle becomes 0 by its mechanical inertia. Previous study said that there is no levitating area nor restoring center for a magnetic top rotating without precession.

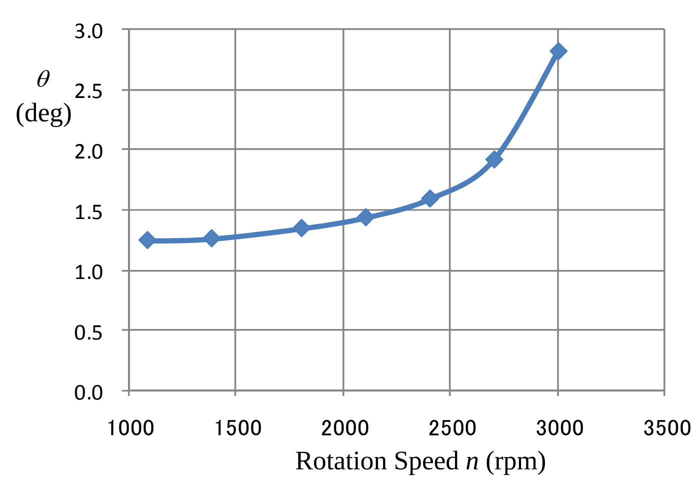
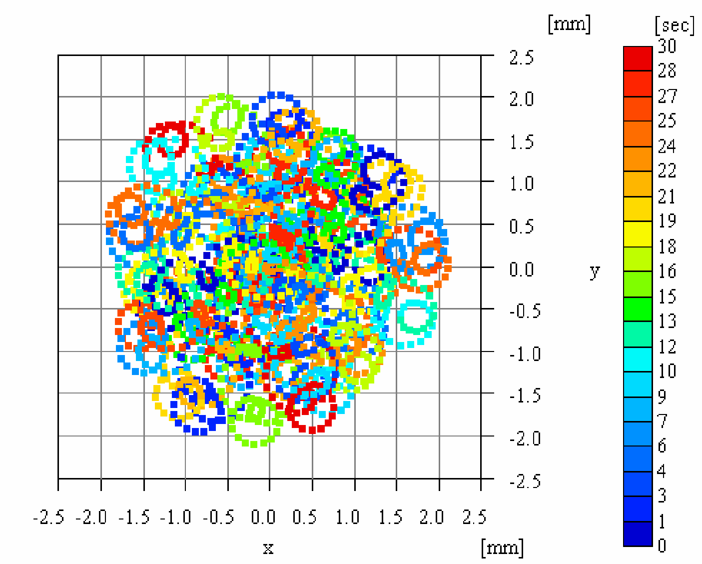
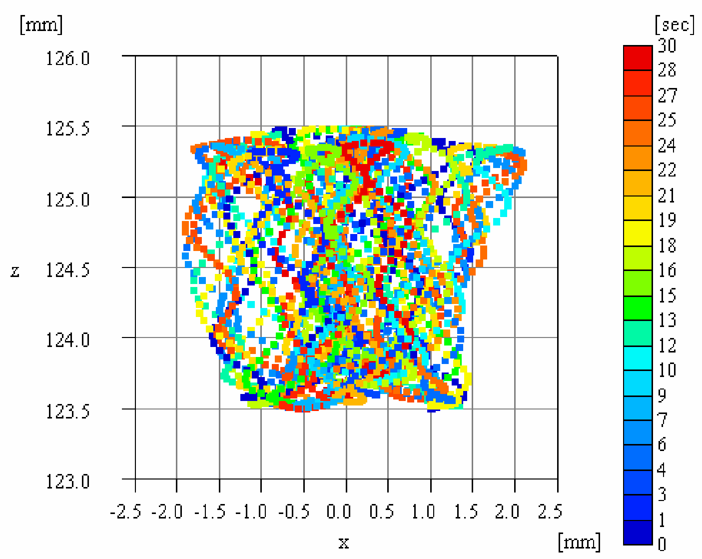


Figure 7. The maximum tilt angle θ v.s. rotating speed.

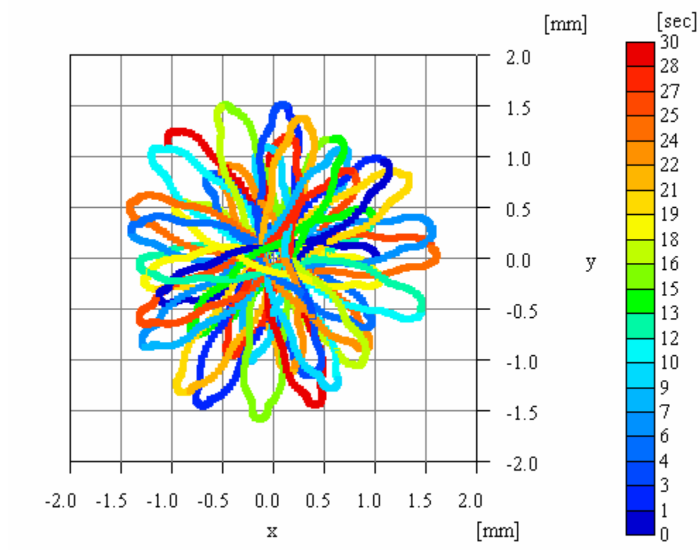


(a) Trajectory in *y*-*x* plane

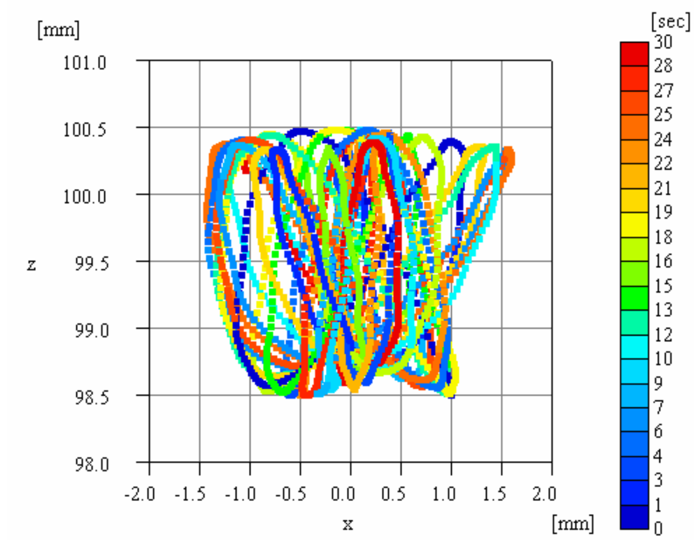


(b) Trajectory in *z*-*x* plane

Figure 8. Trajectory of axis head of a top rotating at 18rps.



(a) Trajectory in *y*-*x* plane



(b) Trajectory in *z*-*x* plane

Figure 9. Trajectory of a magnet center rotating at 18 rps.

Figures 8 and 9 show the simulated trajectory, in 30 seconds, of a 20.37g weight magnetic top rotating at 18rps, i.e., 1080rpm, with the initial point of (1, 0, 98.5). Figure 8 is the trajectory of the head of 25mm long shaft of a top, and Figure 9 is the trajectory of the center of a PM in a rotating top.

Figures 8 and 9 indicate that a levitating magnetic top rotates with both precession and nutation.

Figures 10 and 11 shows the simulated trajectory, in 30 seconds, of a 20.37g weight magnetic top rotating at 50rps, i.e., 3000rpm, with the initial point of (1, 0, 98.5). Figure 10 is the trajectory of the head of 25mm long shaft of a top, and Figure 11 is the trajectory of the center of a PM in a rotating top.

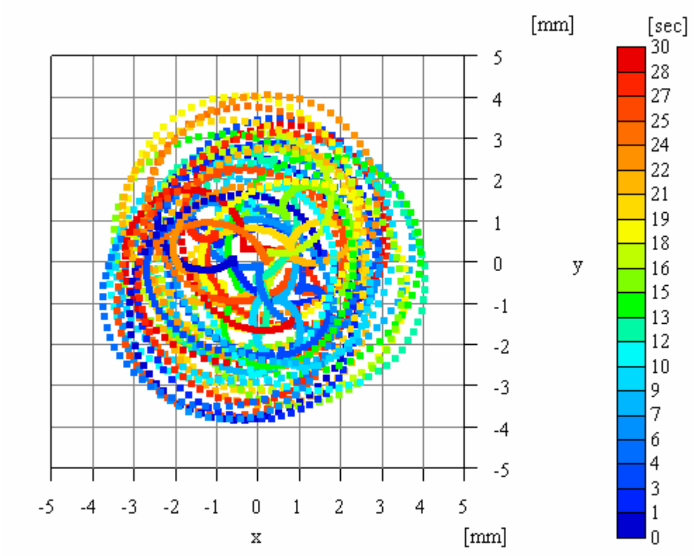
Figures 10 and 11 indicates that a magnetic top, rotating in high speed of 50rps, i.e., 3000rpm, levitates with precession. In this case, nutation mode is hardly observed.

4 CONCLUSIONS

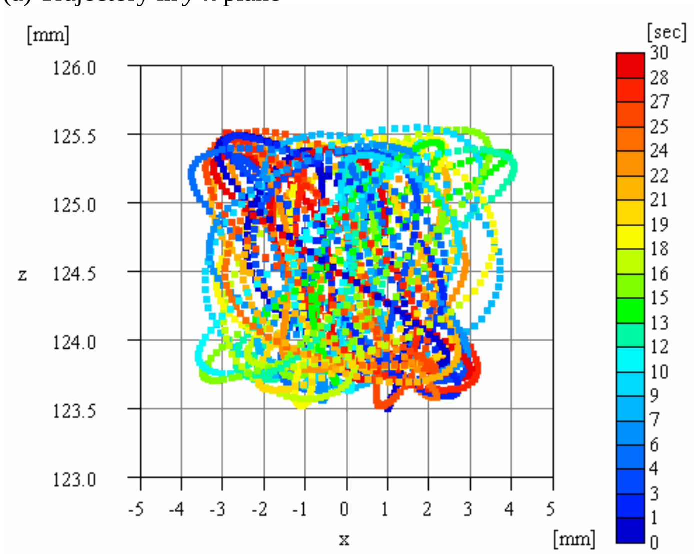
It is difficult for a magnetic top, to repeat experiments in the same condition. However, the simulated results showed good accordance with the experiments obtained from the digital video camera.

A magnetic top may be used as a rotating demonstration model in which some swaying motion can be permitted such as the toys or the relaxation items. The levitation area can be easily designed by the analysis proposed in this paper.

The results of this paper provide the design criteria of a magnetic suspension system for such applications.

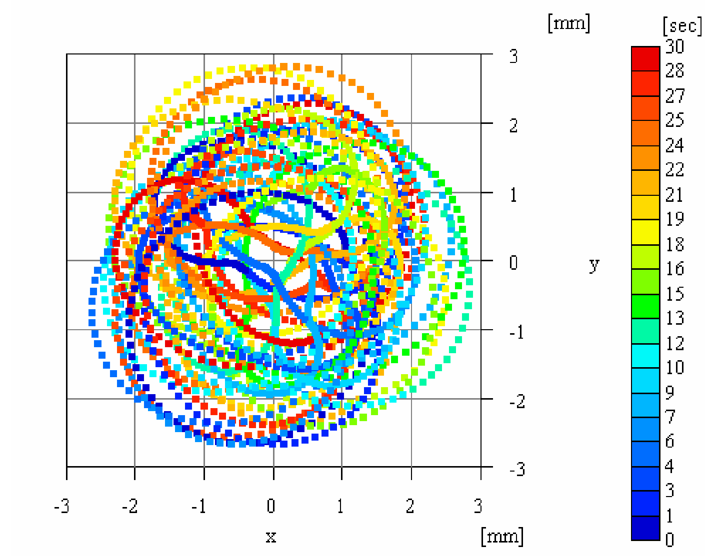


(a) Trajectory in *y*-*x* plane

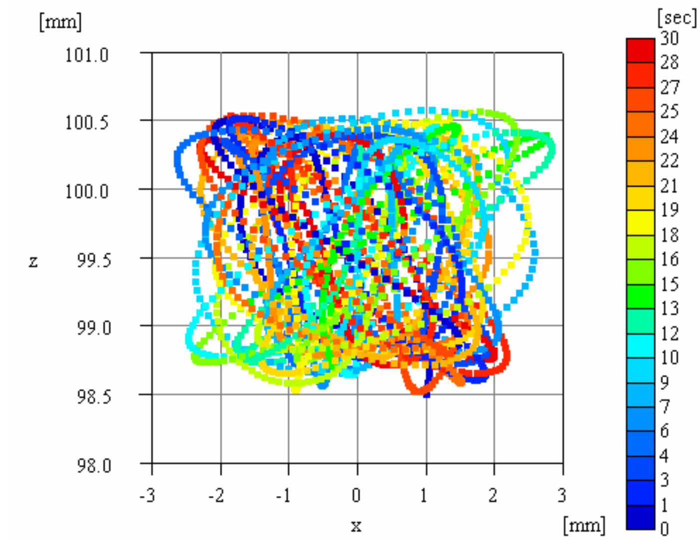


(b) Trajectory in *z*-*x* plane

Figure 10. Trajectory of axis head of a top in 30 sec, 50 rps, started at (1, 0, 98.5)



(a) Trajectory in *y*-*x* plane



(b) Trajectory in *z*-*x* plane

Figure 11. Trajectory of axis head of a top in 30 sec, 50 rps, started at (1, 0, 98.5)

5 ACKNOWLEDGMENT

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