Maple-exploring of a Free Flywheel Suspended by Superconductive Bearing

No. 108

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ABSTRACT: The software Maple as a powerful tool to analyze complicated non-linear dynamic systems is used to explore parametrical conditions guaranteeing the stability of resting or rotating free magnetically levitated flywheel's rotor, to solve the Cauchy problem and to build the phase portraits. The dynamical model based on analytical electromechanics is derived with taking into account six degrees of freedom of the free rotor, constancy of full magnetic fluxes coupled with superconducting rings, and Euler equations of a free body dynamics.

1 INTRODUCTION

Magnetic bearings become a very important element for renewable energy sources, end-use energy efficiency, environmentally preferred advanced generation, and flywheel energy storage. As it is expected, devices using magnetic bearings can be widely implemented, from automobile or helicopter engines to power reserve flywheels to level peak energy consumptions.

We developed a new type of magnetic bearings based on the "Magnetic Potential Well" (MPW) phenomenon [4, 6]. For two constantly oriented closed superconducting loops, the MPW-manifestation signifies that with nearing of these spaced loops, their magnetic attractive force does not increase as usually but decreases, becomes zero and changes into the repulsive magnetic force before the spacing between loops equals zero. A similar picture can be observed in a permanent magnet-closed zero resistant loop pair. The MPW can be also realized in a system with many magnets.

Magnetic bearings have many advantages over ball, gas, and hydro bearings: practically unlimited operating time, absence of lubricant, simplicity and reliability of operation, etc. But many applications of magnetic bearings require solving some problems. One of them is the stability of the free resting or rotating rotor. Trials to suspend a body in the free equilibrium under action of magnetic unregulated forces are fruitless on the basis of Earnshaw's theorem [3]. There are only three exceptions not covered by this theorem. One of them is automation allowing transforming an unstable dynamic system into the stable one by e.g. feed-back control. The second is diamagnetism of a substance particularly bulk superconductors [1, 2, 7]. And finally, the MPW based on zero resistance of a closed current-carrying loop [4, 6]. It should be noted that any of these exceptions assists but doesn't guarantee the free body rest or rotation stability.

Not only stability but also centring force level is important for bearing applications. The stiffness of magnetic bearing must be comparable with one of commercial bearings operating at radial stiffness no less than 10^7-10^8 N/mm. Possibilities to satisfy these data by known magnetic bearings do not look optimistic. Really, the top pressure of "warm" magnetic bearings with automation is restricted from above approximately by $2 \cdot 10^6$ N/m^2 that is determined by magnetic saturation induction of 2-2.5 Tesla for commercial magnetically soft materials. With characteristic bearing size of 0.1m it results in bearing stiffness of $2 \cdot 10^5$ N/m corresponding with published data [10].

The top pressure of 10^3-10^4 N/m^2 for passive superconducting bearings based on ideal diamagnetism of superconductors is restricted by the first critical magnetic field, which is less than 0.2

Tesla for known materials. This is too low value to be estimated as practicable.

In contrast to known magnetic bearings, instead of relying on superconductor diamagnetism or automatic control to keep the air gap within required limits, we propose to use the MPW phenomenon. Manifestation of this phenomenon requires ideal electric conductivity in a closed loop of any shape. This allows using existing high current density superconductors e.g. niobium-titanium, niobium-germanium, and niobium-tin joining. Superconducting magnets using these materials in the form of closed loop and operating in the persistent current mode are in abundance. They are capable of generating constant magnetic field no less than 10-20 Tesla without electric losses that is unreachable for all other modes of magnetic bearings. Before MPW these magnets were used as high magnetic field sources only. Now their applications can be MPW-bearings with pressure of 10^8 N/m^2 and stiffness of 10^7 N/m.

2 FLYWHEEL MODEL

2.1 Coordinate systems

To model the flywheel dynamics, first consider two coaxial immobile superconductive stator's rings magnetically interacting with two rotor's sets of N small planar superconductive circular loops (dipoles) radially symmetrically shifted in relation to the rotor's axis (see Figure 1). The MPW is realized in a set of N dipoles and the nearer superconductive immobile ring. It means that at some rotor/stator coaxiality position, the electric current in each dipole is zero because its full magnetic flux (magnetic linkage) that is constant on the basis of the Faraday's electromagnetic induction law is created by the nearer immobile ring, yet at any another rotor/stator geometrical configuration this current is non-zero to satisfy magnetic linkage constancy of arbitrary zero resistance closed loop.

To determine the potential energy of magnetic interaction, it is convenient to introduce some trihedrons. The origin O of the immobile trihedron Oxyz with unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ is the symmetry center of two coaxial superconductive rings of radius *a*, and the vertical rings' symmetry axis Oz is parallel to the gravity force. The origin O_1 of the second trihedron $O_1x_1y_1z_1$ with unit vectors $\mathbf{i'}_1$, $\mathbf{i'}_2$, $\mathbf{i'}_3$ coincides with the free rotor mass center so that axis O_1z_1 is directed along the rotor's symmetry axis. As an example, see Figure 1 for the case N = 4.

The free rotor is described by six degrees of freedom. The first three of them are Cartesian Oxyz-coordinates *x*, *y*, *z* of the point O_1. Non-dimensional linear coordinates X, Y, and Z are derived by division of corresponding Oxyz-coordinate by value *a*. Other three degrees of freedom are Euler-Krylov angles x_4, x_5, x_6 determining rotor space orientation and x_4 is roll-angle (rotation relatively axis O_1x_1), x_5 is pitch-angle (rotation relatively axis O_1y_1), and x_6 is yaw-angle (rotation relatively axis O_1z_1) at that.

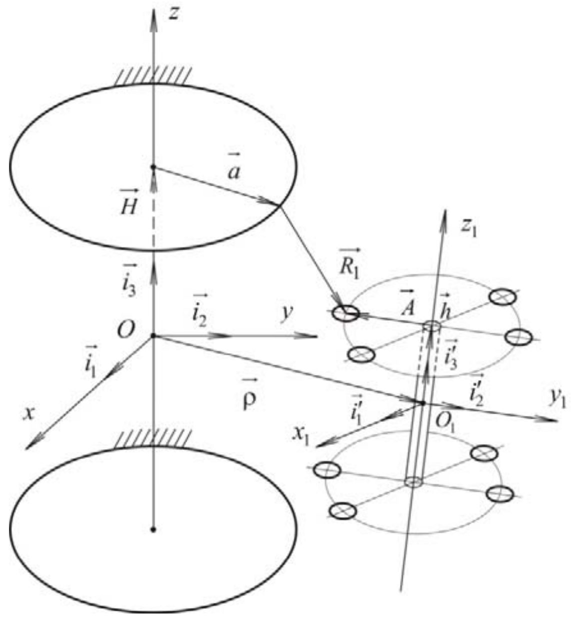

Figure 1. Sketch of the flywheel.

2.2 Potential energy

The potential energy U of the considered dynamic system consists of two parts. One is the gravity represented by the rotor's constant weight force. The second is the sum of magnetic interactions of each j-immobile superconducting ring (j = 1, 2) and N dipoles of upper or lower part of the rotor. Magnetic interactions between dipoles are ignored as infinitesimal.

The MPW-position that is at our disposal corresponds to the coincidence of axes Oz and O_1z_1. At this coaxiality position, the rotor's weight G = mg where m is its mass and g is the gravity acceleration can be balanced at Z = 0 by the difference in attractive forces of the upper part of superconducting loops (one immobile and N on the rotor top) directed above and the lower lesser part directed downward. Such coaxial equilibrium can be accomplished e.g. by the adjusted parameter $k = \Psi_2\Psi_1^{-1} = const > 1$ that is ratio between the upper immobile superconducting ring magnetic linkage Ψ_2 and lower ring linkage Ψ_1.

Below we use the matrix M determining relative space orientations of trihedrons Oxyz and $O_1x_1y_1z_1$ [8].

$$M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (1)$$

or in expanded form

$$M = \begin{pmatrix} c_5c_6 & c_4s_6 + s_4s_5c_6 & s_4s_6 - c_4s_5c_6 \\ -c_5s_6 & c_4c_6 - s_4s_5s_6 & s_4c_6 + c_4s_5s_6 \\ s_5 & -s_4c_5 & c_4c_5 \end{pmatrix} \qquad (2)$$

where $c_r = \cos x_r$, $s_r = \sin x_r$ and elements of the first row present projections of the unit vector along axis O_1x_1 etc.

Introducing radius vector **ρ** of the point O_1, vectors **h**, **A**, $\mathbf{R_1}$, **a**, **H** forming the closed polygon for the upper part of magnetically interacting loops (see Figure 1) and similar vector polygon for the lower part of magnetically interacting loops (not depicted in Figure 1), and using (1) for the components of the vectors connecting arc's elements of rings and dipole centers after transformations one can obtain

$$R_{j1} = X - (-1)^j h a_{31} + A\left(a_{11}\cos t_{ij} + a_{21}\sin t_{ij}\right),$$

$$R_{j2} = Y - (-1)^j h a_{32} + A\left(a_{12}\cos t_{ij} + a_{22}\sin t_{ij}\right), \qquad (3)$$

$$R_{j3} = Z - (-1)^j h a_{33} + A\left(a_{13}\cos t_{ij} + a_{23}\sin t_{ij}\right) + (-1)^j H$$

where index j = 1 is applied to the upper and j = 2 to the lower parts of loops, parameter h is a non-dimensional half-distance between rotor's loops, H is the same for the immobile rings, and the angle t_{ij} together with non-dimensional shift A determine a dipole placement on the rotor.

The mutual inductance of any ring-dipole pair can be derived as the magnetic flux generated by unit electric current in corresponding ring and piercing dipole area. Assuming that each dipole small plane is perpendicular to the rotor axis and its area equals S, taking into account known formulae for the ring magnetic induction (see e.g. [11]) and (3), for the non-dimensional ring-dipole mutual inductance the following formula takes place

$$L_j = \left(b_{j1}\left(a_{31}R_{j1} + a_{32}R_{j2}\right) + b_{j2}a_{33}\right) \qquad (4)$$

where

$$b_{j1} = \frac{R_{j3}}{r_{j1}}\left(-K(k_j) + \frac{1 + r_{j1}^2 + R_{j3}^2}{(1 - r_{j1})^2 + R_{j3}^2}E(k_j)\right),$$

$$b_{j2} = K(k_j) + \frac{1 - r_{j1}^2 - R_{j3}^2}{(1 - r_{j1})^2 + R_{j3}^2}E(k_j), \qquad (5)$$

$$r_{j1} = \sqrt{R_{j1}^{\ 2} + R_{j2}^{\ 2}},$$

where $K(k_j)$, $E(k_j)$ are full elliptic integrals of the modulus

$$k_j = 2\sqrt{r_{j1}\left((1 + r_{j1})^2 + R_{j3}^2\right)^{-1}}. \qquad (6)$$

Any non-dimensional ring-dipole inductance L_0 corresponding to the MPW-position is determined by zero values of all non-dimensional degrees of freedom ($X = Y = Z = x_4 = x_5 = x_6 = 0$) in (4).

On the basis of analytical electromechanics [12], the magnetic energy represented as function of magnetic linkages and mechanical coordinates is the potential energy of an electromechanical dynamic system. Therefore, this supposition and corresponding transformations result for the non-dimensional potential energy

$$u = UU_0^{-1} = \sum_{j=1}^{2}\frac{\Psi_j^2}{\Psi_2^2}\sum_{i=1}^{N}(L_0 - L_{ij})^2 + U_1 Z \ , \qquad (7)$$

where characteristic magnetic energy U_0 and non-dimensional gravity energy U_1 are respectively

$$U_0 = \frac{1}{L}\left(\frac{\mu_0 S}{2a\pi}\frac{\Psi_2}{L_0}\right)^2, \quad U_1 = mgaL\left(\frac{\mu_0 S}{2a\pi}\frac{\Psi_2}{L_0}\right)^{-2}. \qquad (8)$$

3 THE STABILITY PROBLEM

The stability problem for equilibrating or spinning rotor can be investigated on the basis of the Lyapunov's theorem about partial stability [9]. This problem is reduced to finding the positiveness conditions for the potential energy expanded into the Taylor series in corresponding variables. In our case using software Maple [13], the Taylor series for N = 4 can be written as

$$u = B_1\left(X^2 + Y^2\right) + B_2 Z^2 + B_3\left(x_4^2 + x_5^2\right). \qquad (9)$$

where expressions for values B_i (i = 1,…,3) are too cumbersome functions of the geometrical parameters and here are omitted. In accordance with Lyapunov's theorem, the sufficient conditions of the free equilibrium stability are equivalent to positive all parameters B_i. As simulations show, this requirement can be satisfied by a relevant choice of the system parameters.

4 THE DYNAMICS MODEL

The potential energy determined above and Euler equations of a free body dynamics [8] allow deriving initial equations of the rotor motion in the non-dimensional form

$$\frac{d^2X}{dt^2} = -A_1\frac{\partial u}{\partial X}, \frac{d^2Y}{dt^2} = -A_1\frac{\partial u}{\partial Y}, \frac{d^2Z}{dt^2} = -A_1\frac{\partial u}{\partial Z},$$

$$\frac{dn_1}{dt} + k_1n_3n_2 = -A_2\frac{\partial u}{\partial x_4}, \frac{dn_2}{dt} - k_1n_1n_3 = -A_2\frac{\partial u}{\partial x_5},$$

$$\frac{dn_3}{dt} = 0, n_1 = \cos x_5 \cos x_6 \frac{dx_4}{dt} + \sin x_6 \frac{dx_5}{dt}, \qquad (10)$$

$$n_2 = -\cos x_5 \sin x_6 \frac{dx_4}{dt} + \cos x_6 \frac{dx_5}{dt},$$

$$n_3 = \sin x_5 \frac{dx_4}{dt} + \frac{dx_6}{dt}.$$

Here non-dimensional parameter $A_1 = U_0(ma^2w^2L)^{-1}$ is ratio between magnetic characteristic energy U_0 and characteristic kinetic energy (w is the characteristic angular velocity); non-dimensional parameter $A_2 = U_0(I_1w^2L)^{-1}$ determines ratio between magnetic characteristic energy U_0 and rotating kinetic energy with inertia moment I_1 relatively axis O_1x_1 or O_1y_1; non-dimensional parameter $k_1 = (I_3 - I_1)I_1^{-1}$ is ratio of the rotor main central inertia moments (I_3 is the inertia moment relatively axis O_1z_1); n_1, n_2, n_3 are non-dimensional angular velocity components on axis O_1x_1, O_1y_1, and O_1z_1 respectively, and t is the non-dimensional time.

Expressions for partial derivatives of the non-dimensional potential energy u in (10) are too cumbersome and therefore omitted.

The conservative 12th-order system (10) with potential energy determined above is too complicated to be analyzed by known analytical methods. In this case the software Maple 11 [13] is more relevant.

We have developed Maple-based tools that give full analysis of the free rotor dynamics. As an example, we represent solutions for the Cauchy problem and phase portraits as plots in Figures 2-4.

These plots depict the case of H – h = 0.1, $A_1 = 1$, $A_2 = 1$, $k_1 = 0$, $U_1 = 0$ and the following initial conditions: X(0) = 0.05, $X_t(0) = 0$, Y(0) = 0, $Y_t(0) = 0.01$, Z(0) = 0.01, $Z_t(0) = 0$, $x_4(0) = 0$, $x_5(0) = 0$, $x_6(0) = 0$, $n_1(0) = 0$, $n_2(0) = 0$, $n_3(0) = 100$.

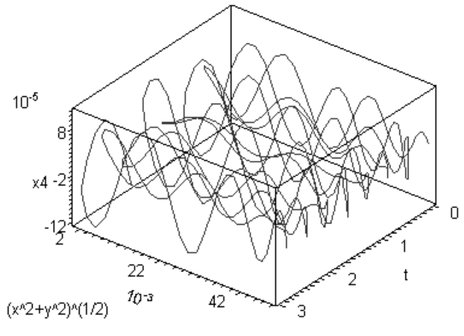

Figure 2. Phase portrait t, $(X(t)^2 + Y(t)^2)^{1/2}$, $x_4(t)$.

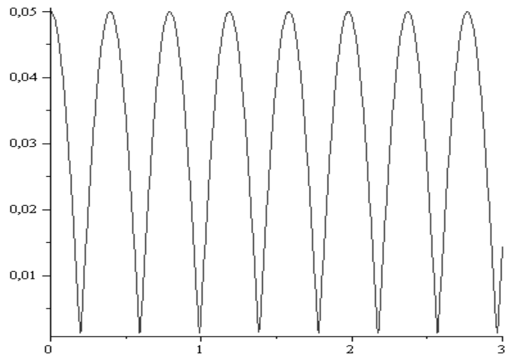

Figure 3. Radial separation versus time.

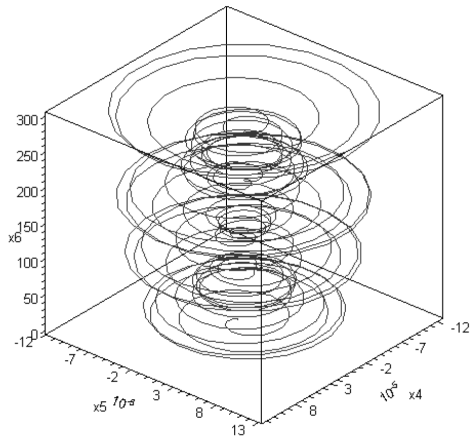

Figure 4. Phase portrait of x_4, x_5, x_6.

5 CONCLUSIONS

The free rotor dynamics problem is considered in the parts of mathematical model construction, substantiation of the free equilibrium stability (i.e. levitation), and dynamic analysis on the basis of the modern computer software.

The mathematical model taking into account six degrees of freedom proved to be complicated non-linear conservative dynamic system of the 12th-order, an analytically unsolvable problem.

The free equilibrium stability i.e. levitation derived on the Lyapunov's theorem is possible by the MPW-manifestation and relevant choice of the magnetic and geometrical parameters.

As examples demonstrate, the software Maple proved to be the effective computer tool to analyze the free rotor dynamics. This approach can be useful to develop superconducting bearings [13].

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