ANALYSIS OF DYNAMICS AND STABILITY OF A HIGH-SPEED VEHICLE WITH ELECTRODYNAMICS LEVITATION AND GUIDANCE

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Abstract. Some methods and numerical results on dynamics and stability of a maglev vehicle with discrete track structure obtained in the Institute of transport systems and technology of the National Academy of Sciences of Ukraine are discussed.

1. **Mathematical model of a vehicle with a discrete track structure**. A levitating vehicle is modelled by a rigid body or a few such bodies connected to each other by means of elastic and dissipative ties. The behaviour of the corresponding generalised co-ordinates $q = (q_1,...,q_n)$ is described by the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q \qquad (1)$$

where $L = L(q, \dot{q})$ is the Lagrange function and $Q = (Q_1,...,Q_n)$ is a vector of non-conservative forces (the last include, in particular, aerodynamical and electrodynamical forces).

The currents in the ground coils are determined by the equation

$$S\frac{di}{dt} + Ri = e(t)\,, \qquad (2)$$

where $S = [s_{ik}]_{i,k=1}^{p}$ is a matrix of inductances and R is a resistance of coils, $i = (i_1,...,i_p)$ and $e = (e_1,...,e_p)$ are vectors of currents and electromotive forces . The number of coils p is determined by the condition that the currents in the dropped coils are negligible. The value $e_k(t)$ depends on the current co-ordinates of exciting magnets and, therefore, on $q(t)$; in their turn, the forces $Q(t)$ depend on the currents $i(t)$. So, equations (1) and (2) are coupled and should be solved simultaneously. Integration is performed on the interval $[0, T = H/v]$ where H is a pitch of coils and v is a vehicle speed, then (in view of the shift of the vehicle) new initial conditions $q(0) = q(T)$, $i_k(0) = i_{k+1}(T), k = 1,\dots, p-1, i_p(0) = 0$ are assigned and the process is repeated.

2. **Stationary motion: calculation of electrodynamical forces.** Under stationary rectilinear motion, the values $q_i(t)$ in a moving co-ordinate system are constants; the vertical co-ordinate of the vehicle z_0 is determined by the condition $F(z) = mg$ where $F(z)$ is the mean value of the total levitation force and m is the vehicle mass.

The currents in neighbouring coils satisfy the relation $i_k(t) = i_{k+1}(t+T)$, so it is sufficient to find the current only in one coil for a fixed z and, therefore, for known $e_k(t) = e_k(t+T)$. There exist some analytical methods for solving this problem (see, e.g. [1,2]). Below two new methods using different ideas are proposed.

The first method is based on the fact that the diagonal elements of the matrix S, $s_{ii} = s_0$ (self inductance of the coil) substantially exceed the off-diagonal elements $s_{ik} = s_q$, $q = |i-k|$ (mutual inductances of the corresponding coils). So it is natural to use a successive approximation method accepting the solution of system (2) with $s_{ik} = 0$ $(i \neq k)$ as the initial approximation. The last may be written as follows

$$i_0^0(t) = \frac{e^{-ht}}{s_0}\left[h\int_{t_0}^{t} e^{ht}\sum_{k=1}^{r} I_k M_{0k}(t)dt - e^{ht}\sum_{k=1}^{r} I_k M_{0k}(t)\Big|_{t_0}^{t} \right], \quad (3)$$

where $h = R/s_0$, I_k is the current in the kth magnet, r is the number of the magnets, M_{0k} is the mutual inductance between the coil and the kth magnet; the value $t_0 < 0$ is taken so that $e_0(t)$ is negligible for $t \le t_0$.

Putting $p = 2q+1$, we find the current $i_0(t)$ in the middle coil. The successive approximations are determined by the formula

$$i_0^{k+1}(t) = \frac{e^{-ht}}{s_0}\left[h\int_{t_0}^{t} L(i_0^k(t))dt - L(i_0^k(t))\Big|_{t_0}^{t} \right], \quad (4)$$

where

$$L(i_0^k(t)) = e^{ht}\left[\sum_{\substack{i=-q \\ s\neq 0}}^{q} s_i i_0^k(t - sT) + \sum_{d=1}^{r} I_d M_{0d}(t) \right].$$

The calculations are continued until $\max_t \left| i_0^{k+1}(t) - i_0^k(t) \right| < e$, where e is a required accuracy.

The method is found to be very effective; the third approximation is practically coincides with the exact solution. In Table 1 the successive approximations $i_0^k(x)$ ($x = vt$ is a current position of the superconducting magnet) are presented (the number of magnets $r = 1$, the current $I_1^0 = 100$ kA, $z_0 = 0{,}15$ m, the sizes of the magnet and coils are 1.0×0,3m and 0,3×0,3m (length×width), the pitch of coils $H = 0{,}36$ m; the coils and the magnet lie in horizontal planes).

Table 1

v m/sec	k	x_m / H							
		-2	-1	0	1	2	3	4	5
50	0	-411	-13710	-8571	-4461	10880	6735	3722	2135
	2	-1168	-13940	-9056	-3688	11430	7727	4047	2031
	4	-1170	-13930	-9057	-3678	11430	7730	4042	2028
100	0	-234	-15410	-12410	-8951	8348	6817	4910	3689
	2	-1287	-6130	-13520	-8592	8886	8223	5909	4098
	4	-1295	-16120	-13520	-8575	8896	8239	5915	4100
150	0	-159	-16020	-14040	-11180	6488	5880	4569	3734
	2	-1338	-16990	-15530	-11220	6761	7226	5718	4357
	4	-1351	-16990	-1554	-11220	6773	7249	5733	4366

The component of the electrodynamical force in a direction g is determined by the known formula

$$F_g(t) = I_1 \sum_{k=1}^{p} i_k(t) \frac{¶M_{0k}}{¶g}, \tag{5}$$

so for $g = z$ and $g = x$, one obtains levitation and drag forces $F_L(t)$ and $F_D(t)$, respectively. Numerical analysis shows that for the mean values of the forces, the accuracy of the approximations is better than that for the currents, especially at a low speed (here even the use of formula (3) for the currents implies satisfactory results for the forces). Thus, coupleness of equations (2) due to mutual inductance of ground coils counts little in the values of electrodynamical forces.

The second method is based on the following consideration. Changing in (2) to the variable $x = vt$, we obtain

$$Si' + mi = -M', \quad ' = \mathrm{d}/\mathrm{d}x, \tag{6}$$

where $m = R/v$. Since under stationary motion v is large, the parameter m is small. So it is reasonable to seek the solution in the form of expansion in m:

$$i = i_0 + m i_1 + m^2 i_2 + \dots \tag{7}$$

Putting (7) in (6) and having made standard operations, we eventually find

$$i(x) = -S^{-1}\left[M(x) + \sum_{1}^{n} (-m)^k \int_{x_1}^{x} \frac{(x-s)^{k-1}}{(k-1)!} S^{-k} M(s)\,\mathrm{d}s \right] \tag{8}$$

In Table 2 the values $\max_t i_0(t)$ calculated for various n in (8) are compared with the exact solution found by direct numerical integration of system (2). As is seen, for practical purposes, a consideration of three terms ($n = 2$) is sufficient for all v. Unlike the above method, here the accuracy increases with the speed, so that for v>100 m/sec, one may take n=1 (this tendency is evident, because $m \to 0$ as $v \to \infty$).

? ?ble 2

v, m/sec	n=0	n=1	n=2	n=3	Exact solution.
50	19670	9059	12320	11780	11700
100	19670	13880	15030	14880	14910
150	19670	15810	16310	16280	16280

3. **Stationary motion: stability analysis.** For an analysis of the Lyapunov stability of stationary motion, the following approach is developed. Systems (1) ? (2) may be reduced to the equation

$$\dot{u} = f(u) \tag{9}$$

where the components $u_i = q_i$, $u_{n+i} = \dot{q}_i$, $i = 1,\dots,n$ and $u_{2n+k} = i_k$, $k = 1,\dots,p$. Let $u(t,u_0)$ be a solution of (9) under the initial condition $u(0,u_0) = u_0$,

$$u^- = u(T,u_0) \tag{10}$$

where $T = H/v$ is the duration of motion between neighbouring coils. As is mentioned above, for the next interval of integration, one should take $q(0) = q(T)$, $i_k(0) = i_{k+1}(T), k = 1,\dots,p-1, i_p(0) = 0$, so the corresponding initial conditions u_k^+ for system (9) are

$$u_k^+ = u_k^- \quad (k \le 2n)$$
$$u_k^+ = u_{k+1}^- (2n < k < p+2n), \quad u_{p+2n}^+ = 0, \tag{11}$$

Relations (10) and (11) determine the map $u_T = F(u_0)$ where $u_0 = u(0)$, $u_T = u^+(T)$. For a stationary motion, $u_T = u_0$; this fixed point (and, therefore, the motion of the vehicle) is asymptotically stable if all the eigenvalues of the corresponding Jacobi matrix $A = F_u(u_0)$ lie within the unit circle. The matrix A is represented in the form $A = CY(T)$ where $Y(T)$ is the monodromy matrix of the variational equation

$$\dot{y} = f_u(u(t))y \tag{12}$$

associated with (9) whereas C is due to the 'shift' $i_r^+ = i_{r+1}$.

Thus, stability analysis is reduced to the calculation of the matrix A and its largest in the modulus eigenvalue.

The results obtained by the above approach, were compared with the results of direct numerical integration of equations (1),(2) with perturbed (relative to their stationary values) initial conditions. It was found that the method enables one to obtain a reliable conclusion on the stability of stationary motion, in any case, when the perturbations are sufficiently small.

The vehicle is symmetric about the longitudinal plane XOZ, so the corresponding oscillations may be treated independently. Numerical results shows that in the case when the vehicle is modelled by a rigid body, the oscillations amplitudes are increasing with time (quite analogously to the case of vertical oscillations of a single magnet [3,4]), i.e. stationary motion is unstable. Note that such an instability takes place for both vertical and horizontal arrangements of superconducting and ground coils within a wide range of their parameters and vehicle speed. Stabilization is reached on inserting sufficiently small dissipative terms in equation (1). For a vehicle of a few rigid bodies, the presence of dissipative ties between them also stabilizes the system.

It should be noted that the above instability cannot be found when the coupleness of equations (1) and (2) is neglected (i.e. the components of disturbances of forces $Q(q,i)$ in equation (1) caused by disturbances of the currents i are dropped).

In studies of transverse oscillations, another type of instability is observed: some generalised co-ordinates increase monotonically with no oscillations (just as in the case of an overturned pendulum). Note that such non-oscillatory instability shows itself for various schemes of the vehicle in a wide range of its parameters. Unlike the oscillatory instability, it cannot be removed by introducing dissipative ties, so it requires a change of the scheme of a system.

To simplify the problem of synthesis of a stable dynamical scheme, an approximate method for stability analysis which does not require integration of differential equations was proposed. It is based on the following considerations. A component F_g of an electrodynamical force F acting on the kth magnet is determined as follows

$$F_g^k = \frac{\partial V_k}{\partial g}, \qquad V_k = I_k^0 \sum_p i_p M_{kp}(x_{kp}, y_{kp}, z_{kp}) \tag{13}$$

where x_{kp}, y_{kp} and z_{kp} are corresponding distances between the magnet and pth coil. Clearly, if the magnets are replaced by elastic constraints with potential functions V_k, the respective forces would coincide with F_g^k. Observing that under stationary motion $x_{pk}(t) = x_{pk}(0) + vt,\ y_{pk} = y_{pk}(q),\ z_{pk} = z_{pk}(q),$ we find that the total potential function of the system may be represented in the form $V = \sum_k V_k = V(q,t) = V(q, t+T)$. Since the frequency $w = 2pv/H$ is much larger than the narural frequencies of system (1), $V(q,t)$ may be replaced by an averaged in t function $V^0(q)$. As a result, the problem is reduced to stability analysis of an equilibrium position q_0 of an autonomous mechanical system with the potential function $V^0(q)$. An absence of non-oscillatory instability is equivalent to positive definiteness of the corresponding quadratic form

$$A(q) = \sum_{i,k=1}^{n} a_{ik}(q_0) q_i q_k, \quad a_{ik} = -\partial^2 V / \partial q_i \partial_k \tag{14}$$

which is easily checked via the known Silvester criterion. Thus, the proposed method does not require calculation of solutions of variational differential equations (12). The function $V(q)$ and, therefore, the coefficients $a_{ik}(q_0)$ may be easily found using any of the above methods for calculating stationary solution of equation (2).

To illustrate the method, consider stability of the system with combined levitation and guidance ground coils arranged on the side walls of the guideway (Fig. 1, where 1- bogie, 2-superconducting magnets, 3- ground coils).

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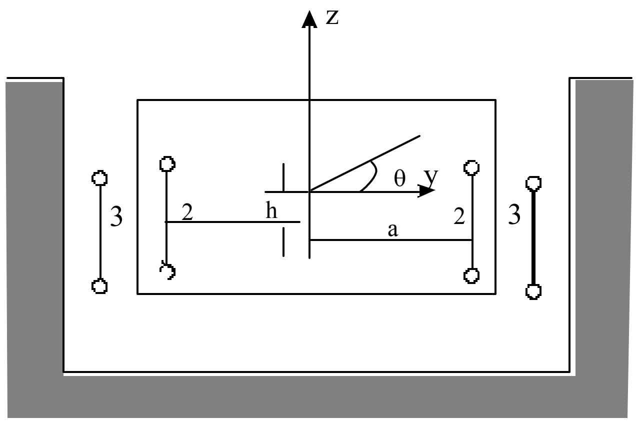

Fig. 1

We assume that the vehicle is symmetric about the transverse plane YOZ, then the corresponding equations (1) are independent and may be written in the form

$$
\begin{aligned}
m\ddot{y} &= \gamma_y \dot{y} + F_y^1 - F_y^2; \\
m\ddot{z} &= \gamma_z \dot{z} + F_z^1 + F_z^2 - mg; \\
J\ddot{\theta} &= \gamma_\theta \dot{\theta} + F_y^2(a\theta - h) + F_y^1(h + a\theta) - F_z^1 a + F_z^2 a
\end{aligned}
\qquad (15)
$$

where F_y^i and F_z^i are the components of electrodynamical forces applied to the magnets, γ_y, γ_z and γ_θ are dissipative coefficients, m and J is the mass and moment of inertia of the vehicle.

Here $V(q) = V(z, y, \theta)$, so the Silvester criterion becomes

$$
\frac{\partial^2 V}{\partial y^2} < 0, \ \frac{\partial^2 V}{\partial z^2} < 0, \ \ \Delta = \frac{\partial^2 V}{\partial y^2}\frac{\partial^2 V}{\partial \theta^2} - \left[\frac{\partial^2 V}{\partial y \partial \theta}\right]^2 > 0. \qquad (16)
$$

The first two inequalities (16) usually hold true, so the system is stable (unstable) when $\Delta > 0$ $(\Delta < 0))$.

In fig.2 the values Δ versus the relative vertical coordinate z_0 (m) of the magnets and coils are plotted (coils 0,8×0,6m, magnets 1,0×0,6m, the distance between coils and magnets planes is 0,15m, $I^0 = 100$ kA, $v = 100$ m/sec). As is seen, the system is unstable for $z_0 < 0{,}45$ m $(\Delta < 0))$ and stable for $0{,}45\text{m} < z_0 < 0{,}53\text{m}$ $(\Delta > 0)$. In fig.3 the corresponding levitation force (H) in the magnet is shown.

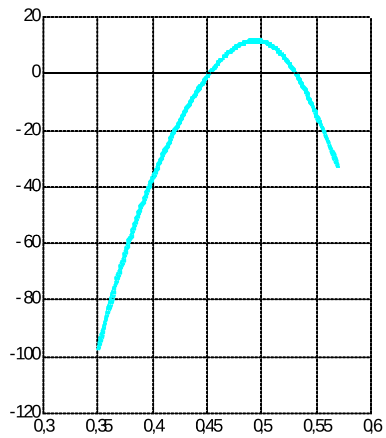

Fig.2

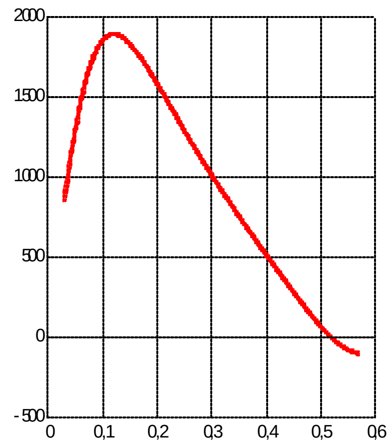

Fig.3

To evaluate an accuracy of the method, vehicle equations (16) were directly integrated along with equations (2) describing the currents in the coils. The oscillations $z(t), y(t)$ and $q(t)$ (curves 1,2 and 3 correspondingly) caused by small initial perturbations of a stationary motion are plotted in fig.4 ($z_0 = 0{,}44$ m) and fig.5 ($z_0 = 0{,}46$ m). As is seen, the oscillations amplitudes increase and decrease, respectively. Thus, the results of stability analysis, obtained by the proposed method, practically coincide with the numerical results.

Note that reliability of the proposed method was confirmed by an analysis of various levitation schemes in a wide range of their parameters. As a result, dependence of stability of motion on a type of levitation system and the number and arrangement of superconducting magnets was investigated.

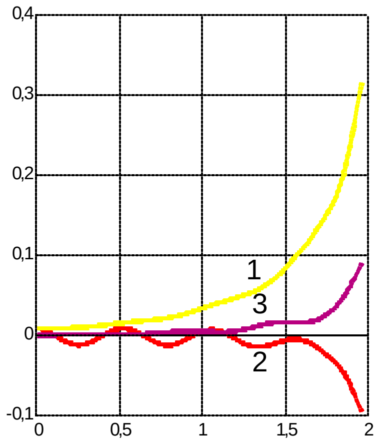

Fig.4

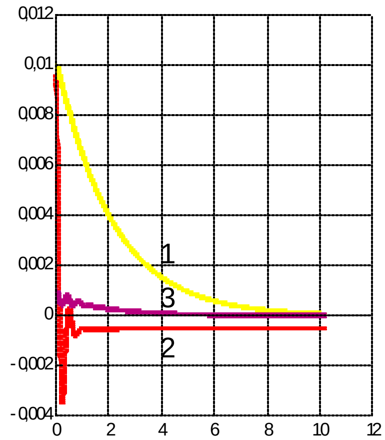

Fig.5

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4. Modified levitation and guidance system. As is seen from fig.2,3, in the stability interval ($0{,}45\,\mathrm{m} < z_0 < 0{,}53\,\mathrm{m}$) corresponding levitation forces are small. It appears that the same conclusion is true for a combined levitation and guidance system composed of two unit coils connected in reverse direction [2]. So, to provide the required levitation force, it is necessary to increase substantially the current in the magnets.

In this connection it is of interest to consider combined levitation and guidance systems which are not subject to the drawback indicated. To this end, some modified systems were worked out, as compared with the known ones, they provide larger levitation forces and ensure stability of the vehicle in a wide range of parameters.

One of the proposed systems is shown in fig. 6. The levitation force is produced, mainly, by the interaction of the elements 1-2 of the magnet and coil while the guidance force is due to the interaction of the elements 3-4 and 4-5.

In fig.7 the values Δ versus z_0 are plotted (coils 0,8×0,15×0,75m, magnets 1,0×0,6m, the distance between coils and magnets planes is 0,15m, $I^0 = 100\,\mathrm{kA}$, $v = 100$ m/sec). The corresponding levitation force is shown in fig.8. The oscillations $z(t), y(t)$ and $q(t)$ (curves 1,2 and 3 correspondingly) for $z_0 = 0{,}12\,\mathrm{m}$ are shown in fig. 9.

A comparison of fig.7,8 with fig.5,6 shows that for the proposed system, the levitation forces in its stability interval ($0,108\,\text{m} < z_0 < 0,145\,\text{m}$) are much larger than that in the conventional system.

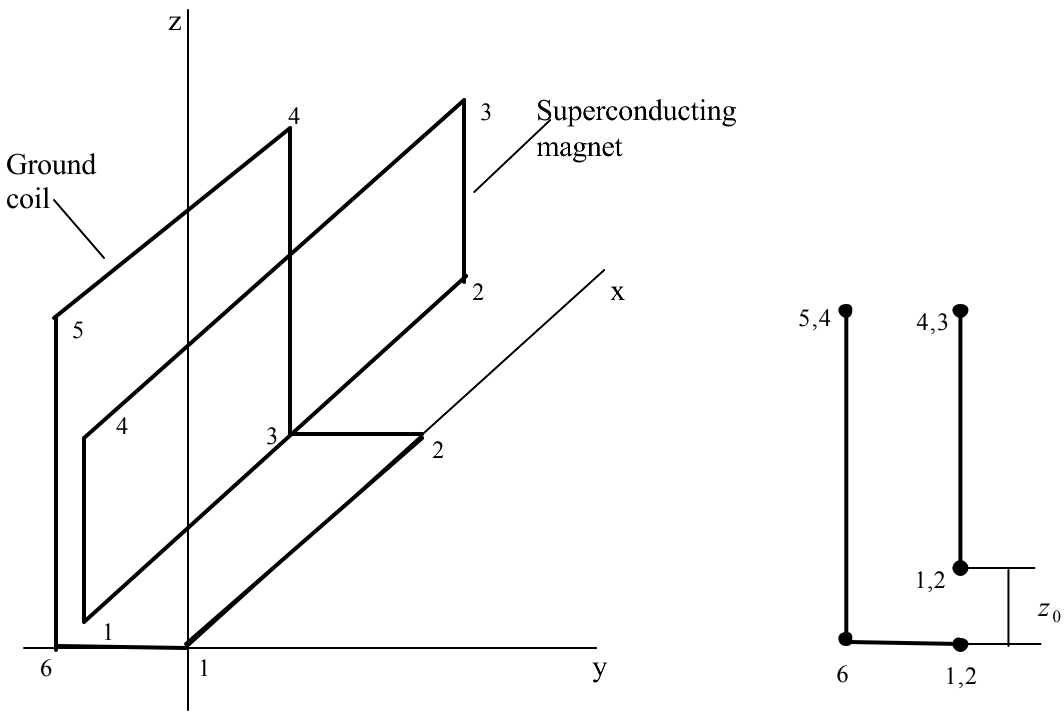


Fig.6

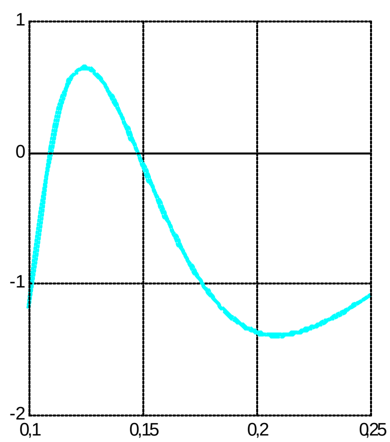


Fig.7

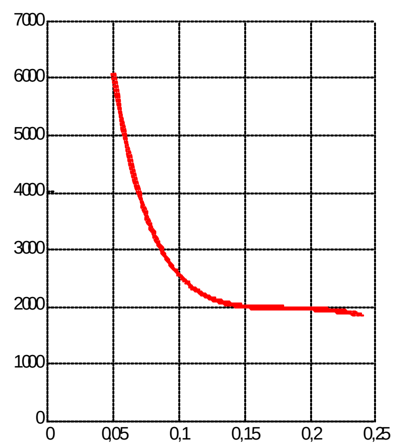


Fig.8

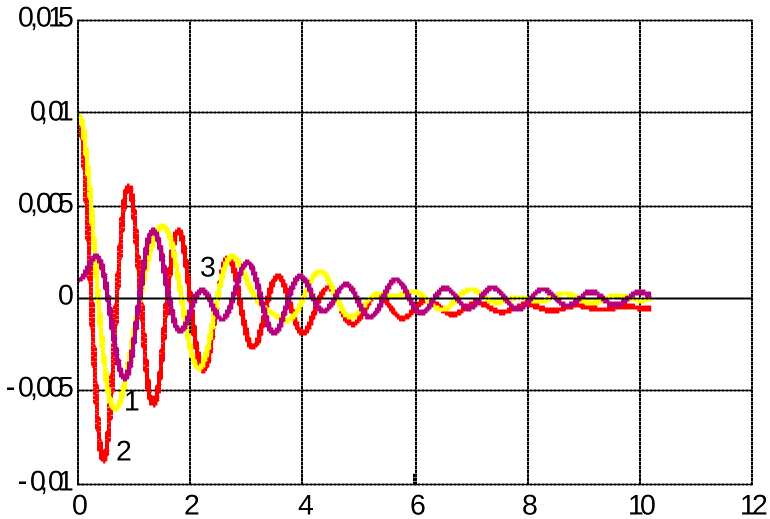


Fig.9

6. **Dynamics of carrying structure.** In this section dynamics of the carrying structure over the duration of the vehicle pass is discussed. This problem is interesting in its own right; moreover, oscillations of the carrying structure with attached ground coils affect the values of electrodynamical forces and, thereby, the vehicle dynamics.

Under stationary motion, the total levitation force may be represented in the form

$$F_L(t) = F_0 + F_c(t) + F_s(t) \tag{17}$$

where F_0 is its mean value, the periodic components $F_c(t) = F_c(t+T)$ and $F_s(t) = F_s(t+T_s)$ are due to periodicity of the ground coils and the carrying structure ($T = H/v$ and $T_s = L/v$ where L is a pitch of the carrying structure). An analysis shows that the corresponding vertical oscillations of the vehicle are small, so it may be assumed that the vertical co-ordinate of the vehicle $z(t) \equiv z_0$; however, the value z_0 should now be found by considering the compliance of the carrying structure.

In general, the vertical displacement $z(x,t)$ of the structure is described by the system of partial differential equations; its right-hand side $P(x,t)$ is the vertical components of electrodynamical forces applied to the attached coils. In its turn, the right-hand side of system (2) about the coil currents depends on relative positions of the coils and superconducting magnets and, therefore, on the co-ordinate $z(x,t)$ of the carrying structure. Thus, the equations describing the dynamics of the structure and the currents are coupled and should be solved simultaneously.

The functions $z(x,t)$ and $P(x,t)$ may be represented in the form

$$z(x,t)=\sum_{i=1}^{\infty} q_i(t) j_i(x), \qquad P(x,t)=\sum_{i=1}^{\infty} P_i(t) j_i(x) \tag{18}$$

where $j_i(x)$ are the oscillation modes of the structure and $q_i(t)$ are the corresponding generalised co-ordinates. The last are governed by the system

$$\ddot{q}_i + w_i^2 q_i = F_i(t), \ i = 1,2,... \tag{19}$$

where w_i are the oscillation frequencies of the structure and $F_i(t)$ are generalised forces. The systems (19) and (2) are solved simultaneously under zero initial conditions (i.e. for $t = 0$,

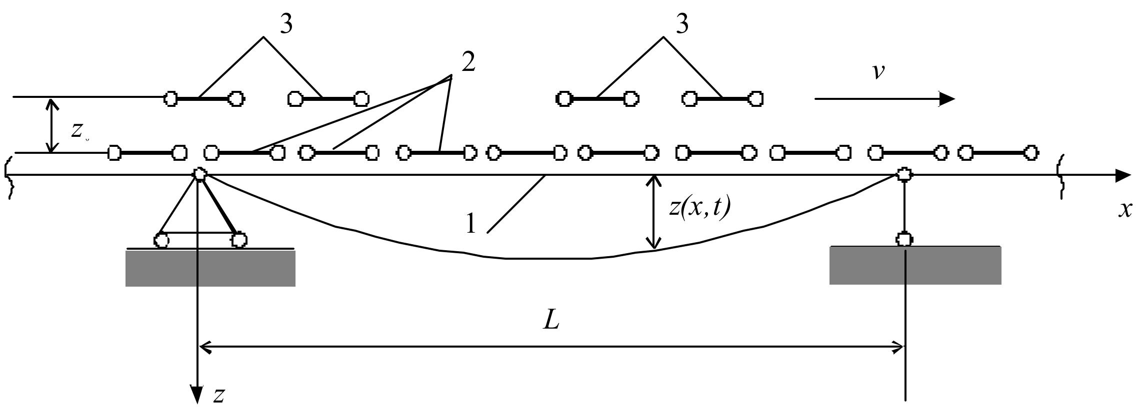
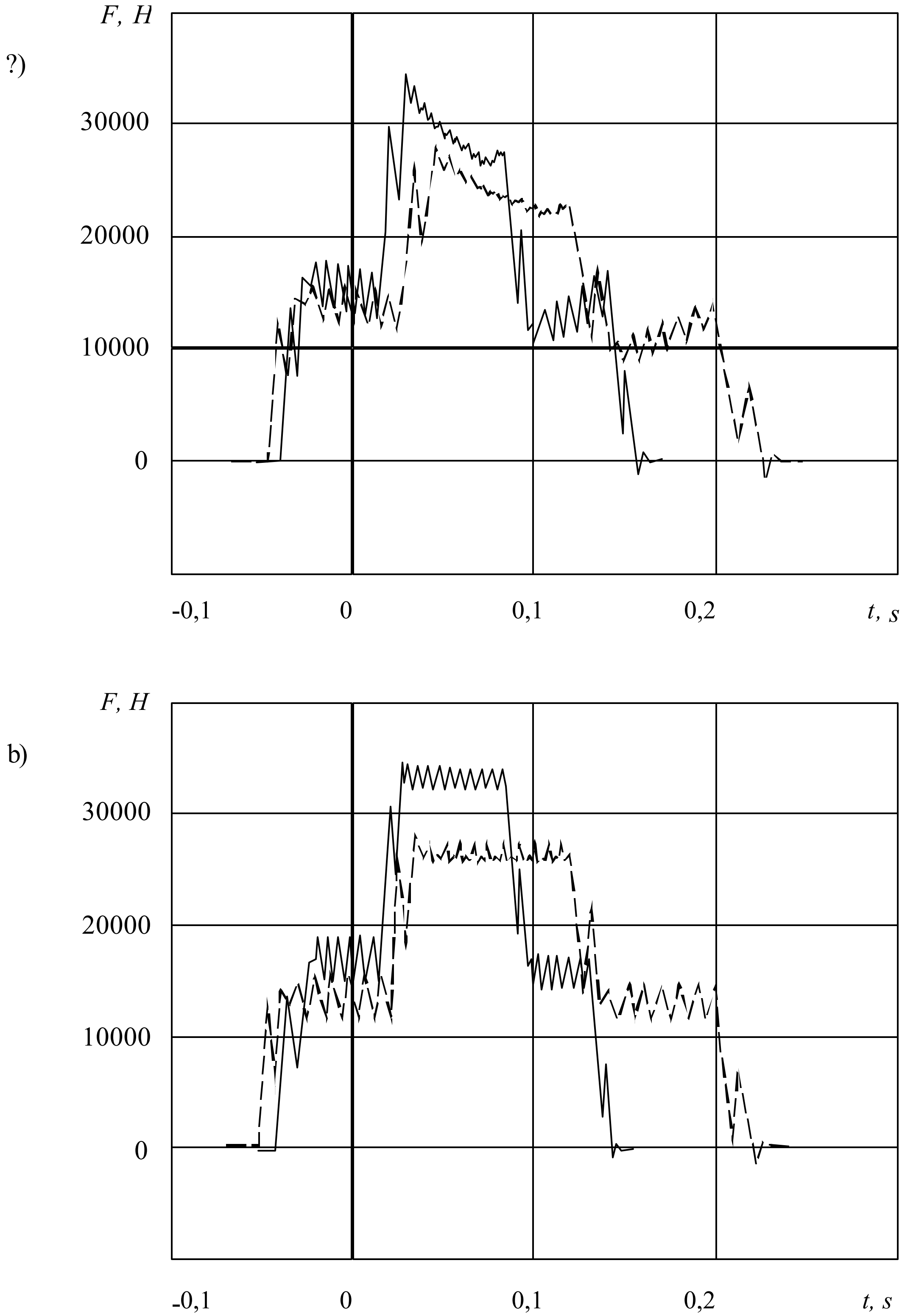


Fig.10

the vehicle is at a distance from the beam so that the currents in the coils are negligible).

To investigate an impact of compliance of a carrying structure to the values of levitation forces, calculations were conducted for the structure of pinned beams with the following set of data: length $L = 18$ m, distributed mass r = 300 N·sec^2/m^2, bending stiffness $EJ = 1{,}62 \cdot 10^2$ N·m^2 (fig.10, where 1-beam, 2-ground coils, 3-superconducting magnets). The data for superconducting coils are as follows: length×width = 1×0,5 ?, mmf=200 kA, the co-ordinates of the magnets on the vehicle are $x_1 = -4{,}25$ m, $x_2 = -3{,}0$ m, $x_3 = 3{,}0$ m and x_4 = 4,25 m. The pitch of the ground coils ? =0,9 ?, length×width = 0,8×0,3 m. Note that for the above data, the static displacement of the beam does not exceed L/1000.



?)
F, H
30000
20000
10000
0
-0,1
0
0,1
0,2
t, s
b)
F, H
30000
20000
10000
0
-0,1
0
0,1
0,2
t, s

Fig.11

In fig.11.a the total levitation force F_L produced by the ground coils lying on one beam is plotted (dotted and solid lines correspond to $v = 100$ m/sec and $v = 150$ m/sec). For fig 11.b, compliance of the structure is not taken into account (the beam is assumed to be absolutely rigid). Note that the values of F_L in the middle part of the plots are close to the total levitation force F_L, because here the components produced by the neighbouring beams are negligible. As is seen, in the beginning, levitation forces for the both cases are closely related, because at the moment the vehicle finds itself nearby the point of beam support where the displacements of the coils are small. While the vehicle passes the middle of the beam, the displacements and, therefore, he distances between the magnets and coils increase; as a result, the force F_L decreases. Such an impact increases in the speed of the vehicle motion.

Thus, the compliance of the structure implies the decrease of the mean value of the levitation force F_L (within 10% – 15% for the cases considered). Moreover, in $F_L(t)$ a noticeable component with the period $T_s = L/v$ appears.

References

[1] Voevodskii K. and Kochetkov V. Theory of superconducting magnet suspension: main results survey. *Cryogenics*, 12, Dec.,1981, pp.719-728

[2] Fujiwara S., Fujimoto T. Characteristics of the combinet levitation and guidance system using ground coils on the side wall of the guideway. Quart. of RTRL, 1989, V.30, 3, pp.123–126.

[3] Dzenzersky V.A., Khatchapuridze N.M., Manashkin A.L., Radchenko N.A.,.Zevin A.A. Dynamical analysis of electrodynamically suspended vehicles. In: *Urban Transport and the Environment for the 21st Century* (Ed. L.J.Sucharov). Computational Mechanics Publications, Southampton, 1995, pp.467-474.

[4] Dzenzersky V., Filonenko M., Gorsky O., Radchenko N.,.Zevin A., Zeldina E. Static and dynamic potential wells with application to high-speed ground transport. Stability analysis of electrodynamically levitayed vehicles. *2.International Conference-Electric Railway Systems*. VDE-VERLAG GMBH. Berlin 1999, pp.389-393.