Analytical Evaluation of the Electromotive Forces in Synchronous Linear Electrical Machines

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Keywords
Analytical evaluation, e.m.f.s with tooth harmonics, Synchronous linear electrical machines.

Abstract
A method for the analytical evaluation of the e.m.f. induced in the stator windings of an EMS Maglev transportation system is presented. The method, based on the reconstruction of the field through superposition of solutions of simple fields (exact - conformal transformations - or approximate), allows to take into account the slots and the interpolar geometry of the ferromagnetic structures, together with the real m.m.f. distribution. The study is applied to an example of system equipped with six-pole levitators and with a three-phase stator winding with one slot/(pole-phase): the analytical solution, that leads to determine the armature e.mf.s, is compared with the results of Finite Element Method simulations (FEM).

1. Introduction

The study of the behaviour of electrical machines is usually performed according to two approaches: analytical (phasor models, Park equations) or numerical (FEM simulations). The first one allows a quick analysis of the machine behaviour both in transient and in sinusoidal or distorted operation, but it ideally considers smoothed ferromagnetic structures; if instead a more realistic modelling of the field distribution is required, taking into account the actual structure conformation and the real distribution of the conductors, it is necessary to employ numerical solutions, definitely more time consuming, and however not always well suited to allow a general approach and to perform parametric analyses.
A method that models in analytical form the ferromagnetic structures slotting and the distribution of the conductors is proposed, suited to take into account these effects in the calculation of the quantities, in any operating condition (transient or steady state), both in design stage and with the aim of control.
This method is applied here to the calculation of the e.m.f. induced in the stator windings of an EMS Maglev transportation system; indeed, in this type of machine, the slotting and the end effects are very intense, for the particular geometry of the armature structure (wide and open slots, with low number of slots/(pole/phase)) and for the discontinuities introduced by the separated levitators. Up to now, the model does not consider the transversal edge effects: thus, also FEM simulations concern 2D analyses.
It should be noted that the cited e.m.f. contribution corresponds only to the mutual fluxes exchanged in the air-gap, while the e.m.f.s due to the leakage fluxes will not be considered.

2. Field Model

Consider the stator and the slider of an iron-core linear electrical machine, where x is the coordinate of a point in a reference frame fixed with the stator, y the coordinate of the same point in a reference frame fixed with the slider, z the relative position between the two systems; it results therefore $x = y + z$.

In a previous paper [1], concerning a rotating machine, it has been shown that, in the hypothesis of magnetic linearity (infinite permeability of the ferromagnetic material), the normal component of the field over a surface disposed at half air-gap width, in a transversal section of the machine, is excellently represented by an expression of the type

$$B(x,y,t) = (\mu_0/g)\cdot(M_S(x,t)+M_R(y,t))\cdot\beta_S(x)\cdot\beta_R(y), \tag{1}$$

where the symbols have the following meaning: μ_0 = vacuum permeability; g = geometric air-gap width; $M_S(x,t)$, $M_R(y,t)$ are functions that express the instantaneous distributions of the m.mf.s respectively produced by the windings of stator and rotor; $\beta_S(x)$ and $\beta_R(y)$ are defined "field functions", because they express the behaviour of the field in the air-gap due to the ferromagnetic singularities of one structure (slots, interpolar zones), considering smoothed the other structure, and vice versa. In the following the expressions of such functions are given, considering that, for a linear synchronous machine, the subscript R (rotor) should be more correctly replaced with f (field).

For the field functions $\beta_S(x)$ e $\beta_f(y)$, their origin is resumed [2], [3]. The following quantities are defined:

- ideal flux density B_i: flux density that would exist between two smoothed and indefinite ferromagnetic structures, separated by an air-gap width g and submitted to a difference of magnetic potential U;
- lost flux density $B_p(x)$ of a real structure (not smoothed), faced to a smoothed one: difference between B_i and the actual flux density $B(x)$, determined along the smoothed structure in presence of the real structure;
- field functions $\beta(x)$, $\beta_p(x)$: ratio between the actual flux density and the ideal one, and between the lost flux density and the ideal one: $\beta(x) = B(x)/B_i = (B_i - B_p(x))/B_i = 1 - B_p(x)/B_i = 1 - \beta_p(x)$.

In a multipart ferromagnetic structure, consisting of several basic structures (a lot of slots, a lot of poles), the principle of superposition of the lost flux density is valid: the total lost flux density in the multipart structure is equal to the sum of the lost flux density functions in the single basic structures, suitably space displaced along the periphery; besides, the real flux density of the multipart structure is obtainable as difference between the ideal flux density and the total lost flux density.

Thus, in order to obtain the field function of a multipart structure, the following procedure can be adopted:

- through analytical study (conformal transformations), the expression of the flux density is obtained in the basic structure; usually, a parametric expression is gained: therefore, to have an explicit function, it is necessary to use an interpolating function (the simplest is a spline function); in alternative, the required function can be constituted by an analytical, approximating expression, for instance fitting the course obtained by a FEM solution;
- the lost flux density of the basic structure is obtained; the single lost flux densities are added for drawing the lost flux density of the multipart structure; the real flux density of the multipart structure is calculated;
- the ratio between the actual flux density and the ideal one gives the required field function.

For the field functions $\beta_S(x)$ and $\beta_f(y)$, the basic structures used are respectively the single slot with indefinite depth (fig.1) and the half-pole of indefinite extension (fig.2), because for such cases the analytical solutions are known; in fig.s 1 and 2, the corresponding courses of the flux density and of the lost flux density are also shown, with reference to the geometry of the Maglev system that will be analysed in the following (fig.9). Applying the described procedure, the field functions of fig.s 3 and 4 are obtained: fig.4 refers to a levitator equipped with 6 poles, with external pole shoe extension equal 0.75 times the internal pole shoe extension.

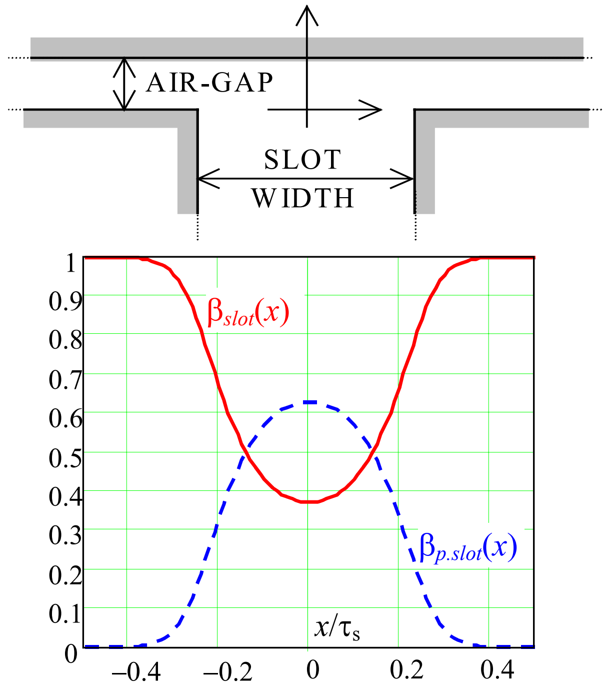

Fig.1: geometry, flux density function and lost flux density function of an indefinitely deep single slot; τ_s is the slot pitch.

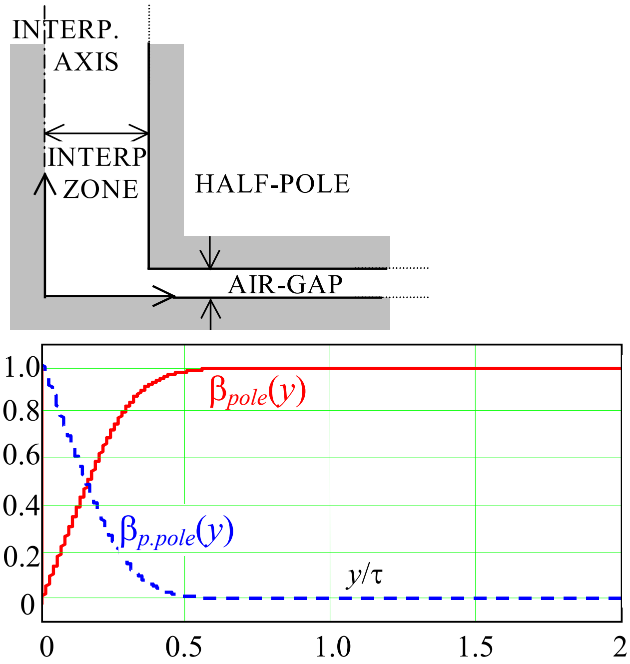

Fig.2: geometry, flux density function and lost flux density function of an indefinitely extended single half pole shoe; τ is the pole pitch.

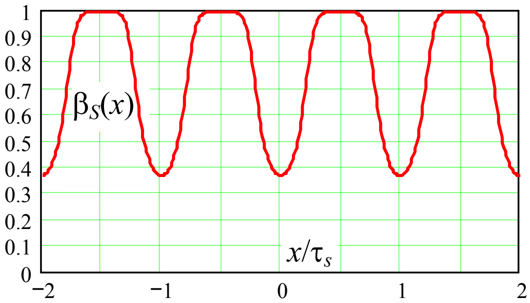

Fig.3: stator field function waveform $\beta_S(x)$

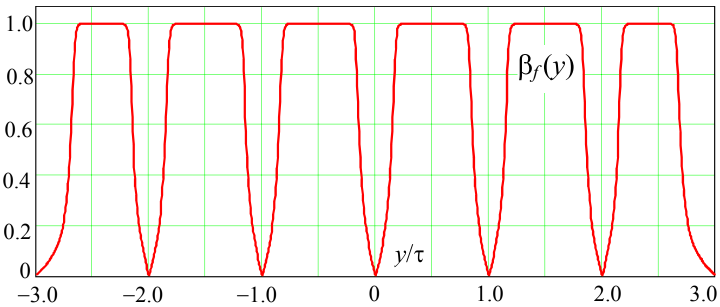

Fig.4: levitator field function waveform $\beta_f(y)$

While the stator field function $\beta_S(x)$ is periodic with a space period equal to the slot pitch τ_S, the levitator field function $\beta_f(y)$ has a space period equal to six times the pole pitch τ and shows six pulses that correspond to the extension of the levitator pole shoes; the field function β_f(y) consists of positive pulses only, because the sign is included in the m.m.f. distribution.

In the following, the m.m.f. functions will be examined. In the general case of a field winding equipped with N_f = N° turns/(field coil) and with a stator three-phase winding (p = 1, 2, 3, phase index), with N_t = N° turns/coil, a = N° of parallel paths, the following expressions are valid:

$$M_f(y,t) = m_f(y)\cdot N_f \cdot i_f(t)\ ; \qquad M_S(x,t) = (N_t/a)\cdot \sum_{p=1,2,3} m_{pS}\left(x-(p-1)\cdot 2\tau/3\right)\cdot i_p(t)\ , \qquad (2)$$

where $m_f(y)$, $m_{pS}(x)$ are the functions that express the m.m.f. space distribution, respectively produced by the field winding and by a stator phase; $i_f(t)$, $i_p(t)$ are the field and the stator phase currents.

The stator phase m.m.f. is given by the sum of the m.m.f.s $m_m(x)$ produced by the single coils; the function $m_m(x)$ can be expressed through a hyperbolic tangent function, that repeats its shape with a space period equal to the double coil pitch; for an integer pitch coil, the space period reduces to the double pole pitch:

$$m_m(x) = \tanh\left(k_m \cdot \cos(\pi \cdot x/\tau)\right); \qquad (3)$$

the coefficient k_m should be set considering the inclination of the curve (fig.5), so as to approach the course of the field in the zone in which the m.m.f. passes from one level to the other.

The field m.m.f. is a square wave with unitary amplitude, also characterised by a period equal to the double pole pitch; defined a step function $\sigma(y)$ as:

$$\sigma(y) = 1 \text{ for } y \geq 0, \qquad \sigma(y) = 0 \text{ for } y < 0, \tag{4}$$

the field m.m.f can be expressed as follows (fig.6):

$$m_f(y) = 2 \cdot \left(\sigma\left(\cos\left(\pi \cdot y/\tau\right)\right) - 1/2\right) \cdot \tag{5}$$

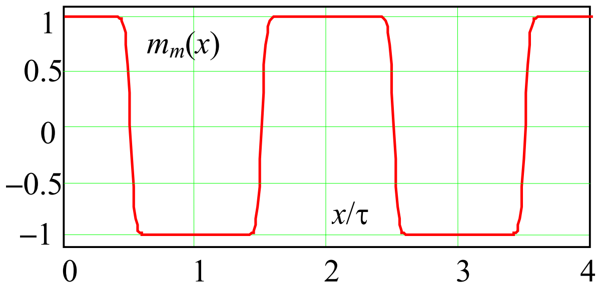

Fig.5: stator coil m.m.f. waveform $m_m(x)$

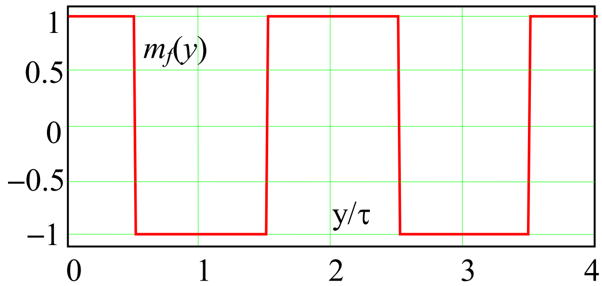

Fig.6: levitator m.m.f. waveform $m_f(y)$

The reason for the different model of the functions $m_m(x)$ and $m_f(y)$ requires some explanations (for further close examinations, please see [1]). Let us consider a slot containing a conductor and the course of the flux density (or the course of the corresponding field function), along a faced, smoothed structure.

If the current does not flow in the conductor, the field in front of the slot has a "basin"-like course (curve $\beta_{slot}(x)$ in fig.7) while, in presence of current, the course is increasing (curve $\beta_{slot+curr}(x)$); the point to point ratio $\beta_{slot+curr}(x)/\beta_{slot}(x)$ can be interpreted as the contribution due to the current only (curve $\beta_{curr}(x)$) and it results to have, with good approximation, a course approaching a hyperbolic tangent.

Now, considering that the armature conductors are not always interested by current (in some instants of operation, or during the no-load operation), it is opportune to separate the effects due to the current from the effects due to the slotting of the structure: therefore, for the stator we have chosen to insert in the field function $\beta_S(x)$ the slotting effect only, attributing the effect of the presence of currents to the m.m.f., that for this reason shows a course like a hyperbolic tangent.

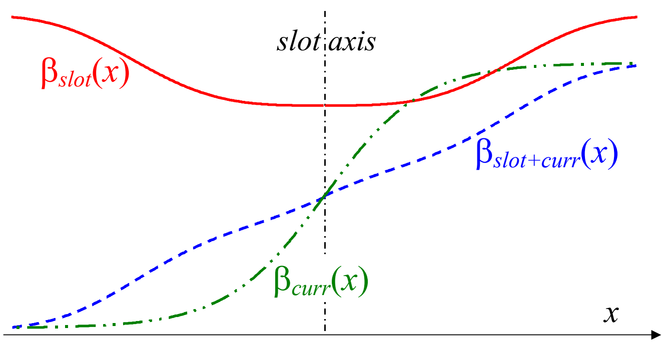

Fig.7: flux density function waveform along a smoothed structure, in front of a slot, with slot current ($\beta_{slot+curr}(x)$) or without slot current ($\beta_{slot}(x)$); the ratio $\beta_{curr}(x) = \beta_{slot+curr}(x)/\ \beta_{slot}(x)$ can be considered as the contribution due to the current only: its shape is well approximated by a hyperbolic tangent function.

Instead, the field winding is always interested by current and therefore the two effects are not distinguished: thus, both are attributed to the field function $\beta_f(y)$, leaving the m.m.f. with step-like course. However, one can observe that the hyperbolic-like course is an inherent property of the flux density, and the armature m.m.f. should be represented by a step-like course: the adoption of a hyperbolic-like course for the m.m.f. is due only to a convenience of the model. The use of such a method allows a satisfactory reconstruction of the field at the air-gap: as anticipated, such flux density is assumed as representative of the normal component of the field distribution along the line at half air-gap width of a transversal section of the system.

The soundness of the method is visualized in the comparison, in a particular instant, between the field obtained by the analytical solution and that obtained by a corresponding FEM solution [4] (fig.8) for the considered six-pole levitator structure (fig.9).

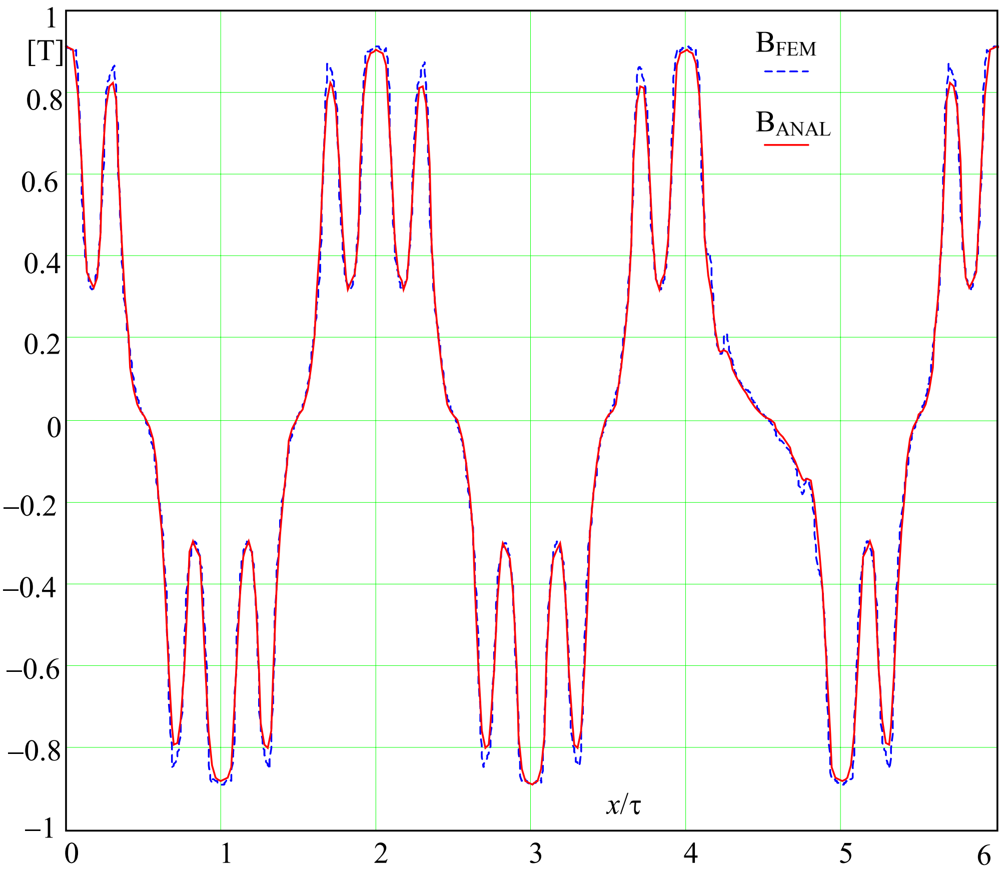

Fig.8: normal component of the flux density along a line at half air-gap width of a transversal section of the considered EMS Maglev system, equipped with six-pole levitators: comparison between analytical and FEM simulations: $N_f \cdot i_f = 6$ kA; $i_1 = \sqrt{2} \cdot 800$ A; $i_2 = i_3 = -i_1/2$.

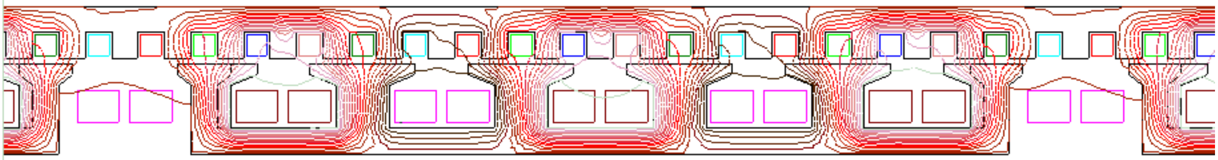

Fig.9: schematic of the analysed EMS Maglev system: $N_f \cdot i_f = 6$ kA; $i_1 = 0$; $i_2 = (\sqrt{(3/2)}) \cdot 800$ A; $i_3 = -i_2$. Maglev system main sizes [mm]: pole pitch: 300; slot width = tooth width = 50; central pole shoe width = 200; ext. pole shoe width = 150; air-gap width = 10; transversal lamination stack length (per side) = 200.

3. General expression of the e.m.f. for a synchronous machine

In the following, the expression of the e.m.f. will be obtained, for a general linear machine, equipped with q slots/(pole-phase): thus, flux linkage and e.m.f. will be evaluated for a group of q series connected coils under each pole. Let y_c be the coil pitch (expressed in number of slot pitches) and x_{ip1} the position of the initial active side of the first stator coil of the p-th phase; the initial and final active side positions of the k-th coil of the same p-th phase are:

$$x_{ipk} = x_{ip1} + (k-1) \cdot \tau_s \ ; \quad x_{fpk} = x_{ipk} + y_c \cdot \tau_s \qquad , \qquad (6)$$

For an unskewed machine of stack transversal size ℓ, the flux linkage of a group of q coils is:

$$\psi_p(z,t) = \sum_{k=1}^{q} \psi_{pk}(z,t) = (\mu_0/g) \cdot \ell \cdot N_t \cdot \sum_{k=1}^{q} \left(\int_{x_{ipk}}^{x_{fpk}} \beta_S(x) \cdot \beta_f(x-z) \cdot \left(M_S(x,t) + M_f(x-z,t) \right) dx \right) \ . \ (7)$$

In eq.(7) the integration is extended along the line positioned at the middle air-gap width.
The corresponding e.m.f. e_p of a group includes a motional and a transformer term; thus, being $z = z(t)$:

$$e_p(t)=\frac{d\psi_p(z(t),t)}{dt}=\frac{\partial\psi_p(z,t)}{\partial z}\cdot\frac{dz}{dt}+\frac{\partial\psi_p(z,t)}{\partial t}=e_{pm}(t)+e_{pt}(t) \quad . \tag{8}$$

Called λ_g the following quantity:

$$\lambda_g=\mu_0\cdot\ell/g\ , \tag{9}$$

and being $v = dz/dt = v(t)$ the speed, by performing the z-derivative under the integral operator, the motional e.m.f. e_{pm} of a group becomes:

$$e_{pm}(t,z(t))=N_t\cdot\lambda_g\cdot v\cdot\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\left(\frac{\partial\beta_f(x-z)}{\partial z}\cdot\beta_S(x)\cdot M_S(x,t)+\frac{\partial\left(\beta_f(x-z)\cdot M_f(x-z,t)\right)}{\partial z}\cdot\beta_S(x)\right)dx\right)\ . \tag{10}$$

On the other hand, considering that $\partial f(x-z)/\partial z = -\,\partial f(x-z)/\partial x$, and performing a "per part" integration of the second term under integral, eq. (10) gives:

$$e_{pm}(t,z(t))=-N_t\cdot\lambda_g\cdot v\cdot\sum_{k=1}^{q}\left(\begin{array}{l}\int_{x_{ipk}}^{x_{fpk}}\left(\frac{\partial\beta_f(x-z)}{\partial x}\cdot\beta_S(x)\cdot M_S(x,t)-\frac{d\beta_S(x)}{dx}\cdot\beta_f(x-z)\cdot M_f(x-z,t)\right)dx+\\+\left[\beta_S(x)\cdot\beta_f(x-z)\cdot M_f(x-z,t)\right]_{x_{ipk}}^{x_{fpk}}\end{array}\right) . \tag{11}$$

As regards the e.m.f. e_{pt} of a group, by performing the time derivation under the space integral operator:

$$e_{pt}(t,z(t))=N_t\cdot\lambda_g\cdot\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\beta_S(x)\cdot\beta_f(x-z)\cdot\left(\frac{\partial M_S(x,t)}{\partial t}+\frac{\partial M_f(x-z,t)}{\partial t}\right)dx\right)\ . \tag{12}$$

The integration of the eq.s (11) and (12) appears cumbersome, because of the heavy expressions of the involved quantities, and because a different integration solution seems to be required for each instantaneous position $z(t)$; moreover, the time dependence of the m.m.f.s (2) seems to complicate the evaluation: on the other hand, it is possible to extract the time dependent factors out of the integrals, leaving inside just the space dependent terms:

$$\begin{array}{l}e_{pm}(t,z(t))=-N_t\cdot\lambda_g\cdot v\cdot(N_t/a)\cdot\left(J_1(z(t))\cdot i_1(t)+J_2(z(t))\cdot i_2(t)+J_3(z(t))\cdot i_3(t)\right)+\\ \quad +N_t\cdot\lambda_g\cdot v\cdot\left(J_f(z(t))-\sum_{k=1}^{q}\left[\beta_S(x)\cdot\beta_f(x-z)\cdot m_f(x-z)\right]_{x_{ipk}}^{x_{fpk}}\right)\cdot N_f\cdot i_f(t)\end{array}\quad ,\ \text{with} \tag{13}$$

$$J_p(z)=\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\left(\frac{\partial\beta_f(x-z)}{\partial x}\cdot\beta_S(x)\cdot m_{pS}\left(x-(p-1)\cdot 2\tau/3\right)\right)dx\right),\qquad p=1,2,3, \tag{14}$$

$$J_f(z)=\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\left(\frac{d\beta_S(x)}{dx}\cdot\beta_f(x-z)\cdot m_f(x-z)\right)dx\right)\quad ; \tag{15}$$

$$e_{pt}(t,z(t))=\sum_{u=1,2,3}L_{pu}(z)\cdot di_u/dt+L_{pf}(z)\cdot di_f/dt\quad ,\quad p=1,2,3,\qquad \text{with} \tag{16}$$

$$L_{pu}(z)=\lambda_g\cdot\left(N_t^2/a\right)\cdot\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\beta_S(x)\cdot\beta_f(x-z)\cdot m_{pS}\left(x-(u-1)\cdot 2\tau/3\right)dx\right),\ p,\ u=1,2,3, \tag{17}$$

$$L_{pf}(z)=\lambda_g\cdot N_t\cdot N_f\cdot\sum_{k=1}^{q}\left(\int_{x_{ipk}}^{x_{fpk}}\beta_S(x)\cdot\beta_f(x-z)\cdot m_f(x-z)\,dx\right)\quad ,\quad p=1,2,3. \tag{18}$$

Equations (13)-(18) suggest the following remarks:

- the space dependent functions $J_p(z)$, $J_f(z)$, $L_{pu}(z)$, $L_{pf}(z)$ (p, u = 1,2,3) can be evaluated off line just once, for a suited number of position z values, subsequently interpolating the calculated points; thus, when the time dependence is to be taken into account, these space quantities can be considered as known functions;

- $J_p(z)$, $J_f(z)$, $L_{pu}(z)$, $L_{pf}(z)$ are able to correctly model the local stator and slider slotting field effects (including the effects of partial slot facings [1]) and the actual field and armature winding structures: this property ensures an accurate modelling of all the e.m.f. harmonic contributions, including the well known toothing e.m.f. harmonics, particularly noisy in synchronous machines;
- of course, the adopted approach is rigorously valid just supposing perfectly unsaturated operation, because it implies the application of the superposition principle;
- moreover, it should be noted that, in general, no closed forms can be directly found for $J_p(z)$, $J_f(z)$, $L_{pu}(z)$, $L_{pf}(z)$ and a numerical integration is required; on the other hand, usually the complexity of the involved functions makes quite heavy the direct calculation of the integrals. Indeed, all the terms in eq.s (14), (15), (17), (18) depend on the space integral of the m.m.f.s distribution, of the field functions and of their derivatives; field functions derivatives are zero, except around the slot and interpolar openings, where sharp, wide variations occur: this pulse-wise behaviour makes troublesome the numerical evaluation of the integrals; a great simplification is achieved if the integrand functions are developed in Fourier series, because, once obtained sinusoidal functions products, the integral can be solved analytically.

4. Expression of the e.m.f. for the Maglev system of the examined case

Let us consider now a single six-pole levitator of a Maglev vehicle, faced to the stator.
The general theory just described can be greatly simplified, thanks to the simplicity of the stator winding (one slot/(pole-phase)); rearranging the previous steps, the following sequence should be considered:
- the e.m.f. $e_{6\tau}$ induced in the portion of the stator winding faced to one six-pole levitator can be obtained as the sum of the e.m.f.s e_{coil} induced in each single coil of that winding portion;
- the e.m.f. of each coil is the time derivative of its flux linkage, and this flux is the integral of the flux density on the surface at half air-gap width;
- considering that the flux density changes weakly in the direction transversal to the motion, the surface integral can be reduced to a line integral along the stator coordinate x, extended between the positions of beginning and end of the considered coil;
- being the stator winding equipped with one slot/(pole-phase), the coils have full pitch: therefore, the beginning and the end of every coil are always in correspondence of multiples of the pole pitch τ;
- remembering that the e.m.f.s induced in coils under adjacent poles have opposite sign, in the calculation of the e.m.f. of the considered winding portion, it is necessary to alternate the sign of the e.m.f.s induced in adjacent coils.

On the basis of these remarks, it follows:

$$e_{6\tau} = \sum e_{coil} = \sum \frac{d\psi}{dt} = \sum \frac{d}{dt} \iint_{A_{coil}} B(x,y,t) \cdot dA = \ell \cdot \frac{d}{dt} \sum \int B(x,y,t) \cdot dx = \\ = \ell \cdot \frac{d}{dt} \cdot \sum_{k=0}^{6} (-1)^k \cdot \int_{k\cdot\tau}^{(k+1)\cdot\tau} B(x, x - z(t), t) \cdot dx \quad (19)$$

The expression of the flux density to be introduced in the integrals is given by eq.(1), detailed for the case under analysis (the system has one slot/(pole-phase), therefore $N_t = 1$, $a = 1$).
It should be observed that, in the developed analysis, a machine without relative inclination between stator slots and levitator pole shoes has been considered; this situation is not realistic, because usually the levitator pole shoes are inclined with respect to the stator teeth, in order to reduce the slotting effects. Nevertheless, this inclination has not been considered here: in fact, the intent is to show the method soundness in a case that, besides the simplicity, has the property to present more evident slotting effects.
In case the pole shoe - slots inclination is of interest, it is possible to transversally subdivide the machine in several “slices”, each without any inclination, but spatially displaced with respect to the others, in the motion direction.

5. Calculation of the no-load e.m.f.

As a simple example of application, the no-load e.m.f. will be evaluated, with moving vehicle, but without stator currents, and with constant field current; in this case, $M_S(x, t) \equiv 0$, and $N_f \cdot i_f(t) = cost = I_f$ occurs; hence, expression (1) simplifies as follows:

$$B(x,y,t) = (\mu_0/g) \cdot I_f \cdot m_f(x - z(t)) \cdot \beta_s(x) \cdot \beta_f(x - z(t)), \qquad (20)$$

in which $y = x - z$ has been already inserted. Moreover, the time dependence affects only the coordinate $z = z(t)$ (just the motional e.m.f. exists); thus, indicated with $v = dz/dt$ the levitator speed:

$$\frac{de_{6\tau}}{dt} = \frac{de_{6\tau}}{dz} \cdot \frac{dz}{dt} = \frac{de_{6\tau}}{dz} \cdot v. \qquad (21)$$

At this point, the solution of the integrals in (19) can be performed.

6. Comparison with FEM solutions

For the FEM solutions, the six-pole levitator with the structure shown in fig.9 has been adopted.
The ratio between the extension of any of the two lateral pole shoes and any of the four central pole shoes has been indicated with σ_e. Two cases have been analysed, in which σ_e equals 0.50 and 0.75 respectively.
In all the cases we have verified a fair agreement between FEM and analytical solutions (fig.10).

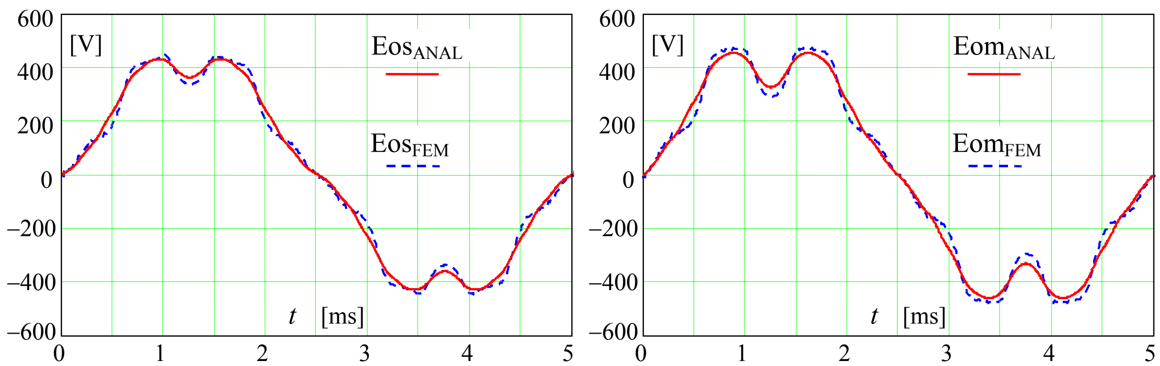

Fig.10: comparison between analytically and FEM evaluated no-load e.m.f. (phase winding portion, consisting of the 6 series connected coils facing one levitator), in the cases $\sigma_e = 0.5$ (E_{os}), $\sigma_e = 0.75$ (E_{om}): field m.m.f.: $N_f \cdot i_f = 6$ kA; vehicle speed $v = 120$ m/s.

7. Conclusion

A method has been illustrated for the analytical evaluation of the waveform of the e.m.f. induced in the stator windings of an EMS Maglev system. The method, based on the reconstruction of the field through suited field functions, allows to take into account the field distribution due to the slotting of the ferromagnetic structures and to the actual m.m.f.s. The obtained analytical solution has been compared with the results of FEM simulations, getting good agreement both in the waveforms and in the numerical values.

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