Towards Self - Sensing EMS Levitation Systems

Kent R. Davey
American Maglev, Inc.
2275 Turnbull Bay Rd., New Smyrna Beach, FL 32168-5941
(386) 426-1215, kdavey@American-Maglev.com
Work supported by American Maglev, Inc.

Topic 8.1

Abstract - Self sensing magnetic bearings maintain a desired air gap without secondary gap transducers by inferring the gap from internal inductance estimates. Maglev systems must account for both the vertical and the lateral gap. This paper examines one technique for predicting both gaps by pooling information from multiple magnets.

Key Words : levitation, inductance, self sensing, inverse problem

Background

Self sensing magnetic bearings have been successfully implemented by several researchers [1][2]. All seek an estimate of inductance usually through a measurement of current in response to a voltage reversal [3]. Care must be exercised in this measurement since the switching transient introduces error in the measurement of current [4].The bearing gap is approximately inversely proportional to the inductance of the magnet winding.

In a maglev system, both the vertical and lateral gap are important. Most EMS systems are laterally stable, but they will have a tendency to oscillate laterally unless appropriately compensated. This is usually approached by offsetting multiple magnets laterally on either side of the bogie [5]. Current can be swapped between magnets on either side of the centerline to suppress an oscillation. In addition, however, the attraction of the vehicle to the track depends on both gaps. *The goal of this research is to predict both the lateral and vertical position of every magnet with respect to the bogie given only the instantaneous inductance of that magnet*.

Bogie Configuration

Shown in Figure 1 is the magnet distribution of 6 magnets on a test bogie. All magnets are displaced a distance δ from the centerline. The angle ξ is fixed by the width and length of the bogie. The bogie is free to travel in the Y direction.

If the inductance can be measure accurately enough, its value at all 6 corners might be used to dictate the lateral and vertical of each magnet. The principle governing equation is

$$V = IR + I\frac{dL}{dt} + L\frac{di}{dt}\,. \tag{1}$$

The voltage V is fixed, and varies only by the length of the line and the resistance drop to the brushes. Its nominal value is ± 720 V. If damping is reasonable, I dL/dt should be small. High accuracy will demand that the resistance R be measured since it will vary with ambient temperature.

If the quantity R + I dL/dt is treated as a single entity, then there are 2 equations and 2 unknowns. Let β = (R+dL/dt)

At two consecutive times,

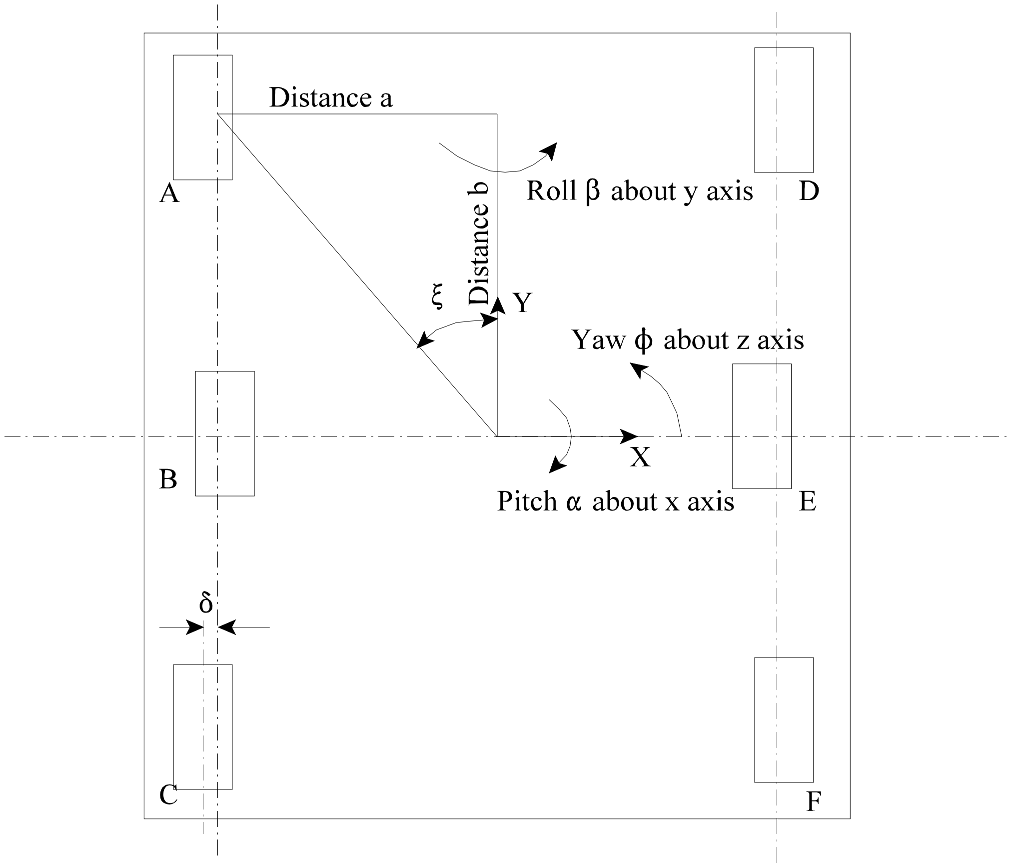

Figure 1 Bogie configuration used for simulation.

$$V_1 = \beta I_1 + L\frac{dI_1}{dt}. \tag{2}$$

$$V_2 = \beta\, I_2 + L\frac{dI_2}{dt}. \tag{3}$$

Solve these two equations for beta and L

$$L = \frac{V_2 I_1 - I_2 V_1}{-I_1 \frac{dI_2}{dt} + \frac{dI_1}{dt} I2}. \tag{4}$$

$$\beta = \frac{-V_1 \frac{dI_2}{dt} + \frac{dI_1}{dt} V_2}{-I_1 \frac{dI_2}{dt} + \frac{dI_1}{dt} I_2}. \tag{5}$$

The quantity β is really of no concern.

Determining Position from Inductance Measurements

It is possible to get the local gap z and horizontal position x of all 6 magnets from the inductance measurements. This inference allows for the operation of a self-sensing electromagnet. The vertical gap can be determined from the inductances by solving a least squares problem fitting the inductances to vertical and horizontal position. For a six magnet bogie, define the matrix

$$\boldsymbol{L} = [L_A \; L_B \; L_C \; L_D \; L_E \; L_F]\,. \tag{6}$$

Define the mean inductance L_{mean} to be 0.6754. Also define the logarithm of L as

$$\boldsymbol{G} = \ln\left(\frac{\boldsymbol{L}}{L_{mean}}\right)\,. \tag{7}$$

Now consider constructing a matrix which would dictate all the gaps at once as

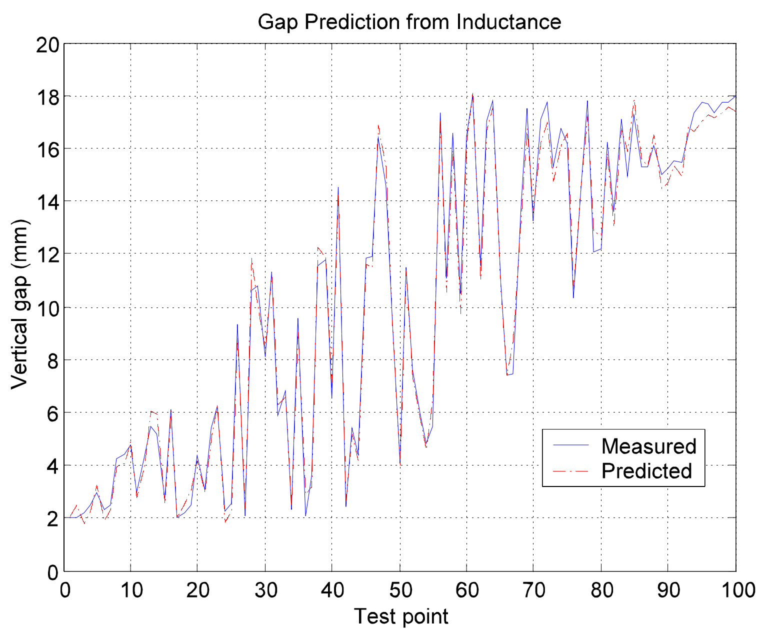

Figure 2 Prediction accuracy for predicting magnet gap given all 6 inductances.

$$[\,1 \;\; \boldsymbol{G} \;\; \boldsymbol{G}^2 \;\; \boldsymbol{G}^3 \;\; \exp(\boldsymbol{G}) \;\; \boldsymbol{G}*\exp(\boldsymbol{G})\,]\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{31} \end{bmatrix} = [\,Z_A \;\; Z_B \;\; \dots \;\; Z_F\,]\,. \tag{8}$$

The unknowns are the vector components u. Note that u is a 31 by 6 matrix of coefficients to be determined. A 10,000 point data base covering the spread of possibilities for x and z with the requirement that they always lie on the plane as defined in Figure 1. For each position, compute the inductances commensurate with that position. The unknowns u are determined in a least squares algorithm to minimize the discrepancy between the predicted gap and the gap from the data. Figure 2 shows the accuracy achieved on 100 new random magnet spreads. The mean error is 0.372 mm, with the maximum gap error being 1.15 mm. The magnets are assumed to be laterally offset from the midline 10 mm. The inductances were computed using a 2D numerical analysis solver.

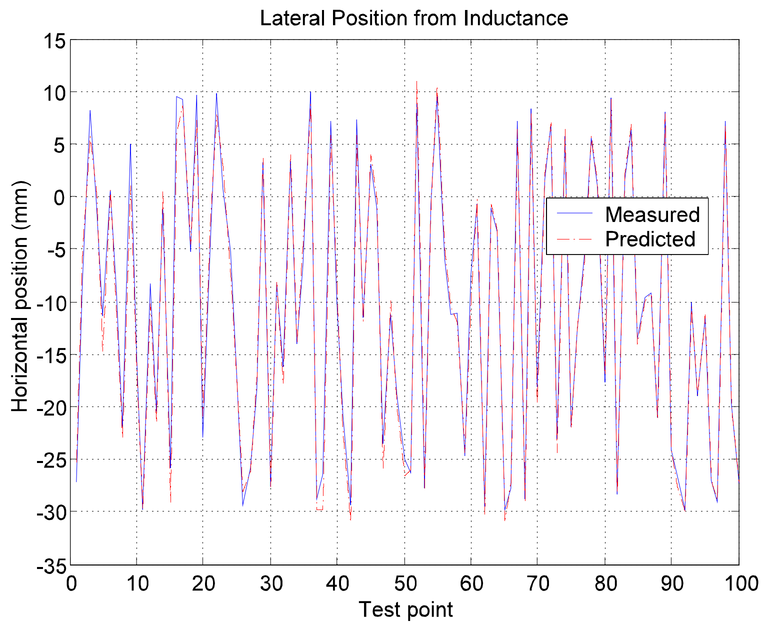

Figure 3 Lateral magnet position prediction using the formulation in (8).

This process can be repeated for the lateral position x. Shown in Figure 3 is a comparison of the predicted and actual positions for a lateral position test set. The mean lateral position error is 0.829 mm, with the maximum being 4.382 mm.

Increasing the Accuracy of the Self-sensing Approach

Depth D

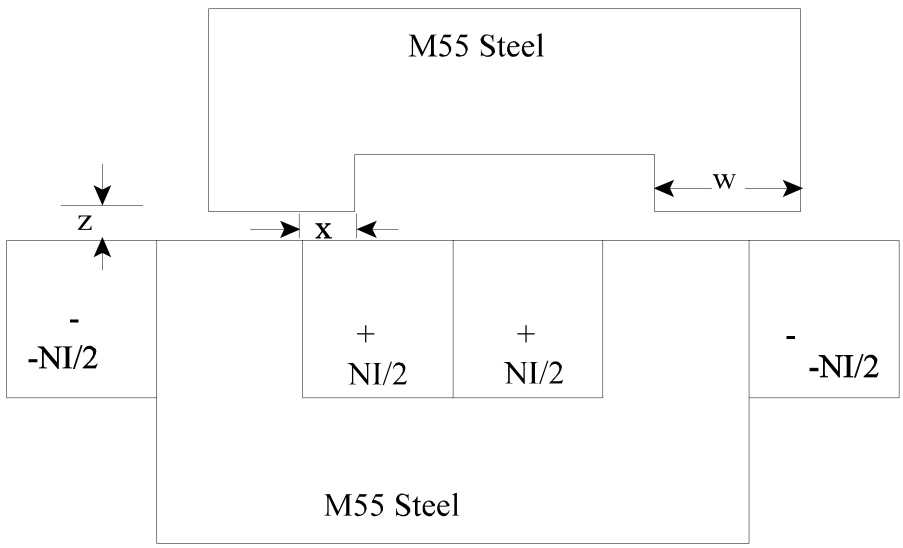

Figure 4 Displacement of the magnet yields natural restoring forces.

The inverse problem accuracy can be further increased by employing the analytical expression for inductance. Given the parameters as defined in Figure 4, the inductance is now a function of both x and z as

$$L_{analy} = \frac{\mu_0 N^2 D}{2}\left\{\frac{w-x}{z} + \frac{4}{\pi}\ln\left(1+\frac{\pi x}{4z}\right)\right\}. \tag{9}$$

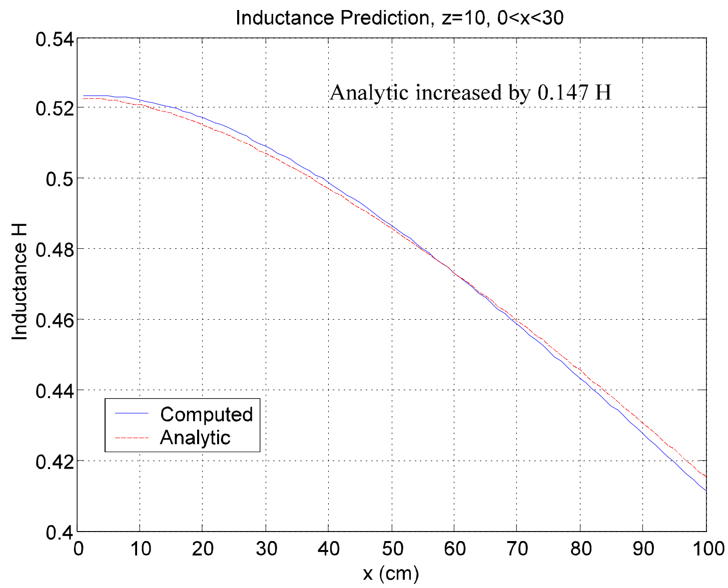

Figure 5 The analytic expression for inductance is quite good when leakage inductance is included.

vertical gap of 10 mm in the range 0<x<30 mm. When the analytic expression is increased by a constant 0.147 H, the fit is very close as shown by Figure 5. This analytic expression can be used to enhance the accuracy of the prediction of x and z from inductance only.

The calculation is approached in two stages. First the position x and z are computed using (8). Next a similar expression to (8) is used to define the difference between the analytic inductance and the true inductance for a range of inductances. Random numbers are employed as before to generate the data necessary to link the measured inductance to its analytic approximation. This is equivalent to fine tuning the offset leakage inductance constant. When this is completed, another vector u is available so that

$$L_{analy} = L_{measured} - \begin{bmatrix} 1 & \boldsymbol{G} & \boldsymbol{G}^2 & \boldsymbol{G}^3 & \exp(\boldsymbol{G}) & \boldsymbol{G}*\exp(\boldsymbol{G}) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{31} \end{bmatrix}. \tag{10}$$

$$x_{(2)} + \left\{ 2\frac{L_{analy}}{\mu_0 N^2} - \frac{4}{\pi} \ln\left(1 + \frac{\pi x_{(1)}}{4 z_{(1)}}\right) \right\} z_{(2)} = w\,. \tag{11}$$

There are 6 inductances available at any time. But since all the magnets lie on a plane, there are 5 unknowns in x and z. Examination of Figure 1 reveals the following relations among the x's and z's,

$$X_D = X_E + \frac{X_A - X_C}{2} + 2\,\delta \tag{12}$$

$$X_F = X_E + \frac{X_C - X_A}{2} + 2\,\delta \tag{13}$$

$$Z_D = Z_E + \frac{Z_A - Z_C}{2} \tag{14}$$

$$Z_F = Z_E + \frac{Z_C - Z_A}{2} \tag{15}$$

$$X_E = \frac{X_A + X_C}{2} = \frac{X_D + X_F}{2} - 2\,\delta \tag{16}$$

$$Z_B = \frac{Z_A + Z_C}{2} \tag{17}$$

Let

$$\tilde{L} = 2\,\frac{L_{analy}}{\mu_0 N^2} - \frac{4}{\pi}\,\ln\left(1 + \frac{\pi x_{(1)}}{4 z_{(1)}}\right)\,. \tag{18}$$

Let the unknowns be Z_A, Z_C, Z_E, X_A, X_C, and X_E.. The following relation results

$$\begin{bmatrix} \tilde{L}_A & 0 & 0 & sign(X_A) & 0 & 0 \\ \frac{\tilde{L}_B}{2} & \frac{\tilde{L}_B}{2} & 0 & \frac{sign(X_B)}{2} & \frac{sign(X_B)}{2} & 0 \\ \tilde{L}_C & 0 & 0 & 0 & sign(X_C) & 0 \\ \frac{\tilde{L}_D}{2} & -\frac{\tilde{L}_D}{2} & \tilde{L}_D & \frac{sign(X_D)}{2} & -\frac{sign(X_D)}{2} & sign(X_D) \\ 0 & 0 & \tilde{L}_E & 0 & 0 & sign(X_E) \\ -\frac{\tilde{L}_F}{2} & \frac{\tilde{L}_F}{2} & \tilde{L}_F & -\frac{sign(X_F)}{2} & \frac{sign(X_F)}{2} & sign(X_F) \end{bmatrix} \begin{bmatrix} Z_A \\ Z_C \\ Z_E \\ X_A \\ X_C \\ X_E \end{bmatrix} = \begin{bmatrix} w \\ w - 2\delta\, sign(X_B) \\ w \\ w - 2\delta\, sign(X_D) \\ w \\ w - 2\delta\, sign(X_F) \end{bmatrix} \tag{19}$$

Alternatively, one can make the unknowns, Z_A, Z_C, Z_E, X_A, and X_C, using the fact that

$$X_E = \frac{X_A + X_C}{2}\,. \tag{20}$$

In this case the governing equations become

$$\begin{bmatrix} \tilde{L}_A & 0 & 0 & sign(X_A) & 0 \\ \frac{\tilde{L}_B}{2} & \frac{\tilde{L}_B}{2} & 0 & \frac{sign(X_B)}{2} & \frac{sign(X_B)}{2} \\ \tilde{L}_C & 0 & 0 & 0 & sign(X_C) \\ \frac{\tilde{L}_D}{2} & -\frac{\tilde{L}_D}{2} & \tilde{L}_D & sign(X_D) & 0 \\ 0 & 0 & \tilde{L}_E & \frac{sign(X_E)}{2} & \frac{sign(X_E)}{2} \\ -\frac{\tilde{L}_F}{2} & \frac{\tilde{L}_F}{2} & \tilde{L}_F & 0 & sign(X_F) \end{bmatrix} \begin{bmatrix} Z_A \\ Z_C \\ Z_E \\ X_A \\ X_C \end{bmatrix} = \begin{bmatrix} w \\ w-2\delta\, sign(X_B) \\ w \\ w-2\delta\, sign(X_D) \\ w \\ w-2\delta\, sign(X_F) \end{bmatrix} \tag{21}$$

In both cases,

$$\boldsymbol{Z} = [Z_A\,,\ \frac{Z_A+Z_C}{2}\,, Z_C\,,\ Z_E+\frac{Z_A-Z_C}{2},\ Z_E\,,\ Z_E+\frac{Z_C-Z_A}{2}]. \tag{22}$$

The position for X follow respectively as

$$\boldsymbol{X} = [X_A\,,\ \frac{X_A+X_C}{2}+2\delta\,, X_C\,,\ X_E+\frac{X_A-X_C}{2}+2\delta\,,\ X_E\,,\ X_E+\frac{X_C-X_A}{2}+2\delta\,]. \tag{23}$$

$$\boldsymbol{X} = [X_A\,,\ \frac{X_A+X_C}{2}+2\delta\,, X_C\,,\ X_A+2\delta\,,\ \frac{X_A+X_C}{2}\,,\ X_C+2\delta\,]. \tag{24}$$

So the process is to use the least square algorithm in (8) to get an estimate of x, z, and the difference between the actual inductance and the analytic expression. This allows a very good computation of $\tilde{L}$. Then (19) or (21) can be employed to improve the estimate. This process is simulated using random numbers in Matlab to simulate possible positions for the bogie. Typical error results are shown in Table I. The numbers range 0<Z<18 and -30<X<30 mm. The uncertainty in Z is usually cut by 4 and that for X by 2.

Table I Errors in a 100 point simulation using 6000 points to build the least square matrix.

	Vertical position Z (mm)	Horizontal Position X (mm)
Max error at step 1	1.0573	5.13
Mean error at step 1	0.375	0.835
Mean error at step 2	0.0714	0.4099
Max error at step 2	0.7289	3.678

One other iteration can be employed. If X is near zero, the estimate of its sign might be wrong. One indicator that this has happened is that the results from (19) and (21) do not agree. When this occurs, the signs for each of the x variables can be sequentially swapped until the results become more closely correlated. This correction has been employed in Table I.

Self-sensing Levitation Results

Assume the 6 inductances for every magnet is measured at every time step. Define

$$L = \ln([L_A \; L_B \; L_C \; L_D \; L_E \; L_F]) \tag{25}$$

So L is a 1 by 6 array. Now assume that we seek the parameters X, Z, α, β, and ϕ defined on page ?. Each of the unknowns will be represented in terms of a simple 3rd order polynomial in L. For example

$$X = c_0 + \vec{c}_1 \cdot L + \vec{c}_2 \cdot L^2 + \vec{c}_3 \cdot L^3. \tag{26}$$

C_0 is a scalar, but all the other unknowns C_1, C_2, and C_3 are 6 by 1 vectors. L in (25) is a 1 x 6 matrix. Another way to think about this operation is to define a 1 by 19 matrix

$$\boldsymbol{A} = [1, \boldsymbol{L}, \boldsymbol{L}^2, \boldsymbol{L}^3]. \tag{27}$$

The output matrix 5x1 is

$$[X, Z, pitch\ \alpha, roll\ \beta, yaw\ \phi] = \boldsymbol{A} * \vec{C}. \tag{28}$$

Where the unknown vector C is 19 by 5. These vectors are determined using a least squares algorithm to minimize the square error between a known array of inputs and L's. This data set typically consists of 6,000 to 10,000 points. It is then tested on 100 points.

The error are dependent on the choice of δ, the offset spacing between magnets. A typical run with a post processing test on 100 points delivers the maximum errors shown in Table II

Table II Typical maximum errors generated by the polynomial fit.

δ (mm)	X error (mm)	Z error (mm)	Pitch error α (deg)	Roll error β (deg)	Yaw error ϕ (deg)
10	2.426	0.935	0.0267	0.0103	0.1569
8	3.059	1.083	0.0353	0.0110	0.254
6	3.58	1.332	0.0465	0.0245	0.264

Similar computations can be performed to compute the mean error. The results are summarized in
Table III

Table III Typical maximum errors generated by the polynomial fit.

δ (mm)	X error (mm)	Z error (mm)	Pitch error α (deg)	Roll error β (deg)	Yaw error ϕ (deg)
10	0.6234	0.2667	0.0084	0.0027	0.036
8	0.6583	0.3556	0.0122	0.0029	0.0401
6	0.8956	0.463	0.0126	0.004	0.0603

Where do the errors occur? Are the problem zones smooth? Shown in Figure 6 is a typical error profile for the vertical gap in mm with the yaw and lateral position picked randomly. Three observations are in order. First the errors increase in the outlier region where either X or yaw ϕ are large. Second the errors increase when the bogie gap is maximum (18 mm). Third, the error is smooth.

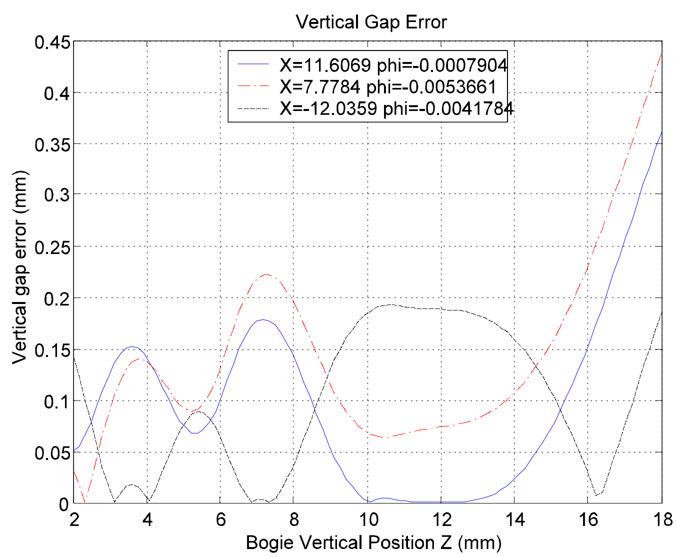

Figure 6 Vertical gap prediction error as a function of gap with α and β set to zero.

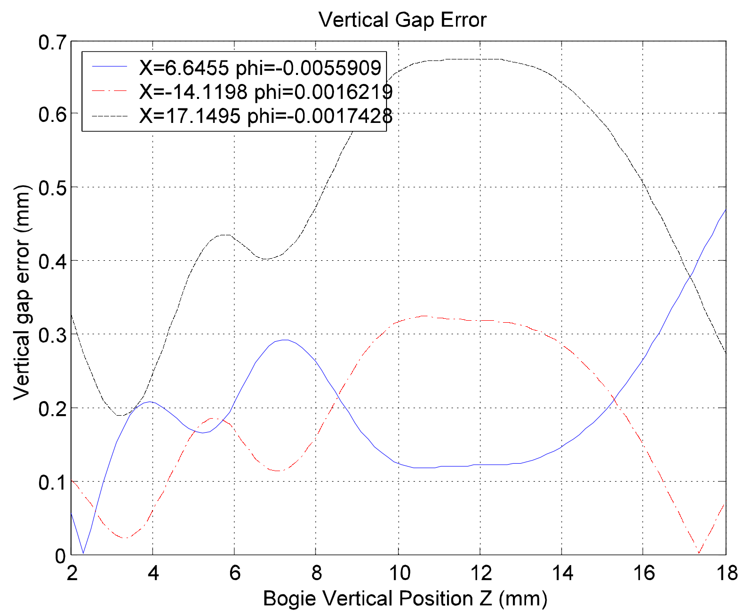

Figure 7 Vertical gap error for random lateral and yaw positions with α and β set to zero.

Since these lateral and yaw positions were chosen randomly, a second plot is in order. Figure 7 shows the same trend, underscoring the fact that large displacements in X lead to larger gap errors.

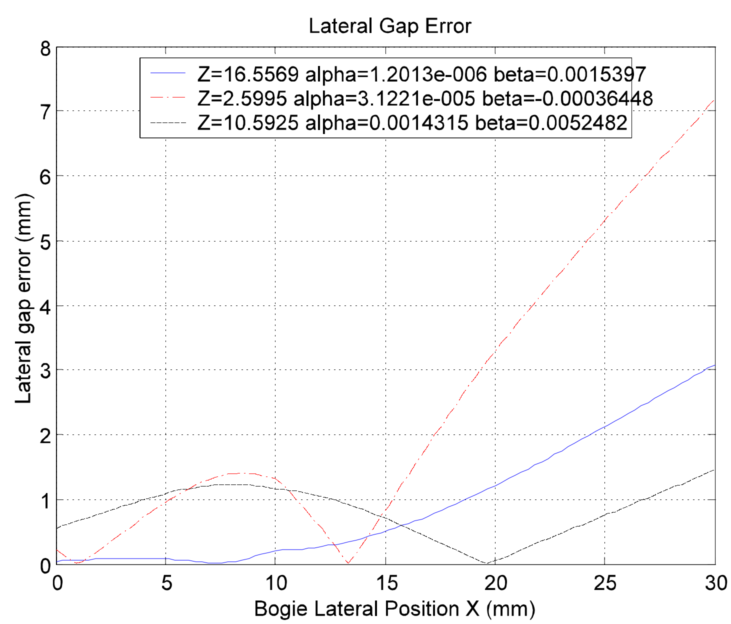

Figure 8 Lateral error with yaw ϕ set to zero.

Shown in Figure 8 is the lateral error resulting from various random positions in vertical gap, roll angle, and pitch angle. Again, the larger errors result at the extreme positions for vertical gap and lateral position, but the changes are smooth.

Conclusions

Among the conclusions to be drawn from this exercise are the following:

1. Obtaining inductance from a simple polynomial is accurate and fast. The computation time on a 1 Ghz computer running uncompiled matlab code is 73 μs.
2. The errors increases at the extremes of the allowed bogie positioning.
3. The errors are smooth, not spiked.
4. A larger offset distance for the magnets will lead to lower errors in the position prediction.

References

1. Dominick Montie and Eric Maslen, "Self Sensing Results for a Magnetic Bearing", *6th International Symposium on Magnetic Suspension Technology*, Turin, Italy, October 7-11, 2001.

2. T. Muzuno, et. Al., "Towards Practical Applications of self sensing magnetic bearings", Proc. ISMB3, Virginia, 1992.

3. T. Mizuno and Y. Hirasawa, "Self sensing magnetic suspension using a hysteresis amplifier with a two quadrant drive", *6th International Symposium on Magnetic Suspension Technology*, Turin, Italy, October 7-11, 2001, pp. 76-80.

4. N. Shricka, N. & R. Markert, "Compensation of disturbances on self sensing magnetic bearings caused by saturation and coordinate coupling", *Proc. of the 7th Symposium on Magnetic Bearings*, Zurich, 2000, pp. 163-170.

5. Aldo D'Arrigo and Alfred Ruffer, Integrated electromagnetic levitation and guidance system for the Swiss Metro project", *Proc. of the 16th International Conference on Levitated Systems and Linear Drives*, Rio de Janeiro, Brazil, June 7-10, 2000, pp. 263-268.