The calculation of the characteristic and non-characteristic harmonic current of the rectifying system

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Abstract The paper analyzes the reasons for producing non-characteristic harmonics in rectifying system. Some formulas for harmonic current calculation are derived. The theoretical and calculative foundations for filter design are provided, we can see that we should pay attention to the non-characteristic harmonic current even if we use 12-pulse converter.

1 Introduce

Because of the electric electron device is used in the power system, the harmonic is becoming more and more heavily, in the maglev system, it is converter that supply power to the vehicle to adjust the speed of the vehicle, the converter will create harmonic current to the power supply system including characteristic harmonic and non-characteristic harmonic, in order to install filter to reduce the harmonic level, we should calculate the harmonic current of the converter.

2 Characteristic harmonic calculation

In order to reduce the harmonic current, in many case, the converter is 12-pulse, its schematic is shown in figure 1[2], in theory, it can contracts 5th harmonic current and 7th harmonic current, then 11th and 13th harmonic current are its characteristic harmonic current

We suppose the electrical source is balanceable as flows:

\[
\begin{align*}
  e_a &= E_m \sin(wt+a) \\
  e_b &= E_m \sin(wt+a-\frac{2\pi}{3}) \\
  e_c &= E_m \sin(wt+a+\frac{2\pi}{3})
\end{align*}
\]

Then the line current of bridge 3 in the power system side is:

\[
i_3 = \frac{2\sqrt{3}}{\pi k} I_d \left[ \sin wt - \frac{1}{5} \sin 5wt - \frac{1}{7} \sin 7wt + \frac{1}{11} \sin 11wt + \frac{1}{13} \sin 13wt + \cdots \right]
\]
Fig1. The schematic diagram of the converter

The line current of bridge 2 in the power system side is:

\[ i_2 = \frac{2\sqrt{3}}{\pi k} I_d \left[ \sin(w t) - \frac{1}{5} \sin(5w t + \pi) - \frac{1}{7} \sin(7w t + \pi) + \frac{1}{11} \sin(11w t + 2\pi) + \frac{1}{13} \sin(13w t + \cdots) \right] \]  

So all the joint current of two controlled bridges in the power system side is:

\[ i_c = \frac{4\sqrt{3}}{\pi k} I_d \left[ \sin(w t) + \frac{1}{11} \sin(11w t + \frac{1}{13} \sin(13w t + \cdots) \right] \]

The joint current of two non-controlled bridges in power system side is:

\[ i_{nc} = \frac{4\sqrt{3}}{\pi k} I_d \left[ \sin(w t + \alpha) + \frac{1}{11} \sin(11w t + \alpha) + \frac{1}{13} \sin(13w t + \alpha) + \cdots \right] \]

The total current in power system side is:

\[ I_n = \frac{4\sqrt{3}I_d \sqrt{1 - \cos(180 - na)}}{nk\pi} \]

3 Non-characteristic harmonic calculation

In fact, the converter can not contracted all the 5th and 7th harmonic current completly because of the transformer ratio which can lead to the unbalance voltage is not balanced, 5th and 7th harmonic current are called non-characteristic harmonic in a 12-pulse rectifiers, in many case, the non-characteristic will be so high that 5th and 7th filter will be needed to ensure the harmonic level.

There are two reasons that lead to the non-characteristic harmonic in the rectifying system, one is the unbalance of the load between two rectifier bridges, the other is the phase conversion process.
3.1 The influence of the load unbalance between two rectifier bridges on the non-characteristic harmonic current

Fig2. the equivalent diagram of two parallel rectifier bridges

Fig2 is the equivalent diagram of two parallel rectifier bridges, from this diagram we can see:

\[
I_{dy}^* = \frac{x_3^*}{x_2^* + x_3^*} I_d^* - \frac{1 - \sqrt{3} \frac{N_y}{N_A}}{K_{X2}(x_2^* + x_3^*)} = \frac{x_3^*}{x_p^*} I_d^* - \frac{1 - \sqrt{3} \frac{N_y}{N_A}}{K_{X2}x_p^*}
\]

\[
I_{dh}^* = \frac{x_3^*}{x_2^* + x_3^*} I_d^* + \frac{1 - \sqrt{3} \frac{N_y}{N_A}}{K_{X2}(x_2^* + x_3^*)} = \frac{x_3^*}{x_p^*} I_d^* + \frac{1 - \sqrt{3} \frac{N_y}{N_A}}{K_{X2}x_p^*}
\]

In this formula, \( K_{X2} \) is the convert coefficient of the reactance, \( K_{X2} = 0.26 \)

\( N_y, \ N_A \) is the number of Y-connected windings and the number of \( \Delta \) connected windings.

The unbalance current between two bridges is:

\[
\Delta I_d^* = I_{dh}^* - I_{dy}^*
\]

\[
= \frac{x_3^* - x_3^*}{x_p^*} I_d^* + \frac{2(1 - \sqrt{3} \frac{N_y}{N_A})}{K_{X2}x_p^*} \sum_r \sin rwt
\]

The harmonic current caused by the unbalance load current is as follows:

\[
i_r = \frac{I_{dh}^* - I_{dy}^*}{\pi} \sum_r \frac{1}{r} \sin rwt
\]

\[
= \frac{\Delta I_d^*}{\pi} \left( \frac{1}{5} \sin 5wt + \frac{1}{7} \sin 7wt + \frac{1}{11} \sin 11wt + \frac{1}{19} \sin 19wt + \cdots \right)
\]

\( r = 6 \times (2k - 1) \pm 1, \ k = 1,2,3\ldots \)

In the most severe situation, \( \Delta I_d^* = I_{dh}^* - I_{dy}^* \)

So the amplitude ratio of non-characteristic harmonic current and the fundamental current is:
\[ I_{\text{r}} = \frac{1}{I_{d_{\text{max}}}} \Delta I_{\text{r}}^{\ast} \]

3.2 The influence of phase conversion on the non-characteristic harmonic current

In reality, when rectifier system is in operation, because of the reactance, the line current is not rectangular waveform but trapezoidal waveform.

The phase conversion process should meet the following equation:

\[
\begin{align*}
L_B \frac{di_c}{dt} + e_{ac} &= L_B \frac{di_a}{dt} \\
e_{ac} &= \sqrt{3}E_m \sin(wt + a) = \sqrt{6}E \sin(wt + a) \\
i_a + i_c &= I_d \\
i_c(0^+) &= I_d
\end{align*}
\]

To solve this equation, we can get:

\[
\begin{align*}
i_a &= \frac{\sqrt{6}E}{2X_B} \left[ \cos a - \cos(wt + a) \right] \\
i_c &= I_d - \frac{\sqrt{6}E}{2X_B} \left[ \cos a - \cos(wt + a) \right]
\end{align*}
\]

In the formula, \( X_B \) is the reactance in AC side, \( X_B = \omega L_B \).

\[ I_d = \frac{\sqrt{6}E}{2X_B} \left[ \cos a - \cos(wt + \gamma) \right] \]

\( \gamma \) is the overlapping angle in phase conversion.

The expression of the current in AC side can be represented in table 1.

<table>
<thead>
<tr>
<th>( \omega t )</th>
<th>( i_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \omega t &lt; \gamma )</td>
<td>( \frac{\sqrt{6}E}{2X_B} \left[ \cos a - \cos(wt + a) \right] )</td>
</tr>
<tr>
<td>( \gamma \leq \omega t &lt; \frac{2\pi}{3} )</td>
<td>( \frac{\sqrt{6}E}{2X_B} \left[ \cos a - \cos(\gamma + a) \right] )</td>
</tr>
<tr>
<td>( \frac{2\pi}{3} \leq \omega t &lt; \frac{2\pi}{3} + \gamma )</td>
<td>( \frac{\sqrt{6}E}{2X_B} \left[ \cos(\omega t + a - \frac{2\pi}{3}) - \cos(\gamma + a) \right] )</td>
</tr>
<tr>
<td>( \frac{2\pi}{3} + \gamma \leq \omega t &lt; \pi )</td>
<td>0</td>
</tr>
</tbody>
</table>

To Fourier analysis the expression of the current, we can get the expression of the effective value of fundamental current and all the harmonic current.
\[ I_1 = \dfrac{3E}{2\pi X_y} \left[ \sin^2 r - 2r \sin r \cos(2a + r) + r^2 \right]^{1/2} = \sqrt{\dfrac{6I_d}{2\pi}} \left[ \sin^2 r - 2r \sin r \cos(2\alpha + r) + r^2 \right] \cos \alpha - \cos(\alpha + r) \]

\[ I_n = \dfrac{3E}{n \pi X_y} \left[ \left( \sin \dfrac{n-1}{2} \dfrac{r}{n-1} \right)^2 + \left( \sin \dfrac{n+1}{2} \dfrac{r}{n+1} \right)^2 - 2(\sin \dfrac{n-1}{2} \dfrac{r}{n-1})(\sin \dfrac{n+1}{2} \dfrac{r}{n+1}) \cos(2a + r) \right] \]

For non-characteristic harmonic current, the phase-angle difference of \( I_{nY} \) and \( I_{n\Delta} \) is 180°, so

\[ I_n = I_{nY} - I_{n\Delta} = \dfrac{3A}{n \pi} \left( \dfrac{E_Y}{X_{BY}} - \dfrac{E_{\Delta}}{X_{B\Delta}} \right) \]

In the formula, \( n=6k+1, k=1, 2, 3, \ldots \)

\[ A = \left[ \left( \sin \dfrac{n-1}{2} \dfrac{r}{n-1} \right)^2 + \left( \sin \dfrac{n+1}{2} \dfrac{r}{n+1} \right)^2 - 2(\sin \dfrac{n-1}{2} \dfrac{r}{n-1})(\sin \dfrac{n+1}{2} \dfrac{r}{n+1}) \cos(2a + r) \right]^{1/2} \]

4 Conclusion

The paper analyzes the reasons for producing non-characteristic harmonics in rectifying system. Some formulas for harmonic current are derived. The theoretical and calculative foundations for administering harmonic are provided. From this paper, we can see that the filter should be installed in the substation to reduce the harmonic current.

Based on a model of power supply system of the Maglev to simulate the distribution of harmonic current, the harmonic current of the Maglev in different situations is calculated. The reasons for producing non-characteristic harmonics in rectifying system are analyzed. Some formulas for harmonic current calculation are derived. Even if we use 12-pulse converter, we should calculate that 5th and 7th harmonic current and install appropriate filter system to make sure that the power level meet the harmonic standard.

REFERENCE:


