Analytical Computation of the Electromagnetic Forces in Synchronous Linear Electrical Machines

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Abstract

A method for the analytical evaluation of the propulsion force of an EMS Maglev transportation system is presented. The method, based on the reconstruction of the field through superposition of solutions of simple fields (exact - conformal transformations - or approximate), allows to take into account the slots and the interpolar geometry of the ferromagnetic structures, together with the real m.m.f. distribution. The study is applied to an example of system equipped with six-pole levitators and with a three-phase stator winding with one slot/(pole-phase): the analytical solution, that leads to the determination of the propulsion force on the vehicle, is compared with the results of Finite Element Method simulations (FEM).

1. Introduction

The study of the behaviour of electrical machines is usually performed according to two approaches: analytical (phasor models, Park equations) or numerical (FEM simulations). The first one allows a quick analysis of the machine behaviour both in transient and in sinusoidal or distorted operation, but it ideally considers smoothed ferromagnetic structures; if instead a more realistic modelling of the field distribution is required, taking into account the actual structure conformation and the real distribution of the conductors, it is necessary to employ numerical solutions, definitely more time consuming, and however not always well suited to allow a general approach and to perform parametric analyses.

A method that models in analytical form the ferromagnetic structures slotting and the distribution of the conductors is proposed, suited to take into account these effects in the calculation of the quantities, in any operating condition (transient or steady state), both in design stage and with the aim of control.

This method is applied here to the calculation of the propulsion force on one levitator of a linear synchronous motor of an EMS Maglev transportation system; indeed, in this type of machine, the slotting and the end effects are very intense, for the particular geometry of the armature structure (wide, open slots, with low number of slots/(pole/phase)) and for the discontinuities introduced by the separated levitators. Up to now, the model does not consider the transversal edge effects: thus, also FEM simulations concern 2D analyses.

2. Field Model

Consider the stator and the slider of an iron-core linear electric machine, where x is the coordinate of a point in a reference frame fixed with the stator, y the coordinate of the same point in a reference frame fixed with the slider, z the relative position between the two systems; it results therefore $x = y + z$.

In a previous paper [1], concerning a rotating machine, it has been shown that, in the hypothesis of magnetic linearity (infinite permeability of the ferromagnetic material), the normal component of the field over a surface disposed at half air-gap width, in a transversal section of the machine, is excellently represented by an expression of the type

$$B(x, y, t) = (\mu_0/g) \cdot [M_S(x,t) + M_R(y,t)] \cdot \beta_S(x) \cdot \beta_R(y), \qquad (1)$$

where the symbols have the following meaning: μ_0 = vacuum permeability; g = geometric air-gap width; $M_S(x,t)$, $M_R(y,t)$ are functions that express the instantaneous distributions of the m.mf.s respectively produced by the windings of stator and rotor; $\beta_S(x)$ and $\beta_R(y)$ are defined "field functions", because they express the behaviour of the field in the air-gap due to the ferromagnetic singularities of one structure (slots, interpolar zones), considering smoothed the other structure, and vice versa. In the following the expressions of such functions are given, considering that, for a linear synchronous machine, the subscript *R* (rotor) should be more correctly replaced with *f* (field).

For the field functions $\beta_S(x)$ e $\beta_f(y)$, their origin is resumed [2], [3]. The following quantities are defined:

- ideal flux density B_i: flux density that would exist between two smoothed and indefinite ferromagnetic structures, separated by an air-gap width g and submitted to a difference of magnetic potential U;
- lost flux density $B_p(x)$ of a real structure (not smoothed), faced to a smoothed one: difference between B_i and the actual flux density $B(x)$, determined along the smoothed structure in presence of the real structure;
- field functions $\beta(x)$, $\beta_p(x)$: ratio between the actual flux density and the ideal one, and between the lost flux density and the ideal one: $\beta(x) = B(x)/B_i = (B_i - B_p(x))/B_i = 1 - B_p(x) / B_i = 1 - \beta_p(x)$.

In a multipart ferromagnetic structure, consisting of several basic structures (a lot of slots, a lot of poles), the principle of superposition of the lost flux density is valid: the total lost flux density in the multipart structure is equal to the sum of the lost flux density functions in the single basic structures, suitably space displaced along the periphery; besides, the real flux density of the multipart structure is obtainable as difference between the ideal flux density and the total lost flux density.

Thus, in order to obtain the field function of a multipart structure, the following procedure can be adopted:

- through analytical study (conformal transformations), the expression of the flux density is obtained in the basic structure; usually, a parametric expression is gained: therefore, to have an explicit function, it is necessary to use an interpolating function (the simplest is a spline function); in alternative, the required function can be constituted by an analytical, approximating expression, for instance fitting the course obtained by a FEM solution;
- the lost flux density of the basic structure is obtained; the single lost flux densities are added for drawing the lost flux density of the multipart structure; the real flux density of the multipart structure is calculated;
- the ratio between the actual flux density and the ideal one gives the required field function.

For the field functions $\beta_S(x)$ and $\beta_f(y)$, the basic structures used are respectively the single slot with indefinite depth and the half-pole of indefinite extension, because for such cases the analytical solutions are known.

In the following, the m.m.f. functions will be examined. In the general case of a stator three-phase winding (p = 1, 2, 3, phase index), with N_t = N° turns/coil, a = N° of parallel paths, and with a field winding equipped with N_f = N° turns/(field coil), the following expressions are valid:

$$M_S(x,t) = (N_t/a) \cdot \sum_{p=1,2,3} m_{pS}(x - (p-1) \cdot 2\tau/3) \cdot i_p(t) \quad ; \quad M_f(y,t) = m_f(y) \cdot N_f \cdot i_f(t) \;, \qquad (2)$$

where $m_{pS}(x)$, $m_f(y)$ are the functions that express the space distribution of the f.m.m. respectively produced by a stator phase and by the field winding; $i_p(t)$, $i_f(t)$ are the stator phase currents and the field current; τ is the pole pitch.

The stator phase m.m.f. is given by the sum of the m.m.f.s $m_m(x)$ produced by the single coils; the function $m_m(x)$ can be expressed through a hyperbolic tangent function, that repeats its shape with a space period equal to the double coil pitch; for an integer pitch coil, the space period reduces to the double pole pitch:

$$m_m(x) = \tanh(k_m \cdot \cos(\pi \cdot x/\tau)); \qquad (3)$$

the coefficient k_m should be set considering the inclination of the curve, so as to approach the course of the field in the zone in which the m.m.f. passes from one level to the other.
The field m.m.f. is a square wave with unitary amplitude, also characterised by a period equal to the double pole pitch; defined a step function $\sigma(y)$ as:

$$\sigma(y) = 1 \text{ for } y \geq 0, \qquad \sigma(y) = 0 \text{ for } y < 0, \tag{4}$$

the field m.m.f can be expressed as follows:

$$m_f(y) = \left(\sigma\left(\cos\left(\frac{y}{\tau}\pi \right) \right) - \frac{1}{2} \right) \cdot 2 \quad . \tag{5}$$

The reason for the different model of the functions $m_m(x)$ and $m_f(y)$ requires some explanations (for further close examinations, please see [1]). Let us consider a slot containing a conductor and the course of the flux density (or the course of the corresponding field function), along a faced, smoothed structure.
If the current does not flow in the conductor, the field $\beta_{slot}(x)$ in front of the slot has a "basin"-like course while, in presence of current, the course is increasing (curve $\beta_{slot+curr}(x)$); the point to point ratio $\beta_{slot+curr}(x)/\beta_{slot}(x)$ can be interpreted as the contribution due to the current only (curve $\beta_{curr}(x)$) and it results to have, with good approximation, a course approaching a hyperbolic tangent.
Now, considering that the armature conductors are not always interested by current (in some instants of operation, or during the no-load operation), it is opportune to separate the effects due to the currents from the effects due to the slotting of the structure: therefore, for the stator we have chosen to insert in the field function $\beta_S(x)$ the slotting effect only, attributing the effect of the presence of currents to the m.m.f., that for this reason shows a course like a hyperbolic tangent.
Instead, the field winding is always interested by current and therefore the two effects are not distinguished: thus, both are attributed to the field function $\beta_f(y)$, leaving the m.m.f. with step-like course. However, one can observe that the hyperbolic-like course is an inherent property of the flux density, and the armature m.m.f. should be represented by a step-like course: the adoption of a hyperbolic-like course for the m.m.f. is due only to a convenience of the model.
The use of such a method allows a satisfactory reconstruction of the field at the air-gap: as anticipated, such flux density is assumed as representative of the normal component of the field distribution along the line at half air-gap width of a transversal section of the system.
As regards the waveforms of the described quantities, see [2], [3], [4].

3. General expression of the electromagnetic force of a synchronous machine

As known, the electromagnetic tangential force of an electrical machine can be expressed as the derivative of the co-energy with respect to the position z of the moving part; in the hypothesis of magnetic linearity (negligible iron core magnetic voltage drops), the co-energy coincides with the magnetic energy, thus it can be evaluated as the integral of the energy density in the air-gap volume. If the additional assumption of invariance of the energy density in the radial and transversal directions is assumed, the volume integral can be reduced to a line integral along the x stator coordinate, by extracting the air-gap g and the transversal size ℓ out of the integral. The integral should be extended along the whole periphery L of the machine.
Thus, the force expression is given by:

$$F(z,t) = \frac{\partial W(z,t)}{\partial z} = \ell \cdot g \cdot \frac{\partial}{\partial z} \left(\int_0^L \left(\frac{B^2(x, x-z, t)}{2 \cdot \mu_0} \right) \cdot dx \right) \quad ; \tag{6}$$

carrying the $\partial/\partial z$ operator under the integral operator $\int(\cdot)dx$, and observing that $\partial f(x-z)/\partial z = -\partial f(x-z)/\partial x$, equation (6) becomes:

$$F(z,t) = \ell \cdot g \cdot \frac{\partial}{\partial z} \left(\int_0^L \left(\frac{B^2(x, x-z, t)}{2 \cdot \mu_0} \right) \cdot dx \right) = -\frac{\ell \cdot g}{\mu_0} \cdot \int_0^L \left(B(x, x-z, t) \cdot \frac{\partial B(x, x-z, t)}{\partial x} \right) \cdot dx \, . \tag{7}$$

Called λ_g the following quantity:

$$\lambda_g = \mu_0 \cdot \ell / g \ , \tag{8}$$

by developing eq. (7) and putting $\partial(\cdot)/\partial x = D_x(\cdot)$, the following three force contributions derive:

- mutual force, due to the simultaneous existence of stator and field m.m.f.s:

$$F_m(z,t) = -\lambda_g \cdot \int_0^L \begin{pmatrix} M_S(x,t)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot \\ \cdot\left(2\cdot M_f(x-z,t)\cdot D_x\beta_f(x-z) + \beta_f(x-z)\cdot D_x M_f(x-z,t)\right)\end{pmatrix} dx \ ; \tag{9}$$

- stator slotting reluctance force, to which the force reduces in case of zero field m.m.f.:

$$F_S(z,t) = -\lambda_g \cdot \int_0^L \left(M_S^2(x,t)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot D_x\beta_f(x-z)\right) dx \ ; \tag{10}$$

- field (levitator) slotting reluctance force, to which the force reduces in case of zero stator m.m.f.:

$$F_f(z,t) = -\lambda_g \cdot \int_0^L \begin{pmatrix} M_f(x-z,t)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot \\ \cdot\left(M_f(x-z,t)\cdot D_x\beta_f(x-z) + \beta_f(x-z)\cdot D_x M_f(x-z,t)\right)\end{pmatrix} dx \ . \tag{11}$$

The integration of eq.s (9), (10), (11) appears cumbersome, due to the heavy expressions of the quantities, and because a different integration solution seems to be required for each instantaneous position $z(t)$; moreover, the time dependence of the m.m.f.s (2) seems to complicate the evaluation; actually, it is possible to extract the time dependent factors out of the integrals, leaving inside just the space dependent terms; thus:

$$F_m(z,t) = -\lambda_g \cdot (N_t/a)\cdot N_f \cdot i_f(t)\cdot \sum_{p=1,2,3} i_p(t)\cdot Y_{mp}(z) \quad \text{, with} \tag{12}$$

$$Y_{mp}(z) = \int_0^L \begin{pmatrix} m_{pS}\left(x-(p-1)\cdot 2\cdot\tau/3\right)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot \\ \cdot\left(2\cdot m_f(x-z)\cdot D_x\beta_f(x-z) + \beta_f(x-z)\cdot D_x m_f(x-z)\right)\end{pmatrix} dx \ ; \tag{13}$$

$$F_S(z,t) = -\lambda_g \cdot (N_t/a)^2 \cdot \sum_{p,u=1,2,3} i_p(t)\cdot i_u(t)\cdot Y_{Spu}(z) \quad \text{, with} \tag{14}$$

$$Y_{Spu}(z) = \int_0^L \left(m_{pS}\left(x-(p-1)\cdot\frac{2\tau}{3}\right)\cdot m_f\left(x-(u-1)\cdot\frac{2\tau}{3}\right)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot D_x\beta_f(x-z)\right) dx \ ; \tag{15}$$

$$F_f(z,t) = -\lambda_g \cdot N_f^2 \cdot i_f^2(t)\cdot Y_f(z) \ , \qquad \text{with} \tag{16}$$

$$Y_f(z) = \int_0^L \begin{pmatrix} m_f(x-z)\cdot\beta_S^2(x)\cdot\beta_f(x-z)\cdot \\ \cdot\left(m_f(x-z)\cdot D_x\beta_f(x-z) + \beta_f(x-z)\cdot D_x m_f(x-z)\right)\end{pmatrix} dx \ . \tag{17}$$

Equations (12)-(17) suggest the following remarks:

- the space dependent functions $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_f(z)$ (p, u = 1,2,3) can be evaluated off line just once, for a suited number of position z values, subsequently interpolating the calculated points; thus, when the time dependence is to be taken into account, these space quantities can be considered as known functions;
- $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_f(z)$ are able to correctly model the local stator and slider slotting field effects (including the effects of partial slot facings [1]), and the actual field and armature winding structures: this property ensures an accurate modelling of all the force harmonic contributions, including the well known toothing and cogging force harmonics, particularly noisy in both rotating and linear synchronous machines;
- of course, the adopted approach is rigorously valid just supposing perfectly unsaturated operation, because it implies the application of the superposition principle;
- moreover, it should be noted that, in general, no closed forms can be directly found for $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_f(z)$ and a numerical integration is required; on the other hand, usually the complexity of the involved functions makes quite heavy the direct calculation of the integrals. Indeed, all the terms in eq.s (13), (15), (17) depend on the space integral of the m.m.f.s distribution, of the field functions and of their derivatives; field functions derivatives are zero, except around the slot and interpolar openings, where sharp, wide variations occur: this pulse-wise behaviour makes

troublesome the numerical evaluation of the integrals; a great simplification is achieved if the integrand functions are developed in Fourier series, because, once obtained sinusoidal functions products, the integral can be solved analytically.

4. Expression of the electromagnetic force for the examined Maglev system

Let us consider now a six-pole levitator of a EMS Maglev vehicle faced to the stator (fig.1).
In this case, the energy integral should be extended to the whole extension of the levitator, between two interpolar axes, i.e. from the interpolar axis preceding the first levitator pole, to the interpolar axis following the last pole of the considered levitator; in the examined case, this extension equals six pole pitches, thus the integral is extended from 0 to 6·τ. On the basis of these considerations, it follows:

$$F_{lev} = \frac{\partial}{\partial z} W_{lev} = \frac{\partial}{\partial z} \iiint_V \frac{1}{2 \cdot \mu_0} \cdot B\left(x, y, t\right)^2 dV = \frac{\ell \cdot g}{2 \cdot \mu_0} \cdot \frac{\partial}{\partial z} \int_0^{6\tau} B\left(x, x-z, t\right)^2 dx . \qquad (18)$$

The expression of the flux density to be introduced in the integral is given by eq.(1), detailed for the case under analysis (the system has one slot/(pole-phase), therefore $N_t = 1$, $a = 1$).
It should be observed that, in the developed analysis, a machine without relative inclination among stator slots and levitator pole shoes has been considered; this situation is not realistic, because usually the levitator pole shoes are inclined with respect to the stator teeth, in order to reduce the slotting effects. Nevertheless, this inclination has not been considered here: in fact, the intent is to show the method soundness in a case that, besides the simplicity, has the property to present more evident slotting effects. In case the pole shoe - slots inclination is of interest, it is possible to transversally subdivide the machines in several “slices”, each without any inclination, but spatially displaced with respect to the others, in the motion direction.

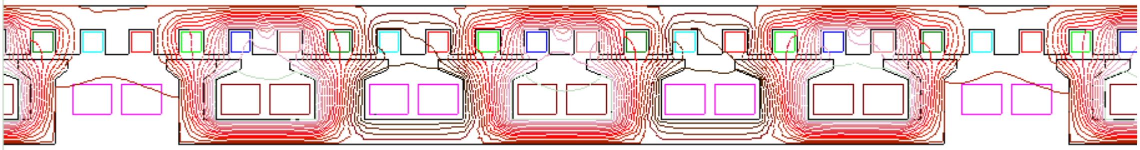

Fig.1: schematic of the analysed EMS Maglev system: $N_f \cdot i_f = 6$ kA; $i_1 = 0$; $i_2 = (\sqrt{3/2}) \cdot 800$ A; $i_3 = -i_2$. Maglev system main sizes [mm]: pole pitch: 300; slot width = tooth width = 50; central pole shoe width = 200; ext. pole shoe width = 150; air-gap width = 10; transversal lamination stack length (per side) = 200.

5. Holding Force Evaluation

As an example of application, the holding force will be evaluated, i.e. the force produced during the vehicle movement, with all the currents imposed and constant: even if this operating condition is not realistic for the Maglev system, it allows to estimate the soundness of the described analytical method. In this case, the time variable included in the stator m.m.f. should be considered equal to a fixed value t_0; therefore, this m.m.f. becomes a function of the coordinate x only; for this reason, in the following it will be indicated as $m_{s0}(x)$.
Putting $\hat{I}_s$ the peak value of stator phase current, and $I_f = N_f \cdot i_f(t) = cost$ the total field current, the flux density expression is given by:

$$B(x,z) = \left(\mu_0 / g\right) \cdot \left(m_{s0}(x) \cdot \hat{I}_s + m_f(x-z) \cdot I_f\right) \cdot \beta_s(x) \cdot \beta_f(x-z) , \qquad (19)$$

and the force acting on one levitator becomes:

$$F_{lev} = \frac{\mu_0 \ell}{2g}\frac{\partial}{\partial z}\int_0^{6\tau}\left(\hat{I}_s^2 m_{s0}(x)^2 + I_f^2 m_f(x-z)^2 + 2\hat{I}_s I_f m_{s0}(x)\, m_f(x-z)\right)\left(\beta_s(x)\beta_f(x-z)\right)^2 dx =$$

$$= \frac{\mu_0 \ell}{2g}\cdot\left(\begin{array}{l} \hat{I}_s^2 \int_0^{6\tau}\left(m_{s0}(x)\beta_s(x)\right)^2 \frac{\partial}{\partial z}\beta_f(x-z)^2 dx + I_f^2\int_0^{6\tau}\beta_s(x)^2\frac{\partial}{\partial z}\left(m_f(x-z)\beta_f(x-z)\right)^2 dx + \\ +2\hat{I}_s I_f\int_0^{6\tau}\left(m_{s0}(x)\beta_s(x)^2\right)\frac{\partial}{\partial z}\left(m_f(x-z)\beta_f(x-z)^2\right)dx \end{array}\right) =$$

$$= \frac{\mu_0 \ell}{2g}\cdot\left(\hat{I}_s^2 J_{css} + I_f^2 J_{cff} + \hat{I}_s I_f J_{csf}\right) \quad . \qquad (20)$$

In order to evaluate (20), the following procedure can be adopted:

- the functions $(m_{so}\cdot\beta_S)^2$, β_f^2, β_S^2, $(m_f\cdot\beta_f)^2$, $(m_{so}\cdot\beta_S^2)$, $(m_f\cdot\beta_f^2)$ are developed in Fourier series;
- β_S^2 has a period equal to the slot pitch τ_S; $(m_{so}\cdot\beta_S)^2$ and $(m_{so}\cdot\beta_S^2)$ have period equal to the double pole pitch $2\cdot\tau$; the other functions have period equal to the extension of the levitator, i.e. $6\cdot\tau$; moreover, τ is multiple of τ_S (with q = 1 slot/(pole-phase), it follows $\tau = 3\cdot\tau_S$): thus, all the periodicities are multiple each others;
- finally, remembering that the integral of the product of sinusoidal functions, extended to the whole period, is non-zero only if the sinusoidal functions have the same period, one obtains some expressions like:

  $$\Sigma_k\, f1coef_k \cdot f2coef_k \cdot \alpha(k) \cdot \sin(k,z,t_0)$$

  with $f1coef_k$ and $f2coef_k$ Fourier series coefficients of the functions $f1$ and $f2$, and $\alpha(k)$ constant.

In the considered case, with $\omega\cdot t_0 = \pi/2$, the expressions become:

$$J_{css}(z) = \sum_k\left[(m_{so}\beta_s)^2 coef\right]_k \cdot\left[\beta_f^2 coef\right]_{3k}\cdot(-3k\pi)\cdot\sin(kz\frac{\pi}{\tau})$$

$$J_{csf}(z) = 2\sum_k\left[m_{so}\beta_s^2 coef\right]_k \cdot\left[m_f\beta_f^2 coef\right]_{3k}\cdot(-3k\pi)\cdot\cos(kz\frac{\pi}{\tau}) \quad . \qquad (21)$$

$$J_{cff}(z) = \sum_k\left[\beta_s^2 coef\right]_k \cdot\left[(m_f\beta_f)^2 coef\right]_{18k}\cdot(-18k\pi)\cdot\sin(18kz\frac{\pi}{\tau})$$

6. Comparison with FEM solution

For the FEM solutions [5], a six-pole levitator with the structure shown in fig.1 has been adopted, in the following operating conditions: $N_f\cdot i_f$ = 6 kA; $i_1 = 0$; $i_2 = (\sqrt{(3/2)})\cdot 800$ A; $i_3 = -i_2$.

Observing fig.2, the analytical holding force has the expected waveform: a roughly sinusoidal average behaviour, with a space period equal to the double pole pitch ($2\cdot\tau$) and a ripple with a space period equal to the slot pitch (τ_s). The comparison with the FEM result shows that the analytical solution has amplitudes equal or lower than the FEM solution, suggesting that the analytically evaluated distribution of the flux density in the air-gap volume is underestimated. Thus, the slotting effect modelled by the previously defined field function of the stator lost flux density $\beta_{pS}(x)$ appears overestimated as regards the force evaluation: this is confirmed also by the comparison between the magnetic energy stored in the air-gap, because the analytical value results quite lower than the FEM value.

This higher inaccuracy evidenced by the described analytical method for the force evaluation compared with the results obtained in a similar situation for the evaluation of the flux density and of the e.m.f. in the stator winding [4] can be justified as follows: to the aim of the e.m.f. evaluation, the flux linkage calculation is performed by line integration along the line at half air-gap width, where the field is reproduced accurately; on the contrary, the electromagnetic force depends on the energy in all the air-gap volume; in fact, as experimented with FEM simulations, the dependence of the flux density on the radial coordinate is not negligible, whilst this dependence has not been analytically modelled.

Thus, a correction coefficient η_s has been looked for, to be applied to the stator lost flux density function. On the basis of the equivalence between the analytically evaluated air-gap energy W_{ANAL} and the FEM evaluated air-gap energy W_{FEM}, a particular situation has been considered: a single slot is faced to a smoothed structure. Remembering that the actual flux density is the difference between the ideal flux density and the lost flux density, the value of the searched coefficient η_s can be obtained by solving the following equation:

$$W_{ANAL} \equiv \int_{-\tau_s/2}^{\tau_s/2} \frac{1}{2\mu_0} \cdot \left[B_i - \eta_s \cdot B_p(x) \right]^2 \cdot \ell \cdot g \cdot dx = W_{FEM} \,. \tag{22}$$

In the examined case $\eta_s = 0.58$ has been obtained; by inserting this correction factor, the agreement between analytical and FEM solutions is greatly better (fig.3).

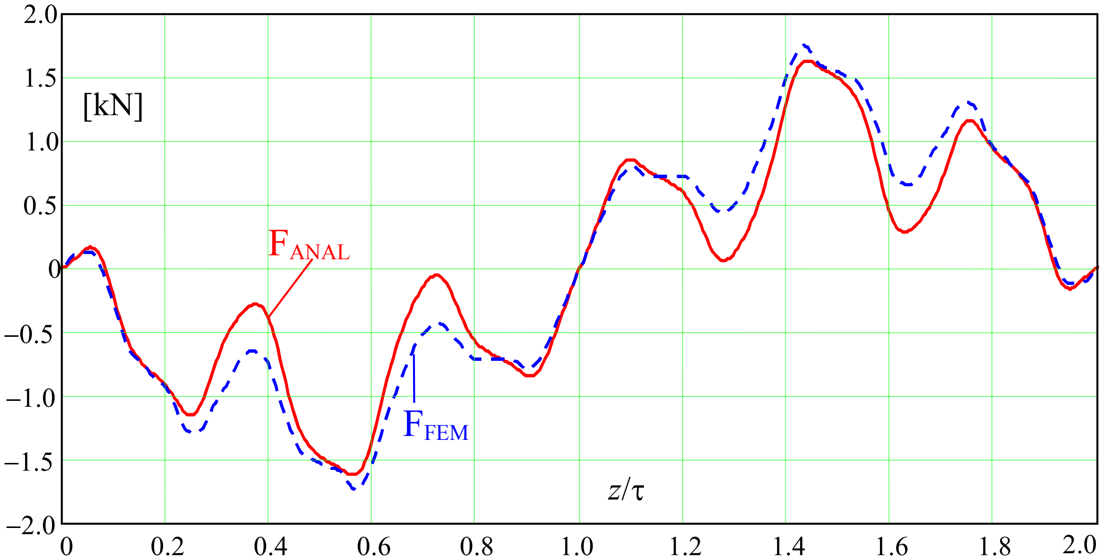

Fig.2: comparison between analytically and FEM evaluated holding force, without correction of $\beta_s(x)$

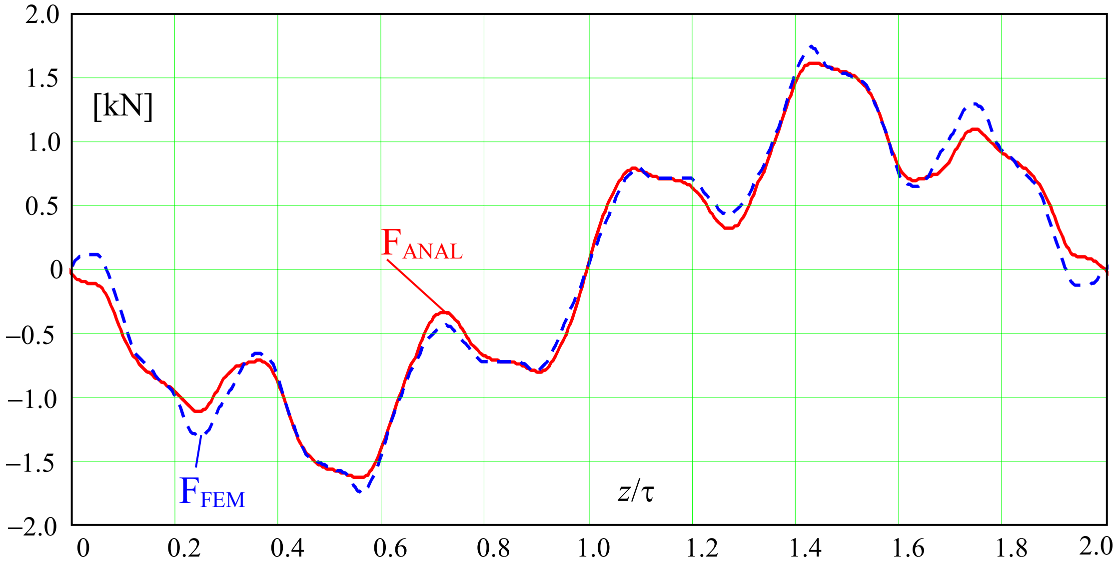

Fig.3: comparison between analytical and FEM holding force, with correction of $\beta_s(x)$ ($\eta_s = 0.58$)

A quantitative index of the deviation between the curves of the two considered cases is given by the average error ε over the pole pitch, expressed as a percentage, referred to the rms value of the FEM evaluated force:

$$\varepsilon = \frac{\frac{1}{\tau}\int_0^{\tau}(F_{ANAL} - F_{FEM})dx}{\sqrt{\frac{1}{\tau}\int_0^{\tau} F_{FEM}{}^2 dx}}. \tag{23}$$

Without the use of the correction coefficient, one obtains ε = 15.1%, while, introducing the evaluated correction factor (η_s = 0.58), the error greatly decreases, becoming ε = 0.8%.

6. Conclusion

A method has been illustrated for the analytical evaluation of the waveform of the propulsion force of an EMS Maglev system. The method, based on the reconstruction of the field through suited field functions, allows to take into account the field distribution due to the slotting of the ferromagnetic structures and to the actual m.m.f.s. The obtained analytical solution has been compared with the results of FEM simulations, getting good agreement both in the waveforms and in the numerical values.

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