

Fault tolerant control of electromagnetic levitation system

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Abstract

In this paper, a fault tolerant control problem is considered for a class of nonlinear system formulated in a gain scheduling form with LMI-based H_∞ control scheme. Key benefits of the proposed method are demonstrated in the simulation of an electromagnetic levitation system with actuator and/or sensor failures. The method is also compared with the conventional output-feedback controller. It is clearly observed that the proposed control scheme shows an improved output performance in comparison with conventional method.

1. Introduction

Modern technological systems rely on more and more sophisticated control functions to meet increased performance requirement as manifested in electromagnetic levitation systems and flight control systems.

Methodology used in conventional control system design often presumes that the actuators and sensors of the system are in good working conditions. As a result, a majority of control system designed using conventional techniques may not be able to maintain a satisfactory performance in the presence of actuator and/or sensor failures. In some cases, even the closed-loop system stability may be in jeopardy. To increase the reliability in the presence of such vulnerability, the control system may need to provide some kind of fault tolerance. Here, a control system designed to tolerate actuator and/or sensor failures, while maintaining an acceptable closed-loop system stability and performance, is called fault tolerant control system [1].

Even though the fault tolerant control system is practically important, only a very few results are

reported in the literature. Shimemura and Fujita [2] described a fault tolerant control problem in the state space representation and proposed a solution technique using Riccati-type equations. Later, Shieh and co-workers [3] developed a design scheme for guaranteeing the control system integrity based on the solution of matrix Lyapunov equations. The design problem for control systems possessing integrity has further been formulated in an H_∞ framework in [4] where the performance of the system can be maintained in an H_∞ optimal sense.

Another technology of fault tolerant controller design has been developed based on the concept of reliable stabilization instead of failure diagnosis and accommodation. Reliable stabilization refers to a control system which retains a desired closed-loop behavior despite any pre-specified subset of actuator and/or sensor failures. Siljak [5] proposed a reliable decentralized controller design method based on the technique of overlapping decomposition. Given any stabilizing controller for a plant, Vidyasagar and Viswanadham [6] gave a procedure for computing a second stabilizing controller such that the sum of the two controllers also stabilizes the plant. Cho and Bien [7] presented a design method in which a redundant adaptive controller operates in parallel with a fixed stabilizing controller. For certain failure modes, the redundant scheme guarantees stability and asymptotic reference tracking. Watanabe and Tzafestas [8] designed a hierarchical multiple model adaptive controller for a discrete-time stochastic system with sensor and/or actuator failures. Shin and Kim [9] analyzed the effect of computation-time delay in the digital computer controller on the overall feedback system performance.

In this paper, we investigate the issue of fault tolerant control for an electromagnetic levitation system model, which represents the essential dynamics of magnetically levitated transport system. The system is highly nonlinear with unstable open-loop dynamics. In electromagnetic levitation system, various failures may take place in the actuators and sensors. Since the failure may result in critical performance degradation and even catastrophe, improvement of reliability and safety of the system has been an essential requirement. It is noted that some results for fault tolerant control of linear system are easily found ([10]-[12]) whereas the system including highly nonlinear and open-loop unstable is yet subject to further investigation for fault tolerant performance. In the present paper, the fault tolerant control methodology is introduced for electromagnetic levitation system and it is demonstrated by using gain scheduling plus LMI-based H_∞ control technique.

The paper is organized as follows. In Section 2, we formulate a fault tolerant control problem. In Section 3, we discuss the design of fault tolerant electromagnetic levitation system, and then we compare the simulation result of gain scheduling plus LMI-based H_∞ controller with that of the conventional feedback controller. Finally, in Section 4, the result is summarized and concluding remarks are given.

2. Modeling of electromagnetic levitation system

One of the major problems of magnetically levitated transport systems is to cope with mass variation and actuator and/or sensor failures. One of the primary causes of its performance deterioration is

known to be the mass variation. The mass variation results from passengers, loading freight, or the failure of any controlled levitation magnet group. Furthermore, the actuator and/or sensor failures in the railway system often cause instability of the magnetically levitated vehicle and tend to increase high regulation air-gap error at the levitated state. Note that disturbances such as lifting force variation due to the actuator and/or sensor failures are a dominant sources which deteriorate the system performance. It is therefore very important to decrease the regulation air-gap error to guarantee safety and enhance ride quality of the railway systems.

We apply the proposed control method to an electromagnetic levitation system shown in Figure 2. The single magnet levitation system [13] with susceptible actuator and sensor failures is described by the following nonlinear dynamic equation:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\frac{\mu_0 N^2 A}{4m} \left(\frac{x_3}{x_1} \right)^2 + g \\
\dot{x}_3 &= \frac{x_2}{x_1} x_3 - \frac{2R}{\mu_0 N^2 A} x_1 x_3 + \frac{2x_1}{\mu_0 N^2 A} (u + u_\zeta), \\
z &= x_1, \\
y &= \begin{bmatrix} x_1 + y_{\zeta 1} & 0 & \dot{x}_2 + y_{\zeta 2} \end{bmatrix}^T,
\end{aligned} \tag{1}$$

where x_1 , x_2 and x_3 are the vertical air-gap z_g , the vertical velocity \dot{z}_g and the magnet current i , respectively, while u denotes the magnet voltage v . z and y represent the controlled output and the measured output. And m is the total levitation mass, N is the number of turns of the coil wrapped around the magnet, A is the effective magnet pole area, μ_0 is the permeability of free space, g is the gravity constant and R is the coil resistance. Here the mass m is taken to be the scheduling parameter, note that the mass m can normally be measured in the electromagnetic levitation system. The actuator failure represents the control input as regular signal plus additional disturbance input and the sensor failure also represents the measurement output as regular signal plus additional disturbance input. The additional disturbance inputs, $\|u_\zeta\| \ll \|u\|$, $\|y_{\zeta 1}\| \ll \|x_1\|$ and $\|y_{\zeta 2}\| \ll \|\dot{x}_2\|$ represent the susceptible actuator failure and the susceptible sensor failure, respectively. For example $u_\zeta \neq 0$ implies an actuator failure and $y_{\zeta 1} \neq 0$ or $y_{\zeta 2} \neq 0$ implies a sensor failure.

For convenience of notation, (1) is rewritten as

$$\begin{aligned}
\dot{x} &= f(x, u, u_\zeta, m), \\
z &= q(x), \\
y &= h(x) + y_\zeta,
\end{aligned} \tag{2}$$

where the functions f , q and h are assumed to be smooth. Here, the component faults are assumed for actuator and sensor failures, respectively.

3.1 Linearization

Let the following deviation variables be defined as

$$\begin{aligned}
x_\delta &= x - x_o(m), \\
u_\delta &= u - u_o(m), \\
z_\delta &= z - r_d, \\
y_\delta &= y - y_o(m).
\end{aligned} \tag{3}$$

where r_d is the reference input. For each fixed m , the corresponding linearized system can be written in the following form:

$$\begin{aligned}
\dot{x}_\delta &= f(x_o(m) + x_\delta, u_o(m) + u_\delta + u_\zeta, m) - \dot{x}_o(m), \\
z_\delta &= q(x_o(m) + x_\delta, u_o(m) + u_\delta + u_\zeta, m) - r_d, \\
y_\delta &= h(x_o(m) + x_\delta, m) - y_o(m) + y_\zeta.
\end{aligned} \tag{4}$$

By using Taylor series expansion, the linearized system is calculated as

$$\begin{aligned}
\dot{x}_\delta &= A(m)x_\delta + B(m)u_\delta + \begin{bmatrix} B(m) & I \end{bmatrix} \begin{bmatrix} u_\zeta \\ o_f \end{bmatrix}, \\
z_\delta &= C_1(m)x_\delta + D(m)u_\delta + \begin{bmatrix} D(m) & I \end{bmatrix} \begin{bmatrix} u_\zeta \\ o_g \end{bmatrix}, \\
y_\delta &= C_2(m)x_\delta + y_\zeta + o_h,
\end{aligned} \tag{5}$$

where

$$A(m) = \frac{\partial f}{\partial x}(x_o(m), u_o(m), w), \quad B(m) = \frac{\partial f}{\partial u}(x_o(m), u_o(m), m), \quad C_1(m) = \frac{\partial q}{\partial x}(x_o(m), u_o(m), m),$$

$$C_2(m) = \frac{\partial h}{\partial x}(x_o(m), u_o(m), m), \quad D(m) = \frac{\partial q}{\partial u}(x_o(m), u_o(m), m).$$

Here $o_f(x_\delta, u_\delta, u_\zeta, m)$, $o_q(x_\delta, u_\delta, u_\zeta, m)$ and $o_h(x_\delta, m)$ denote the higher order nonlinear terms which include the modeling errors and disturbances. The above expression is rewritten for concise representation as follows:

$$\begin{aligned} \dot{x}_\delta &= A(m)x_\delta + B(m)u_\delta + \bar{B}(m)\bar{u}_{\zeta 1}, \\ z_\delta &= C_1(m)x_\delta + D(m)u_\delta + \bar{D}(m)\bar{u}_{\zeta 2}, \\ y_\delta &= C_2(m)x_\delta + \bar{y}_\zeta, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \bar{B}(m) &= [B(m) \quad I], \quad \bar{u}_{\zeta 1} = \begin{bmatrix} u_\zeta \\ o_f \end{bmatrix}, \quad \bar{u}_{\zeta 2} = \begin{bmatrix} u_\zeta \\ o_q \end{bmatrix}, \\ \bar{D}(m) &= [D(m) \quad I], \quad \bar{y}_\zeta = \begin{bmatrix} y_\zeta \\ o_h \end{bmatrix}. \end{aligned}$$

3.2 Construction of the fault tolerant controller

To obtain the smooth functions with a constant scheduling parameter m , (2) can be rewritten as follows:

$$\begin{aligned} 0 &= f(x_o(m), u_o(m), m), \\ r_d &= g(x_o(m), u_o(m), m), \\ y_o(m) &= h(x_o(m), m). \end{aligned} \quad (7)$$

And the nominal state values and input value with a constant m can be given by

$$x_o(m) = \begin{bmatrix} r_d \\ 0 \\ \frac{2r_d}{N} \sqrt{\frac{Gm}{\mu_0 A}} \end{bmatrix}, \quad u_o(m) = \frac{2r_d R}{N} \sqrt{\frac{Gm}{\mu_0 A}} \quad (8)$$

Then, for each fixed m , the corresponding linearized system is given by

$$\begin{aligned} \dot{x}_\delta &= A(m)x_\delta + B(m)u_\delta + \bar{B}(m)\bar{u}_{\zeta_1}, \\ z_\delta &= C_1(m)x_\delta, \\ y_\delta &= C_2(m)x_\delta + \bar{y}_\zeta. \end{aligned} \quad (9)$$

The linearized system coefficients are given by

$$A(m) = \frac{\partial f(x(m), u(m), m)}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2G}{r_d} & 0 & -\frac{N\sqrt{AG\mu_0}}{r_d\sqrt{m}} \\ 0 & \frac{2\sqrt{Gm}}{N\sqrt{A\mu_0}} & -\frac{2r_d R}{A\mu_0 N^2} \end{bmatrix},$$

$$B(m) = \frac{\partial f(x(m), u(m), m)}{\partial u} = \begin{bmatrix} 0 & 0 & \frac{2r_d R}{A\mu_0 N^2} \end{bmatrix}',$$

$$\bar{B}(m) = \left[\frac{\partial f(x(m), u(m), m)}{\partial u} \quad I \right]' = \left[\begin{bmatrix} 0 & 0 & \frac{2r_d R}{A\mu_0 N^2} \end{bmatrix}' \quad I \right]',$$

$$C_1(m) = \frac{\partial g(x(m))}{\partial x} = [1 \quad 0 \quad 0],$$

$$C_2(m) = \frac{\partial h(x(m))}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

To illustrate the proposed control scheme under the mass variation and actuator and/or sensor failures of the electromagnetic levitation system, the simulations were performed with the following information:

- the nominal mass $m_o = 300\text{kg}$, $R = 1\Omega$, $G = 9.8[m/\text{sec}^2]$, $r_d = 10[\text{mm}]$,

$$\mu_0 = 4\pi \times 10^{-7}[H/m], \quad N = 660, \quad A = 0.044[m^2];$$

- the eigenvalues of $A(m_o)$ is given by $\{15.6185, -8.9193 \pm 14.1088i\}$, which means

the nominal system has an unstable mode;

- the parameter γ is set to be 2;
- the reference r_d varies at $t = 1[\text{sec}]$ by step change (from $20[\text{mm}]$ to $10[\text{mm}]$);
- the mass m also varies at $t = 1[\text{sec}]$ by step change (from m_0 to $1.5m_0$);

Now, under the above information, we design a fault tolerant output-feedback controller to cope with susceptible actuator and/or sensor failures. The corresponding LMI formulation consists of minimizing $\text{trace}(X) + \text{trace}(Y)$ with scheduling parameter m as follows:

$$\begin{pmatrix} A'_{cl}(m)X + XA'_{cl}(m) & XB'_{cl}(m) & C'_{cl}(m)^T \\ B'_{cl}(m)^T X & -\gamma I & 0 \\ C'_{cl}(m) & 0 & -\gamma I \end{pmatrix} < 0, \quad (10)$$

$$\begin{pmatrix} YA'_{cl}(m)^T + A'_{cl}(m)Y & B'_{cl}(m) & YC'_{cl}(m)^T \\ B'_{cl}(m)^T & -\gamma I & 0 \\ C'_{cl}(m)Y & 0 & -\gamma I \end{pmatrix} < 0. \quad (11)$$

By solving LMIs (10)-(11) and from (8), the control law of the nonlinear system is obtained by

$$u = u_o(m) + K_o y_\delta, \quad (12)$$

where **Case I** Actuator failure:

$$K_o := \begin{bmatrix} A_\kappa & B_\kappa \\ C_\kappa & D_\kappa \end{bmatrix} = \begin{bmatrix} -3696.3654 & -140.8486 & -95520.1312 \\ -162.5912 & -2146.9656 & 5292.9374 \\ 67.7741 & 433.9179 & 2628.9149 \\ [57.2821 & 529.1612 & 43077.2590] \end{bmatrix} \begin{bmatrix} -24.4546 & -3632.0213 \\ -229.4146 & -2112.4586 \\ 2815.7629 & 504.6076 \\ [0 & 0 & 0] \end{bmatrix}$$

Case 2 Sensor failure:

$$K_o := \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} -14384.2722 & 212.8759 & -95505.6093 \\ 115.4933 & -8292.7723 & 5313.8111 \\ 15.5614 & 458.2624 & -10315.6991 \\ [57.2824 & 529.1618 & 43077.2590] \end{bmatrix} \begin{bmatrix} -28.0055 & -14317.7359 \\ -99.1962 & 708.8512 \\ 10909.6993 & -19.5777 \\ [0 & 0 & 0] \end{bmatrix}$$

Case 3 Actuator and sensor failures:

$$K_o := \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} -3696.2467 & -140.9979 & -95519.5260 \\ -162.4442 & -2143.7552 & 5294.7320 \\ 66.8707 & 430.6484 & -2641.8890 \\ [57.2821 & 529.1612 & 43077.2590] \end{bmatrix} \begin{bmatrix} -24.9678 & -3631.89211 \\ -230.1718 & -2112.2834 \\ 2815.5221 & 501.6097 \\ [0 & 0 & 0] \end{bmatrix}$$

To show the effectiveness of the proposed method, we compare the proposed method with a conventional Output Feedback Controller (OFC) given by [14]

$$u = \frac{2r_d R}{N} \sqrt{\frac{m_o g}{\mu_0 A}} + K'_o y_\delta, \quad (13)$$

where

$$K'_o := \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} -13 & 2500 & -3.198 & 0 \\ 0 & -576.92 & 0 & -100 \\ 10 & -1923.10 & 0 & 0 \\ 0 & 1923.10 & 0 & 0 \\ [0 & 0 & 0 & 100] \\ 0 & 1923.1 & 0 & 0 \\ 10 & 0 & 0 & 0 \\ [-13 & 2500 & -3.198 & 0] \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ [0 & 0] \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Here, the conventional output-feedback controller matrices A_k, B_k, C_k and D_k are obtained by

which the frequency response of the linearized closed-loop system with $m = m_o$ is similar to that with (26).

3.3 Simulation results

The simulation includes the following conditions with susceptible actuator or sensor failures.

Case 1: the actuator failure $u_\zeta(t)$ is assumed to be

$$\square u_\zeta(t) = \begin{cases} 0.5u(t) & \text{where } 5 \leq t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

Figure 3 shows time responses of the nonlinear controlled output $z(t)$ and control input $u(t)$ with actuator failure.

Case 2: the sensor failure $y_\zeta(t)$ is assumed to be

$$\square y_\zeta(t) = \begin{cases} 0.5y(t) & \text{where } 5 \leq t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

Figure 4 shows time responses of the nonlinear controlled output $z(t)$ and control input $u(t)$ with sensor failure.

Case 3: the actuator failure $u_\zeta(t)$ and the sensor failure $y_\zeta(t)$ are assumed to be

$$\square u_\zeta(t) = \begin{cases} 0.5u(t) & \text{where } 5 \leq t \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad \square y_\zeta(t) = \begin{cases} 0.5y(t) & \text{where } 5 \leq t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

Figure 5 shows time responses of the nonlinear controlled output $z(t)$ and control input $u(t)$ with actuator and sensor failures.

The simulation results are given in Figure 3, Figure 4 and Figure 5 which show the time responses of the proposed controller output (solid line) and the conventional feedback controller output (dotted

line), respectively. Note that, in the Figure 3, Figure 4 and Figure 5, the H_∞ controller meets the disturbance attenuation bound γ against susceptible actuator and/or sensor failures while the mass variation of the plant is also engaged in the closed-loop system. It is reported that the proposed control scheme is tested by simulation on the electromagnetic levitation system with the mass variation and actuator and/or sensor failures. The proposed controller shows an improved output performance with the small perturbation in comparison to the conventional one. Moreover, it is clearly observed that the proposed controller also shows a fault tolerant performance against susceptible actuator and/or sensor failures.

4. Concluding Remarks

The fault tolerant control of a class of nonlinear system is studied and applied for an electromagnetic levitation model. The method is shown to be effective for the system not only with mass variation, but also with the susceptible actuator and/or sensor failures. The benefits of this proposed controller are demonstrated via simulation of an electromagnetic levitation system with susceptible actuator and/or sensor failures.

In the present paper, the design was restricted to a nonlinear system with the susceptible actuator and/or sensor failures. To be general, however, the method should be extended to an uncertain nonlinear system with susceptible actuator and/or sensor failures, and be tested and validated by means of actual experiments involving a magnetically levitated vehicle. It is remarked that the study in the paper was originally motivated by experiences in a national project on "Development of a magnetically levitated vehicle System [15]". As shown in Figure 6, an experimental vehicle has been developed by KIMM in Taejon, Korea, and is being under test for various control algorithms. For further studies, the authors wish that the results of the paper be tested for the actual experimental vehicle.

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Figure 4. Proposed FTC and conventional OFC performance against sensor failure

Figure 5. Proposed FTC and conventional OFC performance against actuator and sensor failures

Figure 6. Photograph of experimental vehicle for magnetically levitated transport system

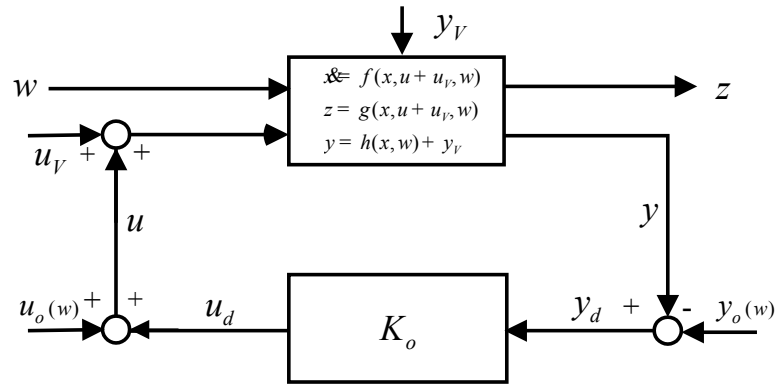


Figure 1. Control system with susceptible actuator and sensor failures

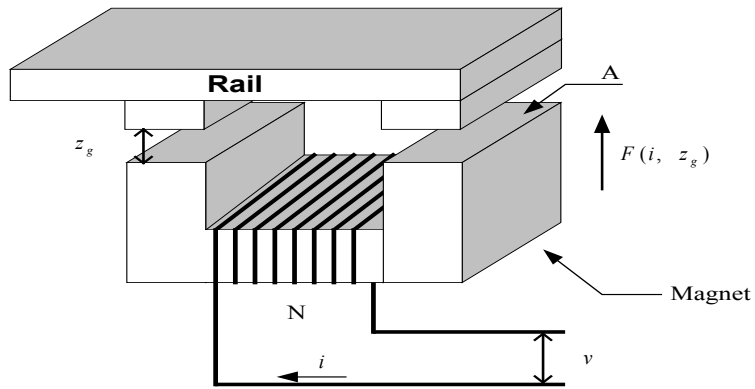


Figure 2. Electromagnet-track configuration

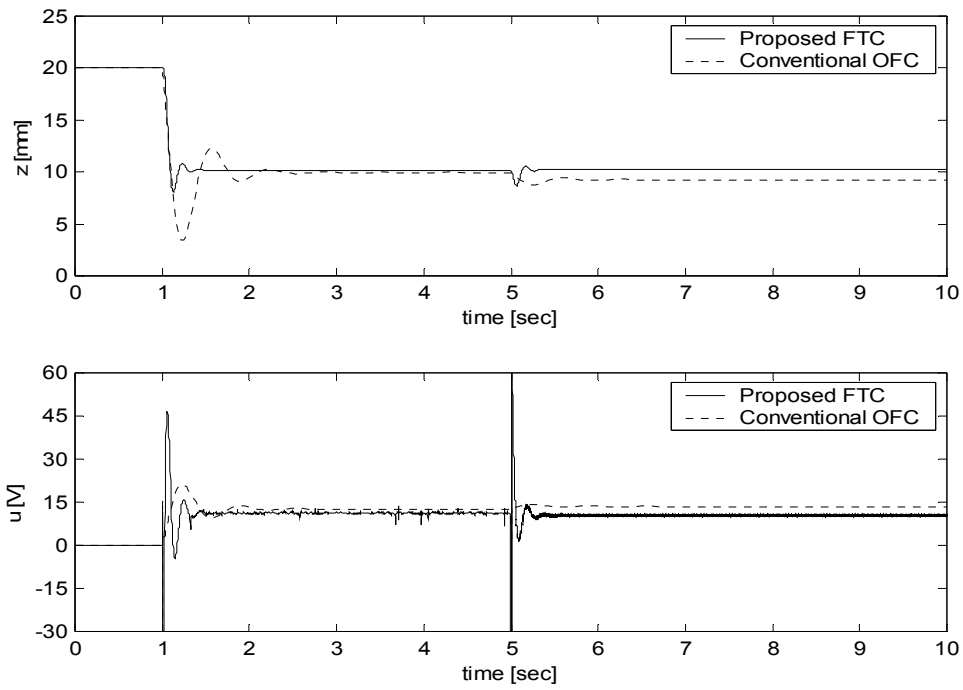


Figure 3. Proposed FTC and conventional OFC performance against actuator failure

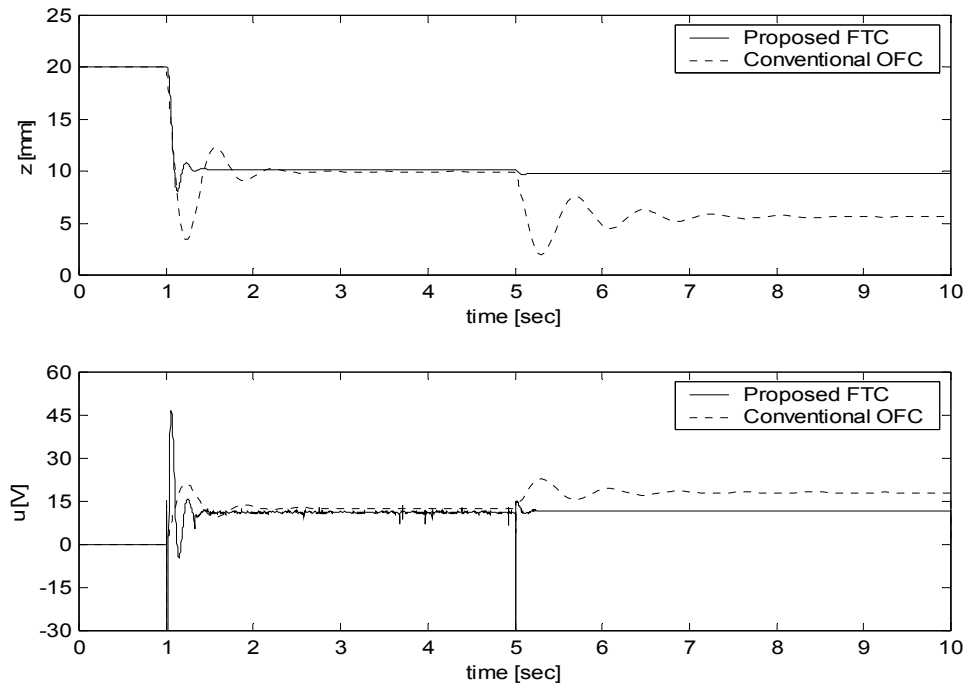


Figure 4. Proposed FTC and conventional OFC performance against sensor failure

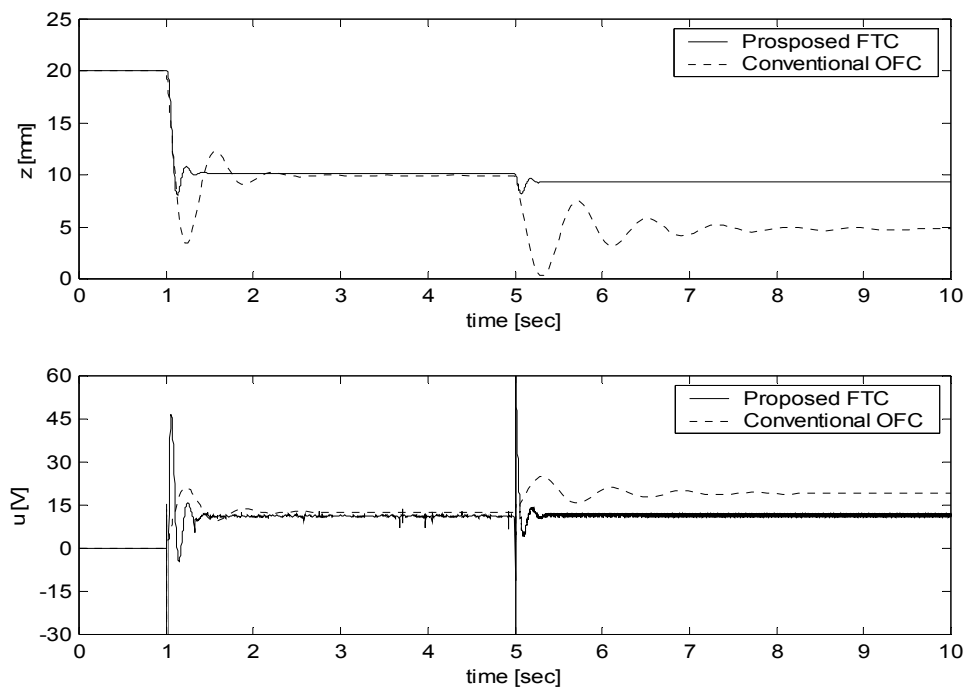


Figure 5. Proposed FTC and conventional OFC performance against actuator and sensor failures



Figure 6. Photograph of experimental vehicle for magnetically levitated transport system