# Swissmetro: design methods for ironless linear transformer

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## Abstract

Among the various contactless electromechanical equipments used in high speed maglev systems, the function of energy transfer is, in practice, not easy to implement. The two main classes of components are the linear generators and the linear transformers. The latter have the advantage of being insensitive to the vehicle speed. They also present interesting potential for the industrial applications.
Nevertheless, if the ironless linear transformer has a particularly simple structure, its design is not trivial. This paper gives some information about the methods to use, to design and calculate a multi-primary linear transformer, considering various external constrains.

## 1. Introduction

### 1.1. General

Maglev systems involve contactless electromechanical devices. This is usual for propulsion, levitation and guidance functions but, in the case of a completely "flying" maglev vehicle, the energy transfer shall also be frictionless. Guaranteeing this function without any influence of the speed of the vehicle is particularly difficult and is limited to the linear transformer approach. The potential and limits of this approach has been described in a previous paper [1]. The present article deals with the modelling, calculation and design methods for ironless linear transformers.

### 1.2. Constraints of the environment

Even if the ironless variant of linear transformer is limited by several factors (see [1] for details), an ironless specification could be imposed either by a large airgap or by constraints of mass (vehicle).
In the domain of high speed systems (maglev trains), classical solutions of energy transfer (friction) are limited by electromagnetic pollution as well as by the overall dimensions (Swissmetro). Interesting variants in the field of inductive contactless solutions belong to the domains of linear generators and linear transformers. In linear transformer systems, according to the specific environment, the most frequently used solutions are either ironless structures or systems with an iron core located at the level of the secondary winding (the winding in motion). Moreover, the environment of the system can impose constraints on the dimension of the transformer, the value of the airgap, the "invisibility" of the system, the acceptable electromagnetic emission areas, etc.

### 1.3. Definition of the main parameters

There is no universal method, available in any case, to design an ironless linear transformer. According to the global environment of the system, different parameters will have to be selected as priority holder. These main parameters can be the mass, the efficiency, the primary voltage, the length of the windings, the volume, etc.
This paper develops the methods for the calculation and presents strategies for inductance calculation and multi-primary architecture. An analyse of the geometrical characteristics of the transformer ensures the choice of the technique and the adequate model of calculation.

# 2. Calculation of mutual inductances

The calculation of the mutual inductances is based on the solving of the integral forms of the Maxwell equations.

## 2.1. Neumann's formula

Neumann's formula is used in the calculation of the mutual inductances which exists between two loops (one turn coils).

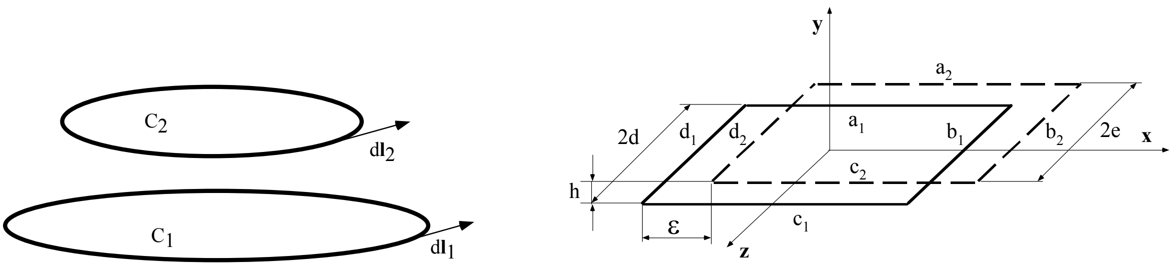


***Figure 1*** *Geometry between two loops (one turn coils) respectively circular and rectangular*

This method takes into account both the finite length of the wire and the longitudinal non-uniformity of the linked magnetic field. The general formula is:

$$M = \frac{\mu}{4\,\pi} \oint_{c_2}\oint_{c_1} \frac{d\vec{l}_2 \cdot d\vec{l}_1}{\left|\vec{r}_2 - \vec{r}_1\right|} \qquad \{1\}$$

For two rectangular coils, one fixed and one in motion, the use of Neumann's formula involves the decomposition into segments of a straight line (the sides and the extremities). The formula is applied to pair of segments.

The coordinates $\mathbf{r}_1$ et $\mathbf{r}_2$ correspond respectively to the primary and secondary straight segments of coils. The integration contour-lines $\mathbf{l}_1$ et $\mathbf{l}_2$ are consequently the primary and secondary coils.
These contour-lines are covered in the following direction:

- for $\mathbf{l}_1$ (primary):

$$d\vec{l}_1 = \underset{\substack{\downarrow \\ a_1}}{dx_1\,\vec{e}_x} + \underset{\substack{\downarrow \\ b_1}}{dz_1\,\vec{e}_z} - \underset{\substack{\downarrow \\ c_1}}{dx_1\,\vec{e}_x} - \underset{\substack{\downarrow \\ d_1}}{dz_1\,\vec{e}_z} \qquad \{2\}$$

- for $\mathbf{l}_2$ (secondary):

$$d\vec{l}_2 = \underset{\substack{\downarrow \\ a_2}}{dx_2\,\vec{e}_x} + \underset{\substack{\downarrow \\ b_2}}{dz2\,\vec{e}_z} - \underset{\substack{\downarrow \\ c_2}}{dx_2\,\vec{e}_x} - \underset{\substack{\downarrow \\ d_2}}{dz2\,\vec{e}_z} \qquad \{3\}$$

For the mutual inductance which links the sides $a_1$ and $a_2$, the terms of equation {1}become:

$$\vec{r}_1 = x_1\,\vec{e}_x - d\,\vec{e}_z \qquad \vec{r}_2 = \left(x_2 + \varepsilon\right)\vec{e}_x + h\,\vec{e}_y - e\,\vec{e}_z \qquad \{4\}$$

$$\left|\vec{r}_2 - \vec{r}_1\right| = \sqrt{\left(x_2 - x_1 + \varepsilon\right)^2 + h^2 + \left(d - e\right)^2} \qquad \{5\}$$

$$d\vec{l}_1 = dx_1\,\vec{e}_x \qquad d\vec{l}_2 = dx_2\,\vec{e}_x \qquad \{6\}$$

$$M_{a_1 a_2} = \frac{\mu}{4\pi} \oint_{c_2} \oint_{c_1} \frac{d\vec{l}_2 \cdot d\vec{l}_1}{\left|\vec{r}_2 - \vec{r}_1\right|} = \frac{\mu}{4\pi} \int_{-\frac{l_1}{2}}^{\frac{l_1}{2}} \int_{-\frac{l_2}{2}}^{\frac{l_2}{2}} \frac{dx_2\, dx_1}{\sqrt{(x_2 - x_1 + \varepsilon)^2 + h^2 + (d-e)^2}} \qquad \{7\}$$

The total mutual inductance between the two coils is the sum of the mutual inductances of the segments:

$$M_{12} = M_{a_1 a_2} + M_{b_1 b_2} + M_{c_1 c_2} + M_{d_1 d_2} + M_{a_1 c_2} + M_{b_1 d_2} + M_{c_1 a_2} + M_{d_1 b_2} \qquad \{8\}$$

## 2.2. The flux method

The flux method considers the sides of the rectangular coil as longitudinally uniform and infinite. Thus, it applies to the calculation of linear mutual inductance between long rectilinear coils.
The basic concept of this method consists in calculating the magnetic field by integrating Poisson's vectorial equation outside the wire:

$$\nabla \times \vec{H} = \vec{e}_z\, J_z \qquad \{9\}$$

Outside the conductor, where the current density is zero, the magnetic field becomes:

$$H_\varphi = \frac{I}{2\pi\rho} \qquad \{10\}$$

The mutual inductance then becomes:

$$L_{12s} = \frac{\mu_0}{\pi} \int_0^{2e} \frac{d - e + x}{(d - e + x)^2 + h^2}\, dx \qquad \{11\}$$

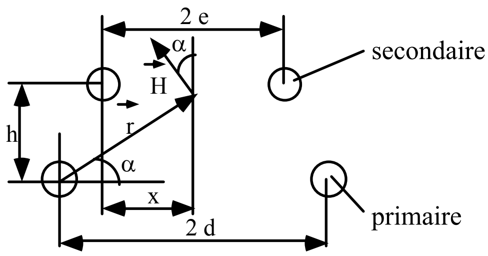


***Figure 2*** *Cross-section*

The wires are actually of finite length and the magnetic field along them is non-uniform. Close to the conductor (length 2d), oriented by the axe z, the magnetic field is (according to Biot-Savart's law):

$$\vec{H}(\vec{r}) = \frac{I}{4\pi\rho}\, \vec{e}_\varphi \left[ \frac{d - z}{\sqrt{\rho^2 + (d - z)^2}} + \frac{d + z}{\sqrt{\rho^2 + (d + z)^2}} \right] \qquad \{12\}$$

This formula shows that the amplitude of the pure azimuthal magnetic field is a function of the length of the conductor (2d). When this length (2d) goes to infinity, {12} simplifies in {10}. Therefore, the use of the simplified formula {10} leads to an inaccuracy, which is inversely proportional to the length of the conductors.

# 3. Calculation of the main inductances

This section is devoted to the calculation of the main inductances of rectangular coils. Various methods are considered according to their ability to consider the following aspects:

- Finite length of the conductors;
- Modelling with and without extremities;
- Coupling between turns of a single winding.

## 3.1. Complete model

The first calculation is based upon the following hypotheses:

- Calculation of the main inductance of a rectangular coil (one turn coil, N=1);
- The external component of the inductance integrates the finite characteristic of the side of the coil;
- The internal component of the inductance is based upon an expression of the magnetic field linked to rectilinear and infinite wire.

By Biot-Savart's formula, the external component of a single turn coil main inductance is:

$$
\begin{aligned}
L_e = \frac{\mu_0}{\pi}\{ & 2\left(\sqrt{(l-r)^2+(2d-r)^2} - \sqrt{(l-r)^2+r^2} - \sqrt{(2d-r)^2+r^2} + \sqrt{2}r - r\ln\left(1+\sqrt{2}\right)\right) \\
& +(l-r)\left(-\ln\frac{l-r+\sqrt{(2d-r)^2+(l-r)^2}}{2d-r} + \ln\frac{l-r+\sqrt{r^2+(l-r)^2}}{r}\right) \\
& +(2d-r)\left(-\ln\frac{2d-r+\sqrt{(l-r)^2+(2d-r)^2}}{l-r} + \ln\frac{2d-r+\sqrt{r^2+(2d-r)^2}}{r}\right) \\
& +r\left(\ln\frac{r+\sqrt{(l-r)^2+r^2}}{l-r} + \ln\frac{r+\sqrt{r^2+(2d-r)^2}}{2d-r}\right)\}
\end{aligned}
\qquad \{13\}
$$

This external component is a function of the section of the wire (r is the radius).

The internal component of the main inductance is calculated thanks to the magnetic energy formulas:

$$L_i = \frac{\mu l}{8\pi} \qquad \{14\}$$

Deeper investigations in internal component of the main inductance show that the hypothesis of infinite length is widely valid. The sole restriction appears when the diameter of the wire is close to its length. In that very particular case the flux is:

$$
\begin{aligned}
\Phi_i &= \iint_S B \cdot dS = \int_{y=-d}^{d}\int_{x=0}^{r} \mu_0 \frac{ix}{4\pi r^2}\left(\frac{d-y}{\sqrt{x^2+(d-y)^2}} + \frac{d+y}{\sqrt{x^2+(d+y)^2}}\right) dx\, dy \\
&= \int_{y=-d}^{d}\int_{x=0}^{r} \mu_0\, I \frac{x^2}{r^2}\frac{x}{4\pi r^2}\left(\frac{d-y}{\sqrt{x^2+(d-y)^2}} + \frac{d+y}{\sqrt{x^2+(d+y)^2}}\right) dx\, dy
\end{aligned}
\qquad \{15\}
$$

Finally, the main inductance is the sum of the two components (internal and external):

$$L = L_i + L_e \qquad \{16\}$$

### 3.2. Linear main inductance

When the geometry of the coils is such that the value of the length is way larger to the value of the width, then the linear approximation is valid.

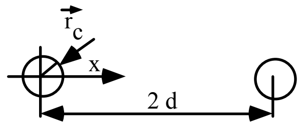


***Figure 3*** *Cross section of a linear main inductance*

The well known associated formula is:

$$L' = N^2 \frac{\mu_0}{\pi} \left( \frac{1}{2} + \ln \frac{2\,d}{r_c} \right) \qquad \{17\}$$

### 3.3. Main inductance of a N turns rectangular coil

The two ways to calculate the main inductance are:

- To add the sum of the main inductances linked to each turn to the sum of the mutual inductances between turns of a single winding;
- To determine the position of an equivalent central turn and to multiply its main inductance value ($L_{eq}$) by the square of N, the number of turns.

$$L_{bob} = \sum_{i=1}^{N} L_i + \sum_{i,j=1}^{N} M_{i,j} \qquad L_{bob} = N^2 L_{eq} \qquad \{18\}$$

The ratio of the winding section versus the overall dimension is relevant to choose a calculation method.

## 4. Results: sensitivity analysis

### 4.1. Mutual inductance

**Finite length**

The first result illustrates the influence of the finite length of the coil. For a primary coil of one meter long and a secondary length, which varies from 0.1 m to 10 m, the difference between the flux' and Neumann's methods is (width = 0.3 m) given in the following figure

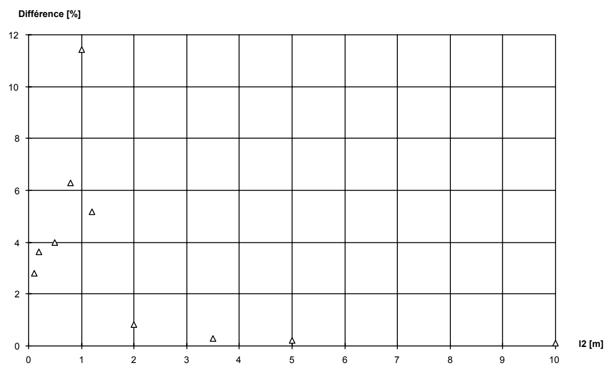


***Figure 4*** *Difference the flux' and Neumann's methods*

The maximum error occurs when the lengths of the coils are equal. Thus, when the difference between the lengths increases, the error is reduced. A narrow structure will impose a greater length difference between the coils in order to reduce the error of the flux method.

**Extremities**

The table below illustrates the influence of considering the extremities of the coils (2 coils of one meter long, 0.3 m in width, airgap 15 mm, perfectly overlapped):

| method | Flux_sides | Flux_sides + extremities | Neum_sides | Neum_sides + extremities | Flux2D_ sides | Measured value |
|---|---|---|---|---|---|---|
| $L_{12}$ [mH] | 1,19 | 1.69 | 1.09 | 1.404 | 1.16 | 1.41 |
| Normed error [%] | 15.6 | 19.8 | 22.7 | 0.5 | 17.7 | - |

***Table 5*** *Difference between theory and measures, with and without extremities*

**Conclusion**

The flux method can be used when both the difference between primaries and secondary is important (factor of 3) and the difference between the length and the width exceed 2. It is also relevant when the primary and secondary lengths are similar and the ratio between length and width surpass 4.
When the structure of the windings is long, the extremities are important in Neumann's formula if the ratio between the primary and secondary lengths is inferior to 5 (the extremities are negligible beyond this value). In the flux' method to consider the extremities represents a source of error in most cases.

## 4.2. Main inductances

First of all, it is important to notice that, in the case of the main inductance, it is difficult to establish general rules due to the large number of factors, to their imbrication and to their possible mutual error compensation.

Table 6 shows for a coil (1 m long, 0.3 m width, wires radius of 1.75 mm and N=4), the value of the main inductance calculated by various methods, as well as measured.

| | $L_{lin}$ | $L_{eq}$ | L (turn by turn) | $L_{measured}$ |
|---|---|---|---|---|
| inductance [mH] | 34.6 | 43.1 | 36.07 | 36.20 |
| Normed error [%] | 4.5 | 19 | 0.5 | - |

***Table 6*** *Comparison between theoretical and measured values*

The sophisticated method, consisting in considering each turn (turn by turn), yields a high level of accuracy compared with the approach based on an equivalent turn centered in the winding. The good performance of the simplest method (linear inductance $L_{lin}$) is due to an error compensation effect (neglect of the extremities and equivalent turn).

With a rough estimate, the simple linear approach is valid for long structures and a low number of turns. When both the winding geometry is close to a square and the number of turns is rather high, the sophisticated method gives excellent results (calculations turn by turn, application of Biot-Savart's formula for both internal and external components of the inductance; calculation of the mutual term by the Neumann's formula). Some configuration could crate problems in terms of numerical calculations (convergence difficulties). More complex decision procedures and trees are given in [2].

# 5. Design and calculation of a multi-primary linear transformer

## 5.1. The transformer model

Ref [1] introduces a multi-primary transformer model based on voltage equations. It also shows the conditions when the mutual inductances between primary can be neglected. It also demonstrates the lack of sensitivity of linear transformers versus speed. The power distribution between the various primaries is illustrated in [3].
To compensate the reactive power or to tune the circuitry for a HF supply by frequency converter, it is necessary to connect an input capacitor in parallel with the two primaries (see Fig. 7). $C_2$ capacitor allows an exchange of reactive power (magnetisation of the transformer).

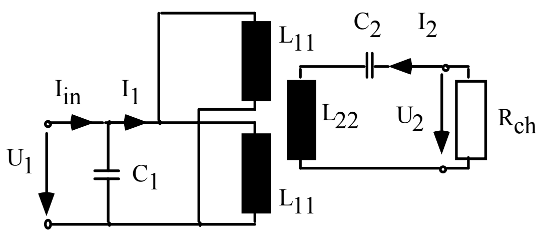


***Figure** 7 Electric scheme of the multi-primary transformer*

The expression for the input impedance $\underline{Z}_{in}$ is:

$$\underline{Z}_{in} = \frac{1}{\frac{1}{\underline{Z}_{eq}} + \frac{1}{\underline{Z}_{C1}}} \qquad \{19\}$$

where $\underline{Z}_{eq}=R_{eq}+jX_{eq}$ is the impedance corresponding to two primaries and the secondary referred to the primaries;
and, $\underline{Z}_{C1}=jX_{C1}$ is the impedance of the capacitor $C_1$.

$C_1$ is calculated so that the reactance $X_{C1}$ the imaginary part of $\underline{Z}_{in}$ cancel each other

$$X_{C1} = \frac{R_{eq}^{\ 2} + X_{eq}^{\ 2}}{X_{eq}} \qquad \{20\}$$

Ref [1] analyses the influence of respectively the frequency, the type of wire, the airgap, the capacities at the primary and secondary, the position of the secondary, the speed, the coupling between contiguous primaries and the number of turns.

## 5.2. CAD software and comparison theory - measurement

Based upon the specifications of the secondary, the CAD software enables to design the transformer by calculating the primary main values (voltage, current, efficiency, power factor). It also evaluates the sensitivity of the transformer characteristics to the main parameters (geometrical, electrical,...).
The parametric curves showed below illustrate the results obtained on an energy transfer test bench of low power (two 1 m long primaries, a 0.9 m long secondary, width of the transformer 0.3 m).
The validation of the method has been done on a low voltage and high frequency (100 kHz) transformer bench designed for electric vehicles application.
The structure of the test bench is constituted by two contiguous primaries connected in parallel to a frequency converter. The secondary winding, connected through a rectifier to a load resistance, slides over the primaries.

For a transformer with the following parameters:

- $l_1(a) = l_1(b) = 1$ m, $\delta = 55$ mm, width = 300 mm ($\delta$ is the airgap)
- $N_1=4$, $N_2=14$
- $U_1=350$ V, $U_2=215$ V

The calculated and measured values of the power as a function of secondary capacitor for reduced voltage supply (dc 50 V) are, for Litz wires:

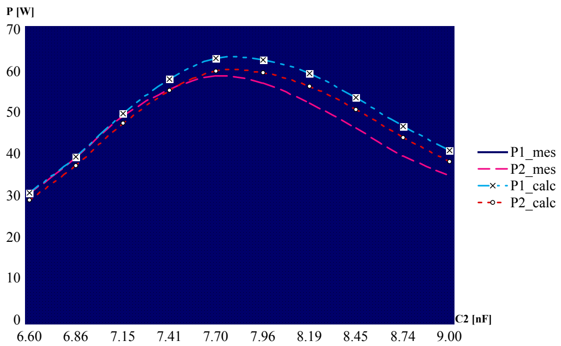


***Figure 8*** *P=f(C$_2$)*

The theoretical and measured evolution of the secondary voltage versus the airgap is given below:

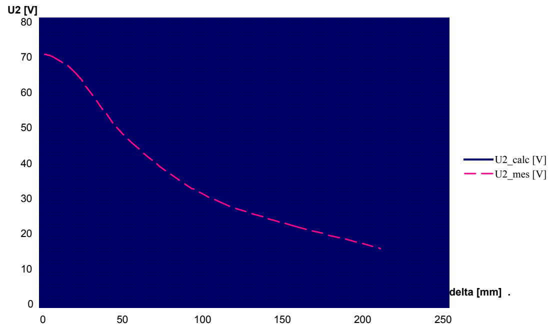


***Figure 9*** *Secondary voltage versus airgap value*

The concordance between the characteristics calculated by the program and measured shows that these calculation methods are valid and the results puts to light the interesting potential of this energy transfer technique. The best configuration leads to an efficiency of 94% for the transformer and 95% for the frequency converter. This corresponds to a global efficiency close to 89%.

## 6. Conclusion

This paper describes the best way to model and calculate a ironless linear transformer comprising various primary windings. This method and its CAD software have been successfully applied in various industrial and transport applications over the last years. The linear transformer for the Swissmetro application was designed by these method and tools.


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