Equivalent Circuit Model with Parameters Depending on Secondary Speed for High-Speed Transport ---
A Smart Combination of Classical Modeling and Numerical Electromagnetic Calculation

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ABSTRACT: A linear induction motor (LIM) has advantages of low cost, robust structure, direct drive etc. Japanese HSST and Linear-Metros use LIMs for its thrust system. But the end-effect will be serious in high-speed applications of LIM. Electromagnetic numerical calculation methods are useful for the design of new LIMs', which requires consideration of end-effect. The disadvantage of such numerical methods is relatively long computational time. Authors propose a smart approach combining classical equivalent circuit method and such numerical methods. This new equivalent circuit will be useful to investigate advanced drive controls as well as to design LIMs for high-speed transport system.

1 INTRODUCTION

The great development of power electronics and control techniques brought very good prospects to use Linear Motors in railway applications, such as JR Maglev and HSST system in Japan, Transrapid in Germany and Linear Metro. In these linear motor applications, linear induction motors (LIMs) have advantage of low cost, robust structure, direct drive etc., so HSST system and Linear Metro use linear induction motors for its thrust system.

The top speed of these systems is assumed to be about 100km/h today, and the application to the urban transportation whose maximal speed is over 200km/h is being proposed for near future. As it is known, the end effects have influence on LIMs’ performance, at not only such higher speeds mainly but also region of slip in low-speed. Consequently, its influence must be appropriately taken into account in the development of the analytical or calculation model.

On the other hand, electromagnetic numerical calculation methods like Finite Element Method (FEM) and Finite Difference Method (FDM) have been widely used for the design of electromagnetic machines, and their methods are useful for the design of new LIMs which requires substantial consideration of this end-effect. But the disadvantage of such numerical methods is relatively long computational time at especially three-dimensional case, since one needs calculations on many operating points to know various states of the operation. And when motor constants change because of end-effects, the control of LIMs becomes difficult.

Authors propose a smart approach combining classical equivalent circuit method of induction motors and such numerical methods, for the purpose of advanced drive controls and a design of new LIMs for high-speed transport system as well as the reduction of calculation time of its electromagnetic analysis that considers end-effects. The new method is based on a parameter-fitting of an equivalent circuit of a LIM from numerical calculations in a few actual operational speeds. This parameter-fitting method of an equivalent circuit stands on electromagnetic analysis in this paper, moreover it is possible to determine constants by using actual LIMs’ testing result.

2 LIM’S EQUIVALENT CIRCUIT MODEL AS AN INDUCTION MOTOR

The end-effect can assume to depend mainly on LIM's operational speed. So, when the equivalent circuit of rotary induction motors is applied to LIM, a difference of the circuit of conventional rotary induction motors and linear induction motors is whether their constants depend on speed or not.

An equivalent circuit used in this paper, is shown in Figure 1. This circuit is the same as rotary induction motor. In rotary type, each equivalent circuit constants, such as $r_0$, $L_0$, and so on are treated as constant value which depends on nothing, on the
other hand, in LIM’s case, these circuit constants become functions of speed because of including LIM’s end-effect, i.e., these become \( r_0(v), L_0(v) \) and so on as functions of speed.

\[
Z = r + jL
\]

\[
Z_0 = r_0 + jL_0
\]

\[
Z_1 = r_1 + jL_1
\]

\[
Z_2 = r_2 + jL_2
\]

Figure 1: An equivalent circuit of induction motor without winding impedance at one phase

3 IDENTIFICATION METHOD OF CIRCUIT CONSTANTS

Because it is assumed that the end-effect depends on LIM’s operational speed in this paper, under the condition of constant speed, frequency characteristics of impedance are thought and it is possible that equivalent circuit constants are set under the constant speed. These constants are approximated as a function of speed, thus an equivalent circuit of rotary induction motors can be applied to that of LIMs.

This sequence of the identification of LIM’s circuit constants is summarized as shown in Figure 2.

3.1 General Identification Technique

In general rotary induction motor, equivalent circuit constants are set from conditions that slip \( s \) is 0 and 1. The slip \( s = 0 \) is no-load state and \( s = 1 \) is constraint state. If circuit constants are identified from this two operating point, this is useful for reduction of analysis or testing cost. But the both state \( s = 1 \) and \( s = 0 \) is not actual operating condition and especially in LIM’s case, this identification technique is badly constructed.

3.2 New Identification Technique

Equivalent circuit constants can be identified using two operation points. In this paper, as new identification technique, however, these constants are set using the value of slip that is used for practical operation, and in order to be approximated with high accuracy. And these constants are identified using a total of six operational values of slip including three acceleration states and three regenerative braking states at the condition of each constant speed. These six slip values are set from the region of actual use.

Because equivalent circuit parameters are set from electromagnetic analysis result or actual testing result, in this paper, using impedance that is calculated from these results, their parameters as functions of speed are identified. Compared with old analytical method of LIM for example space harmonic technique, one of the feature of this new identification method is the utilization of impedance of a result from electromagnetic analysis or actual testing, instead of the utilization of the flux density in LIM’s air gap, which is used by a classical harmonic method.

Under the condition of some constant speed, for example they are set to 5 km/h, 10 km/h, 20 km/h, ..., 70 km/h, and slip at six operational points, absolute value of impedance \(|Z|\) and power factor \( \cos \phi \) are analyzed by electromagnetic analysis. Then, as shown in Figure 3., when circuit constants \( r_0, L_0, r_2, L_2 \) are assigned, \(|Z|\) and \( \cos \phi \) represent the curve in Figure 3 as function of frequency of power supply \( f \), and frequencies at these six slip points are set to \( f_1, f_2, ..., f_5, f_6 \).

\[
|Z| = \sqrt{1 + \left( \frac{r}{L} \right)^2}
\]

\[
\cos \phi = \frac{v_2}{v_1} = \frac{\text{cos} \phi_2}{\text{cos} \phi_1}
\]

Under constant speed, electromagnetic analysis or actual testing results Input

Decision technique of circuit constants under constant speed

Making function of speed of circuit constants

Under each constant speed, circuit constants

Circuit constants as "function of speed"

Output

Figure 2: The sequence of decision of equivalent circuit constants depending on speed

Figure 3: Analytical or actual value and equivalent circuit result

Circuit constants \( r_0, L_0, r_2, L_2 \) are identified on the basis of parameter-fitting method; there constants are set by minimizing error of the curve from circuit constants and analytical value at six points and each constant speed. Therefore equivalent circuit constants are assumed to be a parameter, this method becomes optimization problem which minimize error of absolute value of impedance \(|Z|\) and power factor \( \cos \phi \) at six slip points.

This optimization problem can be formulated as follows:

\[
F = \sum_{i=1}^{6} \left( \frac{Z_{ai} - |Z_i|}{|Z_{ai}|} \right)^2 + (1 - \alpha) \left( \frac{\cos \phi_{ai} - \cos \phi_i}{\cos \phi_{ai}} \right)^2
\] (1)
In Equation 1, the evaluation function $F$ is the index of amplitude of error and absolute value of impedance $|Z|$ and power factor $\cos \phi$ from analytical value at six operation points are $|Z_{mi}|$ and $\cos \phi_{mi}$, those from equivalent circuit are $|Z|$ and $\cos \phi$, respectively. And $\alpha$ is weighting coefficient for converting multipurpose problem to monopurpose problem.

Minimizing this evaluation function $F$, circuit constants $r_0$, $L_0$, $r_2$, $L_2$ are determined, that is to say, equivalent circuit constants are set by the method where error between analytical value and calculation result from equivalent circuit becomes minimum at six slip point. And these can be identified by using this method at each constant speed.

Equation 1 is minimized by using optimization toolbox of MATLAB and there is a problem of initial value. It is assumed that values of speed for determination of circuit constants is set to $v_1$, $v_2$, ..., $v_n$ and there is relation of $v_1 < v_2 < \ldots < v_n$.

Initial values of circuit constants $r_0(v_1)$, $L_0(v_1)$, $r_2(v_1)$, $L_2(v_1)$ are determined from the same method of rotary induction motor, i.e., those values are decided by values when slip $s = 1$ and $s = 0$. This is because LIM’s end-effects depend on its operation speed, so the effect hardly occurs at the case of lowest speed $v_1$.

For $v_2 < \ldots < v_m < \ldots < v_n$, circuit constants $r_0(v_m)$, $L_0(v_m)$, $r_2(v_m)$, $L_2(v_m)$ are identified as initial values which are $r_0(v_{m-1})$, $L_0(v_{m-1})$, $r_2(v_{m-1})$, $L_2(v_{m-1})$, by using Equation 1.

4 ANALYTICAL MODEL

For the purpose of identification of circuit constants, electromagnetic numerical analysis like FEM and FDM or actual testing result is needed. So, a LIM is analyzed using two-dimensional FDM, which is the one of the electromagnetic analytical method and can take end-effect into account. Because of two-dimensional analysis, LIM’s transverse edge-effect is not taken into account, but substantial consideration of its end-effect can be exactly analyzed.

4.1 A Model as Rotary Induction Motor

As first, to simplify identification problem and to confirm the approach combining classical equivalent circuit method and numerical methods, a model of a rotary induction motor is considered. This model is shown in Figure 4. In Figure 4, left and right boundaries are set to cyclic boundary condition.

4.2 A Model of Linear Induction Motor

It is said that end-effect doesn’t occur in this rotary model. So in this case, utilizing classical identification method, i.e., the impedance at slip $s = 0$ and $s = 1$, the equivalent circuit is made up. And it is confirmed that there is no problem by using classical identification method.

4.3 Parameters of two models

Other parameters of these two models are summarized in Table 1.

Table 1: Other Parameters of Analytical Models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>RIM</th>
<th>LIM</th>
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<tbody>
<tr>
<td>Nominal current (A)</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Slip frequency (Hz)</td>
<td>3</td>
<td>12.5</td>
</tr>
<tr>
<td>Turns of coil</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Material of Secondary Conductor</td>
<td>Aluminum</td>
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</tr>
<tr>
<td>Magnetic Resistance of Primary Core</td>
<td>7.958*10^2</td>
<td></td>
</tr>
<tr>
<td>Conductivity of Secondary Conductor</td>
<td>2.290*10^7</td>
<td></td>
</tr>
<tr>
<td>Magnetic Resistance of Sec. Conductor</td>
<td>7.958*10^3</td>
<td></td>
</tr>
<tr>
<td>Magnetic Resistance of Back Iron</td>
<td>7.958*10^2</td>
<td></td>
</tr>
</tbody>
</table>
5 CHARACTERISTICS BY ELECTROMAGNETIC ANALYSIS AND EQUIVALENT CIRCUIT

5.1 Rotary Induction Motor

Figure 5 shows flux lines and the distribution of vector potential when secondary speed is 30 km/h, slip frequency is 3 Hz.

Using the vector potential shown in Figure 5, the impedance can be calculated and equivalent circuit constant can be identified.

In this rotary type model in Figure 4, as described in section 4.1, its constants are set from the impedance at slip $s = 0$ and $s = 1$ and frequency of power supply $f = 15$ Hz. Figure 6 shows speed-thrust characteristics of the result of FDM analysis and calculation of equivalent circuit at some constant conditions of frequency.

However, when the frequency of power supply increases, thrust of the result of the circuit is slightly decreasing in FDM case. This is because the secondary part of this rotary model consists of only aluminum plate and the resistance of secondary becomes bigger than that of conventional rotary induction motor, which has cage-shaped conductors as the secondary.

Therefore, as the result of this rotary case, it is shown that the approach combining classical equivalent circuit method and numerical electromagnetic methods is adequacy.

5.2 Linear Induction Motor

Under the assumption that circuit constants depend on speed in LIM’s case, LIM’s equivalent circuit is identified by the use of the new identification method described in section 3.2.

5.2.1 Electromagnetic Analytical Result

Figure 7 and Figure 8 show frequency characteristics of absolute value of impedance and power factor respectively using FDM under the condition of constant speed. These conditions of constant speed are set to 10 km/h, 20 km/h, 40 km/h, 60 km/h, ..., 240 km/h.

In this rotary model, it is said that equivalent circuit model is in good agreement with FDM using classical identification method of an induction motor in any region of slip and frequency of power supply.
5.2.2 Equivalent Circuit Constants as Function of Speed

LIM’s equivalent circuit constants are identified using analysis results of impedance and power factor as shown in Figure 7 and Figure 8. The region of slip frequency for actual use is chosen from -12.5 Hz to 12.5 Hz and 6 points of slip frequency for identification are set to 13.5, 12.5, 11.5 Hz in accelerative operation, and -13.5, -12.5, -11.5 Hz in regenerative braking.

These circuit constants $r_0$, $L_0$, $r_2$, $L_2$ that depend on LIM’s operational speed are summarized in Figure 10. Circle indications show data-produced circuit constants under the constant speed and curved solid lines link these by quadratic function as the function of speed using values at all points of speed which is used for the identification of circuit constants.

In order to decrease calculation cost of electromagnetic analysis, the speed function of circuit constants are approximated by using three conditions of speed which are chosen as 10, 140 and 240 km/h, as shown in Figure 11. In this case, it is assumed that the function of speed for approximation is a quadratic function.

In addition, in Figure 10 and Figure 11, $R_0$ becomes zero in all speed regions, because in FDM analysis the conductivity of LIM’s primary core is set to 0, i.e., it is assumed that the iron loss of the primary does not exist.

5.2.3 Characteristics of LIM calculated using circuit constants as a function of its speed

Figure 12 and Figure 13 show power supply frequency characteristics of the absolute value of impedance and the power factor based on circuit constants as a speed function approximated by all points of speed in Figure 10.

Figure 14 and Figure 15 show frequency characteristics of the absolute value of impedance and the power factor based on constants as the speed function approximated by only three points of speed in Figure 11.
The region for actual use is shown in these figures as an example, in the case of nominal speed 140 km/h. Seeing results of these characteristics, in the outside of actual region, these characteristics calculated from the equivalent circuit do not conform to the result of two-dimensional FDM. On the other hand, in the actual region, these fit into FDM within the error of 10%.

The Figure 16 shows the actual traction force calculated by using FDM and circuit constants based on speed functions approximated by all points and only three points.

In the result of this traction force, results of equivalent circuit constants of the both speed functions are nearly corresponding. Compared with two cases of speed functions, it is found that differences of characteristics from their function are small. It is said that this is useful for decreasing of calculation cost of FDM. These decrease from 78 operational points to 18 points in this case.

However, the result of traction force from the equivalent circuit becomes larger than that of the analysis of the FDM with the increase of its speed, because of errors of the absolute value of impedance and the power factor. It is thought that this error between circuit constants and numerical electromagnetic analysis is caused from “exit end-effect” and this exit-effect cannot be taken into account by an equivalent circuit model shown in Figure 1. This problem is clear if the case of the state of the synchronous speed is thought; in general, LIMs generate braking force at the synchronous speed because of exit-end-effect.

On the other hand, at the constant current mode in low speed, thrust is decreasing with the increase of the velocity in both cases of FDM and equivalent circuits. Almost all of the reason of this decreasing is the influence of entrance end-effect, and it is
found that this effect can be taken into account, even if the classical circuit model is applied to LIM.

6 CONCLUSION

In this study, a smart approach combining classical equivalent circuit method of induction motors and numerical electromagnetic analysis is proposed for advanced drive controls and a design of new LIMs for high-speed transport. It is assumed that LIM’s end-effect depends mainly on its operational speed, and the equivalent circuit of rotary induction motor is applied to LIM. These circuit constants functions of speed because of including the end-effect.

Equivalent circuit constants are identified by the impedance which is calculated from electromagnetic analysis such as FDM. Because of the utilization of impedance for identification of its constants, this identification method can adapt to actual testing result.

The result shows characteristics of LIM such as the absolute of impedance, power factor and traction force calculated from the equivalent circuit fir into the result calculated by using FDM within the error of 10%. Moreover, it is found that the circuit constants as the speed function are approximated by using three conditions of the speed.

It is shown that this new combination approach is effective and useful for realizing the identification of the equivalent circuit model of LIM for there are control system and design, even if the classical circuit model is applied to LIM.

And it is thought that this error of 10 % result from LIM’s exit end-effect, which cannot be taken into account by using the classical induction motor model. In order to consider this effect and build up the model with more accuracy, extra constants may be needed to the classical circuit model.

7 REFERENCES


