# Controllability and Observability of 2 DOF Permanent Magnet Maglev System with Linear Control

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**Abstract:** A new type of 2 DOF (degree of freedom) magnetic levitation system for multi-DOF levitation is proposed. In this system, the force of permanent magnets are used for levitation and controlled by adjusting the reluctance of the magnetic circuit. Using permanent magnets, the feature of this system is effective for saving energy and avoiding heat generation. First, the principle of the levitation system and typical reluctance control methods are described. Second, an experimental device based on the principle is introduced. finally, the feasibility of this system is considered from linear control theory.

**Keywords:** permanent magnet, magnetic levitation system, linear control, 2 DOF.

## 1. Introduction

Magnetic suspension system which controls attractive forces by adjusting the air gap has been developed[1]. The feature of this suspension mechanism is use of permanent magnets and linear actuators. To control supporting forces, the system adjusts air gap lengths by permanent magnet movements. There were some 1DOF developments of this suspension mechanism [2]-[4].

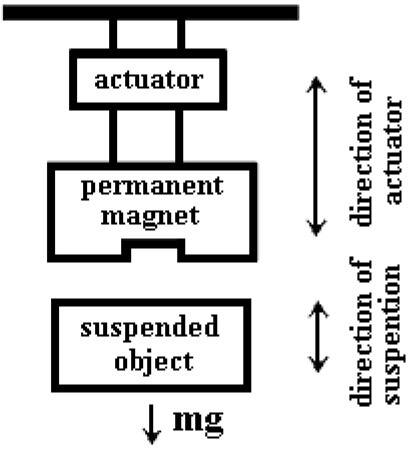
As the suspension mechanism uses permanent magnets not electromagnets, there is no heat generation, no need of volume for a coil, and easier tele-operation by using long rod for permanent magnet movement. As the results, this suspension mechanism may be suitable for micromanipulation. However, when this suspension mechanism is used for the operation of manipulation, the performance of a 1 DOF system is insufficient on the points of the stability of passive control directions, countermeasures for various shape of the suspended object, controllable area, and so on. We need multi-DOF suspension systems.

In this paper, we study the 2 DOF suspension system that manipulate the object in the vertical plane, as a one step of multi-DOF micromanipulation. The principle of suspension system is explained and a 2 DOF system is introduced.

## 2. Principle of suspension system

A suspension system with a permanent magnet and linear actuator is proposed as shown in Fig.1. A ferromagnetic body is suspended by an attractive force from a permanent magnet, which is driven by an actuator, positioned above. The direction of levitation is vertical, and the magnet and the object move only in this direction. The equilibrium position of the ferromagnetic body is determined by a balance between the gravity force and the magnet force.

If the actuator does not actively control the magnet's position, the levitated object will either fall or adhere to the magnet. However servo-control of the actuator can make this system stable. Because there is a smaller attractive force for a larger air gap between the permanent magnet and object, the actuator drives the magnet upwards in response to object movement from its equilibrium position towards the magnet. Similarly, the actuator drives the magnet downwards in response to object movement away from the magnet. In this way, the object can be stably suspended without contact. In comparison to the electrical control method of electromagnetic suspension systems, this system is a mechanical control maglev system [3].



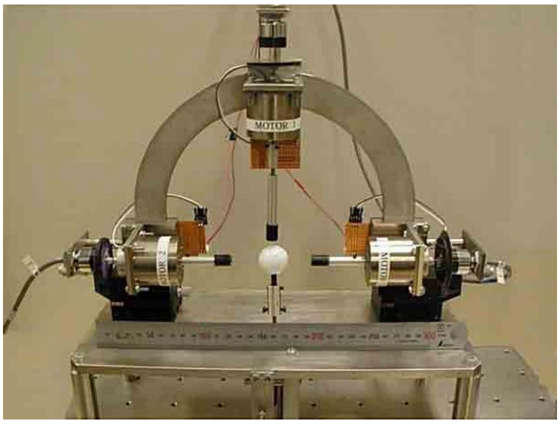


**Figure 1. Outline of suspension mechanism**

**Figure 2. Photograph of the prototype of**

## 3. 2 DOF suspension system

### 3.1 Experimental setup

A photograph of a prototype 2 DOF suspension system is shown in Fig. 2. There are three voice coil actuators which drive permanent magnets. The actuators are installed to semicircle rail and directions of the magnet movements can be adjusted bye the installed position. The suspended object is an iron ball, and is manipulated by the movements of the three magnets. The motions of the magnets and the iron ball are sensed by gap sensors and a photo sensor, respectively.

The iron ball has a diameter of 19 mm and a weight of 63 g. the permanent magnets have cylindrical shape with a diameter of 8 mm and a length of 10 mm. The moving part with the magnet has a weight of 375 g. The actuator has a stoke length of 15 mm and a rated propulsive force of 10 N at a coil current of 2 A. The gap sensor sensing the permanent magnet movement, which is located behind the actuator, is eddy current type and has a sensing range of 10 mm and a resolution of 10 μ m. The movement of the iron ball is sensed by 2 axes photo sensor and has a sensing range of 20 mm and a resolution of 5 μ m. Controller is a digital DSP controller with 12 bit resolution A/D converters and 16 bits D/A converter

### 3.2 Modeling and theoretic analysis of system

As shown in Fig. 2, it is considered that the motions of the iron ball and the magnets divide into two directions, vertical and horizontal, movement. It is consumed that two motions are independent of each other. The analysis of vertical motion has been already investigated [1]. Here, the horizontal motion, which involves the motions of an iron ball and two permanent magnets driven by actuators, are mainly investigated. Fig. 3. shows the model of horizontal motion of system. As can be seen, the symbols used in the following study are

$z_0$: displacement of the iron ball,
$z_1, z_2$:displacements of the left and right permanent magnets,
$m_0$: mass of the iron ball,
$m_1, m_2$: mass of the magnets,
$f_{a1}, f_{a2}$:generating forces of the left and right actuators,
$f_{m1}, f_{m2}$: attractive forces between the ball and the magnet,
$d_1, d_2$: left and right side air gap lengths about the ball.
$k_{m1}, k_{m2}$: the coefficients of magnetic fields.
$k_1, k_2$: the spring coefficients of the left and right actuators,
$\xi_1, \xi_2$:the damping coefficients of the left and right actuators.

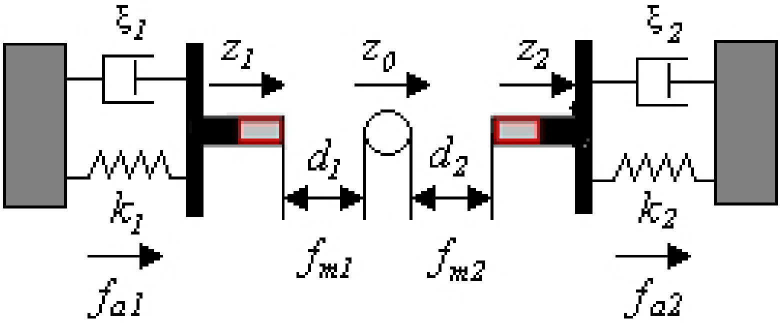


**Figure 3. The model of horizontal motion**

The displacements and forces are considered positive when they act in the right side direction. And it is consumed that the friction of the guide of the actuator and viscosity of the air are negligible. The equations of the motion of the suspended object and the magnets are

$$m_0 \ddot{z}_0 = f_{m2} - f_{m1} \quad (1)$$

$$m_1 \ddot{z}_1 = -\xi_1 \dot{z}_1 - k_1 z_1 + f_{m1} + f_{a1} \quad (2)$$

$$m_2 \ddot{z}_2 = -\xi_2 \dot{z}_2 - k_2 z_2 - f_{m2} + f_{a2} \quad (3)$$

where $f_{m1}=k_{m1}/d_1^2$, $f_{m2}=k_{m2}/d_2^2$. Due to the relationship between magnetic force and the air gap is nonlinear, the equations, (1), (2) and (3) must be linearized. After linearizing, the horizontal motions can be expressed by the following state space model:

$$\dot{x} = Ax + bu \quad (4)$$

$$y = cx \quad (5)$$

There are three situations of this system in terms of the relation of magnets and inputs. They are discussed in the following context.

### 3.2.1 Two forces input system

In the model of Fig.3, the magnets are independent and $f_{a1}$ and $f_{a2}$ are the system inputs. If $f_{a1}$ is not equal to $f_{a2}$ and $m_1$ is not equal to $m_2$ then the system has two inputs, i.e. this system is two forces input system. Where

$$x = \begin{pmatrix} \dot{z}_0 & z_0 & \dot{z}_1 & z_1 & \dot{z}_2 & z_2 \end{pmatrix}'$$

$$A=\begin{bmatrix} 0 & \frac{k_{m1}}{m_0 d_{01}^3}-\frac{k_{m2}}{m_0 d_{02}^3} & 0 & -\frac{k_{m1}}{m_0 d_{01}^3} & 0 & \frac{k_{m2}}{m_0 d_{02}^3} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k_{m1}}{m_1 d_{01}^3} & -\frac{\xi_1}{m_1} & \frac{k_{m1}}{m_1 d_{01}^3}-\frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{k_{m2}}{m_2 d_{02}^3} & 0 & 0 & -\frac{\xi_2}{m_2} & \frac{k_{m2}}{m_2 d_{02}^3}-\frac{k_2}{m_2} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b=\begin{pmatrix} 0 & 0 & 1/m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/m_2 & 0 \end{pmatrix}'$$

$$c=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u=\begin{pmatrix} f_{a1} & f_{a2} \end{pmatrix}'$$

Since the controllability matrix of system is not a square matrix, we have to use the rank of it to determine the controllability of system. $Rank\begin{bmatrix} b & Ab & A^2b & A^3b & A^4b & A^5b \end{bmatrix}$ = 6, so the system is controllable. And the determinant of the observability matrix,

$$\det\begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' & (A')^4c' & (A')^5c' \end{bmatrix} = -\frac{\mathrm{km2}^6}{\mathrm{d02}^6\,\mathrm{m0}^4\,\mathrm{m1}^2}-\frac{\mathrm{km2}^6}{\mathrm{d02}^6\,\mathrm{m0}^4\,\mathrm{m2}^2}+\frac{2\,\mathrm{km2}^6}{\mathrm{d02}^6\,\mathrm{m0}^4\,\mathrm{m1}\,\mathrm{m2}}$$

is not equal to 0, so the system is observable.

### 3.2.2 Same force input system

Also in the model of Fig.3, the magnets are independent, but if $f_{a1}=f_{a2}=f_a$ and $m_1=m_2$, in fact the system has only one input $f_a$. We call this system the same force input system. Where

$$x=\begin{pmatrix} \dot z_0 & z_0 & \dot z_1 & z_1 & \dot z_2 & z_2 \end{pmatrix}'$$

$$A=\begin{bmatrix} 0 & \frac{k_{m1}}{m_0 d_{01}^3}-\frac{k_{m2}}{m_0 d_{02}^3} & 0 & -\frac{k_{m1}}{m_0 d_{01}^3} & 0 & \frac{k_{m2}}{m_0 d_{02}^3} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k_{m1}}{m d_{01}^3} & -\frac{\xi_1}{m} & \frac{k_{m1}}{m d_{01}^3}-\frac{k_1}{m} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{k_{m2}}{m d_{02}^3} & 0 & 0 & -\frac{\xi_2}{m} & \frac{k_{m2}}{m d_{02}^3}-\frac{k_2}{m} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b=\begin{pmatrix} 0 & 0 & 1/m & 0 & 1/m & 0 \end{pmatrix}'$$

$$c=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u=f_a$$

The determinants of the controllability matrix, $\det\begin{bmatrix} b & Ab & A^2b & A^3b & A^4b & A^5b \end{bmatrix}$

and the observability matrix $\det\begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' & (A')^4c' & (A')^5c' \end{bmatrix}$

are not equal to 0 (here, we omit the results of the determinants), so the controllability and observablity are confirmed and it is theoretically verified that the levitation is feasible.

However, under such condition that $\frac{k_{m1}}{d_{01}} = \frac{k_{m2}}{d_{02}}$. The

$$\det\begin{bmatrix} b & Ab & A^2b & A^3b & A^4b & A^5b \end{bmatrix} = 0,$$

$$\det\begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' & (A')^4c' & (A')^5c' \end{bmatrix} = 0.$$

So the system cannot be controlled and observed.

### 3.2.3 Two magnets connecting system

In the model of Fig.3., if the two magnets are dependent, i.e. they are connected together. This system in which $z_1 = z_2 = z$, $m_1 = m_2 = m$ is two magnets connecting system.

As a result,

$$x = \begin{pmatrix} \dot{z}_0 & z_0 & \dot{z} & z \end{pmatrix}'$$

$$A = \begin{bmatrix} 0 & \frac{k_{m1}}{m_0 d_{01}^3} - \frac{k_{m2}}{m_0 d_{02}^3} & 0 & \frac{k_{m2}}{m_0 d_{02}^3} - \frac{k_{m1}}{m_0 d_{01}^3} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\xi}{m} & -\frac{k}{m} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b = \begin{pmatrix} 0 & 0 & 1/m & 0 \end{pmatrix}'$$

$$c = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$$

$$u = f_a$$

In this case, there is only one input, so it has the same problem of the same force input system. That is, when $\frac{k_{m1}}{d_{01}} = \frac{k_{m2}}{d_{02}}$, the determinant of the controllability matrix, $\det\begin{bmatrix} b & Ab & A^2b & A^3b \end{bmatrix} = 0$,

and the determinant of the observability matrix $\det\begin{bmatrix} c' & A'c' & (A')^2c' & (A')^3c' \end{bmatrix} = 0$,
so the system is uncontrollable and unobservable.

However, when $\frac{k_{m1}}{d_{01}} \neq \frac{k_{m2}}{d_{02}}$, the determinants of controllability matrix and observability matrix are not equal to 0. The system is controllable and observable.

## 4. Conclusion

Aiming noncontact micromanipulation, a 2 DOF magnetically suspended system has been investigated. The following are conclusions of this paper. 2 DOF prototype suspension system has been made. To make a model of the system, the controllability and observability have been analyzed with linear control theory. As the result of the analysis, it has been found that the system is controllable and observable.

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