CONTACT FREE STATIC STABLE EQUILIBRIUM IN THE GROUND AND SPACE SYSTEMS

S.S. Zub
Kharkiv State Educational University, Blukher St.2, Kharkiv, Ukraine,
370873 / 218840, stah_z@yahoo.com

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Abstract
The work considers applicability of the Earnshaw's Theorem to superconductive magnetic systems. Differences between static contact free magnetic equilibrium systems and systems of magnetic levitation are discussed. In the scope of quasi-stationary approximation for linear circuits we use electromechanical analogy to create version of Lagrange formalism, which describes the magnetic interaction and motion of the solid bodies system, that consists of superconductive and permanent magnets. This approach provides the complete description of dynamics of the considered systems, allows to derive unambiguously the potential energy of magnetic interaction expression and to carry out the detailed investigation of the stability problem. We disclose, analyze, and rigorously substantiate the idea of obtaining the contact free static equilibrium of a body in the system of superconductive and permanent magnets in the presence and in the absence of the gravity force. The simplest physical model is used to derive fully analytical proof of equilibrium stability.

Introduction

The question of contact free equilibrium of the bodies charmed for a long time human imagination and that was reflected in myths and legends of various civilizations. We do not find prototypes of this phenomenon in a surrounding world, instead of flying – the other dream of the human society. Nevertheless the phenomenon, produced by the force of imagination, has received the special name – levitation. The attempts of ancient naturalists to realize the «divine idea» by using miraculous properties of the ferromagnetic materials encountered a problem of stability equilibrium. This problem has remained the «bedrock» of the mentioned idea.

Mathematically this problem as the «impossibility theorem» was formulated by Earnshaw [1, p.79]. "Ether particles could have no stable equilibrium position if they interacted by any type or combination of $1/r^2$ force.

Earnshaw's theorem applies to a test particle, charged and/or a magnet, located at some position in free space with only divergence- and curl-free fields. No combination of electrostatic, magnetostatic, or static gravitational forces can create the three-dimensional potential well necessary for stable levitation in free space. The theorem also applies to any array of magnets or charges" [2, pp.4-5].

Earnshaw was the first who proved mathematically the impossibility of stability for the considered system of point charges.

The opinion, that Earnshaw's Theorem, proved for the point charges, could be generalized on all electrical and magnetic systems had being spread for a long time. Hundred years after Braunbeck has shown that expansion of «impossibility theorem» on the whole class of lengthy magnetic bodies was illegal [3]. He has shown that the system with diamagnetic bodies ($m<1$) does not fall under conditions of Earnshaw's Theorem.

The fact is that the proof of the theorem is built on the harmonicity of potential energy of interaction function. The well known fact is that harmonicity functions have no minimum in internal points of harmonicity area.

On the contrary, expression of the potential energy of interaction in system with diamagnetic has summand which are not a harmonicity functions. These nonlinear summands appear because of interaction of magnetic bodies with self-inducted polarized currents on the diamagnetic.
Let's notice, however, that inharmonicity of the trial body potential energy function, itself, as a function of its coordinates when other bodies of the system have the fixed position, does not mean the achievement of the stable equilibrium, but removes only the ban on such equilibrium existence.
Braunbeck has not only created the theory, but has also received for the first time experimentally magnetic levitation of a body, based on the effect of diamagnetic repulsion. He predicted the multiple amplification of this effect for the type I superconductors ($m = 0$).
Magnetic levitation above a superconductor as the ideal diamagnetic body was received for the first time in Kapitsa's test, which was carried out in the Institute of Physical Problems of the Academy of Sciences of the former USSR [4, p.304].
In the existent approach to magnetic levitation a gravity force is considered as a harmful factor, which is necessary to overcome, and the magnetic force is considered as a tool of its overcoming.
In traditional systems the magnetic force is a counterbalance of the gravity in some disposition, because it changes with a distance on amount, when the gravity keeps permanent. It is paradoxically, but the gravity, which is the obstacle to surmount, at the same time is needed to reach equilibrium in such systems.
On the one hand, magnetic levitation gives the answer to a question of the contact free equilibrium possibility; on the other hand the question of equilibrium, got only by magnetic forces, remains open.
It is clear that in traditional systems when the gravity force is absent the necessary condition of stability cannot be realized ($\sum F \neq 0$).
In connection with aforesaid, it is evident, that the question of possibility of the contact free static stable equilibrium existence in the system of the bodies, interacting only by magnetic forces, is very important.
If there is a configuration of magnetic bodies in which contact free static stable equilibrium of the body or the bodies, interacting with other bodies of the system only by magnetic forces, is realized, then such configuration can be named the magnetic potential well (MPW).
First of all we shall note that any of the known magnetic levitation systems does not answer the formulated requirements for the MPW.
It should be said in addition that our statement of the MPW problem differs from one of the Kozoriz (see [5,6] and MAGLEV 2002 Proceedings). Instead of the magnetic potential energy minimum with respect to one linear coordinate we consider the minimum with respect to all coordinates (but without the cyclic ones).
For the MPW existence such conditions as an existence of zero force in some point, which is sufficiently far from the other bodies of the system, firstly, and the minimum of magnetic potential energy of interaction, realized just in this point, secondly, are necessary.
Let's notice, that in the point of equilibrium the MPW existence requires minimum of the potential energy of interaction on all generalized coordinates of a body. As it is known for a free solid body there are six of them.
As it has been marked, to obtain the equilibrium, based only on the interaction by magnetic forces, in the system, the force must change a sign on the interval of interaction. Kozoriz discovered this kind of force type in 1975. Therefore we will name it the Kozoriz's effect. It was shown experimentally, that if some conditions are met, the existence of minimum of the magnetic potential energy of interaction as a function of distance between two magnetic elements, which are the superconductive coils, is possible [5,6].
We proved theoretically, that a fully analogous effect arises also in the system, consisting of ideally conducting coils and a permanent magnet [7, pp.95-100].
So, one can try to build a system, in which free of contact equilibrium can be reached only by magnetic forces. But the question of such stable equilibrium stays. The problem is to provide stable equilibrium

on all generalized coordinates of the body, but not only as a function of distance between two magnetic elements.
The most common approach to investigate the electromagnetic interaction of the bodies is the field-theoretic approach based on using differential system equations in partial derivatives. In a case of the system stability investigation in the systems of many bodies this approach results into the unjustified difficulties, because the electromagnetic field is a dynamic system with infinite number of freedom degrees.
The essential simplification of our task is possible to be reached by using quasi-stationary approximation to describe the electromagnetic field. Thus we come to the description of the bodies interaction by the system of ordinary differential equations, to the dynamic system with the finite degrees of freedom. Such approach allows using the powerful mathematical apparatus of the Stability Theory.
Using of quasi-stationary approximation is certainly admitted, as the considered task about equilibrium is, in fact, quasistatic. Just such approach was used in the work [8] for the investigation of the magnetic levitation stability based on the Kozoriz's effect.
It is known that the equations for quasi-stationary circuits can be considered from the point of view of the Lagrange approach to dynamics [3; 4, pp. 783-787]. A class of models, selected for MPW research, [5-8] includes ideally conductive coils and permanent magnets. However such models have the features, which are badly taken into account by available version of Lagrange formalism.
First of all the constancy of permanent magnet moment at its modeling by a current circuit guesses constancy of a current in a circuit.
In a context of an electromechanical analogy the requirement of constancy of a current is naturally described by some constraint reaction. To use the Lagrange approach for describing of such system it is necessary to take into account the constraints of this type for exception of dependent variables.
Besides, the presence of ideally conductive circuits in the system leads to the cyclic recurrence of the corresponding electrical coordinates. Nevertheless, the velocities (currents), which are corresponding to the cyclical coordinates, depend on time and obviously presents on the base formulas of known above Lagrange formalism [10, pp. 783-787].
The basic requirement to the developed below Lagrange formalism is the coincidence of Lagrange equations of motion and the known equations of quasi-stationary circuits [10 pp. 783-787; 11 pp. 278-308] in application to the viewed class of model bodies.

Lagrange Formalism

The common approach to the deduction of the equations of Lagrange in independent coordinates [12, pp.47-66] guesses, firstly, the expressing of a kinetic energy through the independent variables, secondly, the projection of forces on a space, restricted by constraints (the corresponding projections Q_i termed as generalized forces). Thus the equations of Lagrange look like:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=Q_i . \qquad (1)$$

Let's consider the system of n circuits, which we have numbered by letters $i, j, k, l = 1,..n$. Among them ν of circuits are ideally conductive, and $n-\nu$ have the specified value of current. Circuits of the first type we shall number by small Greek letters $\alpha, \beta, \gamma = 1,..\nu$, and the second type ones by the first small characters of the Latin alphabet $a, b, c = (\nu+1),..n$.
Let's designate by q_α a charge that is flowing through the fixed section of an ideally conductive circuit, i.e. $q_\alpha = \int I_\alpha dt$, and q_a is a corresponding variable for a circuit with a given current, i.e. $q_a = \int I_a dt = I_a t$.

Let's suppose, that the mechanical motion of coils (circuits) without the account of magnetic interaction, i.e. at zero currents, is already described by a set of independent variables of Lagrange X_I, where the capital Latin letters transverse values $I, J, K, L = 1, \dots M$.
For the selected variable complete kinetic energy of the system looks like:

$$T = T_{magn} + T_{mech}, \tag{2}$$

where T_{mech} is the kinetic energy of a mechanical motion of coils that dependents on X and dX/dt, and T_{magn} is the electrical component of energy. Requirements of current constancy in coils we shall consider as holonomic constraints dependent on time:

$$q_a = I_a \cdot t . \tag{3}$$

We consider that $T_{mech}(X, \dot{X})$ is already expressed in independent Lagrange variable, therefore all further transformations will affect only T_{magn}. Let's write energy T_{magn} in independent variables X, q_a , dq_a/dt, using constraints (3):

$$T_{magn} = \frac{1}{2}\sum_{a,b} L_{ab}\dot{q}_a\dot{q}_b + \sum_{a,a} L_{aa}\dot{q}_a I_a + \frac{1}{2}\sum_{a,b} L_{ab} I_a I_b . \tag{4}$$

Accordingly to the division of circuits into two types, the matrix of inductance L_{ik} is divided into 4 blocks. Thus L_{ab} - square symmetrical matrix of inductance of ideally conductive circuits, L_{ab} is a square symmetrical matrix of inductance of circuits with a specified current, and L_{aa} is the rectangular matrix of inductance describing interaction of circuits of a different type and is corresponding to the right upper block of an initial matrix of inductance.
The quantities I_a are not the dynamic variable, and represent fixed numbers - parameters of constraints (3).
The expression (4) in a general case is true for the linear conductors, the thickness of which we can neglect. But for the superconductors this restriction, as it is shown in [11], can be taken off.
The equations of Lagrange in the shape (1) also are broken into two groups, as the coordinates q_a are eliminated by constraints.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_a}\right) - \frac{\partial T}{\partial q_a} = Q_a , \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}_I}\right) - \frac{\partial T}{\partial X_I} = Q_I . \tag{6}$$

In the equation (5) generalized forces Q_a coincide the initial force effective in a -circuit, because the projection of independent variables on a subspace does not affect a -components. At the same time in the equations of the Kirchhoff for the ideally conductive circuits we assume the resistance and EMF equal to zero, therefore in the equation (5) we have $Q_a = 0$.
In the equation (6) generalized forces Q_I coincide initial force, because the mechanical variables are not affected by projection. As at zero currents the mechanical force is equal to a derivative with the opposite sing from potential Π of a not electromagnetic origin on mechanical coordinate, so $Q_I = -d\Pi/dX_I$.
The force of an electromagnetic origin is correctly described by the term $-dT_{magn}/dX_I$ of the equation (6) in complete conformity with the formula for the force from the theory of quasi-stationary circuits.
Hence, the final Lagrange function of our system in the variables X_I, dX_I/dt , q_a and dq_a/dt looks like:

$$T_{mech} - \Pi + T_{magn}, \tag{7}$$

and the variables q_a do not enter obviously this expression and consequently are cyclical coordinates. To eliminate the variables dq_a/dt, we shall take the advantage of the Raus method [12, pp. 270-277].

Let's note, that mechanical variable X_I and function T_{mech} play a passive role in exception of dq_a / dt, and to complete the exception of cyclical coordinates in our system, thus, it is enough to apply the Raus method to an electrical part of the energy of the system.

Let's find the expression for a kinetic energy in Raus variables p_a. For this purpose we shall express all $\dot{q}_a$ through p_a, using (4):

$$p_a = \frac{\partial T_{magn}}{\partial \dot{q}_a} = \sum_b L_{ab}\dot{q}_b + \sum_b L_{ab} I_b \,. \tag{8}$$

Let's consider, that $\det(L_{ab}) \neq 0$, then from (8) we find:

$$\dot{q}_a = \sum_b L^{-1}_{ab}\left(p_b - \sum_b L_{bb} I_b\right), \tag{9}$$

where L^{-1}_{ab} is an inverse matrix for a matrix L_{ab}.

That case, when there are no superconductive circuits $n = 0$ in the system, and, hence, such elements as L_{ab} are absent in an initial matrix of inductances L_{ik}, will be considered below.

Substituting expression (9) in the formula (4), we shall receive expression T_{magn} in Raus variables:

$$T_{magn} = \frac{1}{2}\sum_{a,b} L^{-1}_{ab} p_a p_b + \frac{1}{2}\sum_{a,b} L^{*}_{ab} I_a I_b \,, \tag{10}$$

where $L^{*}_{ab} = \left(L_{ab} - \sum_{a,b} L^{-1}_{ab} L_{aa} L_{bb}\right)$.

Let's calculate Raus function:

$$R = p_a \dot{q}_a - L = \frac{1}{2}\sum_{a,b} L^{-1}_{ab} p_a p_b - \frac{1}{2}\sum_{a,b} L^{*}_{ab} I_a I_b - \sum_{a,b} \Gamma_{ab} p_a I_b + \Pi - T_{mech}, \tag{11}$$

where $\Gamma_{ab} = \sum_b L^{-1}_{ab} L_{bb}$.

According to the equality (11), the function $(-R)$ (which plays a role of a Lagrange function for mechanical coordinates) is equal

$$T_{mech} - V, \tag{12}$$

where V is a generalized potential, defined by equality

$$V = \frac{1}{2}\sum_{a,b} L^{-1}_{ab} p_a p_b - \frac{1}{2}\sum_{a,b} L^{*}_{ab} I_a I_b - \sum_{a,b} \Gamma_{ab} p_a I_b + \Pi \,. \tag{13}$$

Let's consider now any motion of initial system. In this motion

$$p_a = const = c_a \,, \tag{14}$$

and the change of mechanical coordinates $X_I = X_I(t)$ can be defined from the differential equations:

$$\frac{d}{dt}\frac{\partial R}{\partial \dot{X}_I} - \frac{\partial R}{\partial X_I} = 0, \tag{15}$$

in which p_a is necessary to change on c_a everywhere. These equations are the equations of Lagrange (with a Lagrange function $(-R) = T_{mech} - V$) for some auxiliary system with M-degrees of freedom that has a kinetic energy as a mechanical kinetic energy T_{mech} and generalized power potential

$$V = \frac{1}{2}\sum_{a,b} L^{-1}_{ab} c_a c_b - \frac{1}{2}\sum_{a,b} L^{*}_{ab} I_a I_b - \sum_{a,b} \Gamma_{ab} c_a I_b + \Pi(t, X_I). \tag{16}$$

From expression (16) it is visible, that generalized potential depends only on mechanical coordinates, as all other factors are of stationary values in virtue of the imposed constraints I_α, or as integrals of a motion $(p_\alpha = c_\alpha)$. Obtained auxiliary system we shall term as a reduced system. Thus, the change of mechanical coordinates X_I determines a motion of reduced system with M degrees of freedom, with a kinetic energy T_{mech} and generalized potential V.

At the corresponding motions of the initial and the reduced system their total energies are equal. After defining of mechanical coordinates X_I of the reduced system from the equations (15), cyclical coordinates of the initial system are determined under the formula:

$$q_\alpha = \int \frac{\partial R}{\partial c_\alpha} dt + c'_\alpha . \tag{17}$$

Hertz has shown, that the potential energy of a conservative system can be always viewed, as a kinetic energy of the latent motions [12, pp. 281-285]. The motions, which make change only in cyclical coordinates, are meant by the latent motions. It is possible to term the concept of the kinetic origin of the potential energy (and consequently, the kinetic origin of the forces) as the *Hertz principle*.

In our case it is visible, that all electrical part of a kinetic energy of the system T_{magn} has transferred in a potential energy of the reduced system with smaller number of degrees of freedom. The role of the latent motions, obviously, is executed by electrical processes. Not going deeply in a philosophical aspect of this concept, we shall mark, that the formally magnetic interaction in our system corresponds completely with Hertz principle.

In works [8,13] on dynamics of electromechanical systems there is an ambiguity of interpretation of mechanical sense of magnetic field energy as kinetic, or potential. In the offered approach this question is solved in complete conformity with a Hertz principle, namely: the magnetic energy by its form is kinetic for the initial system, and potential for the reduced one.

Last deduction has the special sense in research of mechanical stability of the systems, as the "theorem of the energy minimum" deals with the potential energy. The obtained formalism provides a universal method of finding of the potential energy for the systems of a viewed type.

Let's consider, at last, the case, when there are no superconductive circuits $(n = 0)$ in the system. In this case after the constraints accounting, the system contains only the mechanical variables. In combined equations (5,6) remains only last. It can be written as follows:

$$\frac{d}{dt}\left(\frac{\partial T_{mech}}{\partial \dot{X}_I}\right) - \frac{\partial T_{mech}}{\partial X_I} = -\frac{\partial}{\partial X_I}\left(\Pi - T_{magn}\right). \tag{18}$$

From this equation of Lagrange it appears, that the role of kinetic energy of the system, as well as it is in a general case, is played by mechanical kinetic energy T_{mech}, and the potential energy of a not electromagnetic nature gets the addition equal to the energy of direct currents with the minus sign. Thus, the Lagrange function of the system after the account of constraints looks like:

$$L = T_{mech} - \left(\Pi - T_{magn}\right) = L_{mech} - V , \tag{19}$$

where L_{mech} is Lagrange function of a mechanical system with zero currents, and $V = -T_{magn}$ is a potential energy of magnetic interaction.

Comparing (19) with (12), we see, that the obtained energy of direct currents enters also the common expression (13), but it takes into account as well an indirect interaction of direct currents through the ideally conductive circuits (see the expression for $L^*_{\alpha\beta}$).

Thus, the ideally conductive circuit is an adequate model of the superconductive coil [8, pp.19], and the circuit (coil) with the specified direct current is an adequate model of a permanent magnet, if the field is considered outside of the magnet, and the change of its magnetization under activity of the eigen-fields of the investigated system can be neglected [14, pp.269].

MPW existence proof

One of the most interesting problems solved with the help of the given Lagrange formalism is the problem of MPW. In spite of the available proofs of the MPW existence [15-17], completely analytical expression in elementary functions for the MPW has not been given till this time.

Physical Model

Let us consider the system, consisting of two magneto-interacting bodies, which are contained in the homogeneous external permanent magnetic field. First of them is a system of three mechanically fastened and galvanically free concentric superconductive coils, situated in the perpendicular planes. Second is the little current coil with the permanent moment (permanent magnet model). The first body specifies in fact the coordinates system normals to the coils planes are the orts of the system), and the external homogeneous magnetic field B_0 is directed along z axis.

Three inductances (L_{11}, L_{22}, L_{33}), located in a point of origin, represent superconductive rings, and the fourth ring (free) with the direct current simulates small permanent magnet modeling.

In a coil, which is perpendicular to the *z* axis, the magnetic flows are "frozen" $\Psi_3 \neq 0$. Other two coils (stabilizers) carrying the zero "frozen" magnetic flows $\Psi_1 = \Psi_2 = 0$. The fourth element of the system (the permanent magnet) is a free body and is characterized by its magnetic moment. So, the stabilizers are numbered by the indexes 1 and 2, basic coil by the index 3, and the free body by the index 4.

For obtaining completely analytical solution we shall view the expressions for the mutual inductance, which enter the formula for energy, in dipole approach [18, pp.8-11]. The dipole approach for the mutual inductances will be valid, when the distances between elements are much greater than the sizes. As we show below it is always possible to be reached by the relevant choice of parameters (B_0, Ψ_3). As to mutual inductances of the first three superconductive rings with conterminous centers, they are equal to zero, as they are perpendicular.

Mathematical Model

The expression of a potential energy through inductances and flows (16) looks like:

$$V = \frac{1}{2}\left(\frac{(I_4 M_1)^2}{L_{11}} + \frac{(I_4 M_2)^2}{L_{22}} + \frac{(I_4 M_3 - \Psi_3 + \Phi_3)^2}{L_{33}} - I_4^2 L_{44}\right) - m n_z B_0 , \tag{20}$$

where I_4, L_{44} is a current and inductance of the 4-th coil with direct current (magnetic dipole); B_0 is a homogeneous magnetic field directed along an axis z; Ψ_3 is a complete flow through the 3-rd circuit; Φ_3 is a flow of an exterior magnetic field through the third circuit; $m = p y_0^2 I_4$ is a magnetic moment of the 4-th coil.

In a Cartesian coordinates, connected with the concentric mutually orthogonal rings, the mutual inductances of superconductive rings and a free body (permanent magnet) in the dipole approach looks like [16]:

$$\begin{cases} M_1 = \dfrac{m_0 p \cdot x_{01}^2 y_0^2}{4} \dfrac{\left(3n_y xy + 3n_z xz + n_x\left(2x^2 - y^2 - z^2\right)\right)}{\left(x^2 + y^2 + z^2\right)^{5/2}}; \\ M_2 = \dfrac{m_0 p \cdot x_{02}^2 y_0^2}{4} \dfrac{\left(3n_x xy + 3n_z yz + n_y\left(2y^2 - x^2 - z^2\right)\right)}{\left(x^2 + y^2 + z^2\right)^{5/2}}; \\ M_3 = \dfrac{m_0 p \cdot x_{01}^2 y_0^2}{4} \dfrac{\left(3n_x xz + 3n_y yz + n_z\left(2z^2 - x^2 - y^2\right)\right)}{\left(x^2 + y^2 + z^2\right)^{5/2}}, \end{cases} \tag{21}$$

where x_{01} - radius of the 1-st superconductive ring (stabilizer); x_{02} - radius of the 2 superconductive ring (stabilizer); x_{03} - radius of the 3-rd superconductive ring (base); y_0 - radius of a current coil simulating a permanent magnet; m_0 - magnetic penetrability of vacuum; x, y, z - coordinates of a magnet. Thus, variables x, y, z, n_x, n_y - are the complete set of the generalized coordinates of a magnetic dipole, and $n_z = \sqrt{1-(n_x^2 + n_y^2)}$. Expression of a potential energy in elementary functions through generalized coordinates of a free body is obtained by substitution (21) in a common expression (20). Further analytical calculation has been carried out with a help of the standard functions of the famous mathematical system of Computer Algebra Maple V (Taylor series expansion, positive definiteness of the quadric form testing and etc.)

A necessary condition of the potential energy minimum by generalized coordinates corresponds the vanishing of the first derivative of the potential energy of the system. Obviously, that to a coaxial position of bodies there will correspond such position of a free body, at which $x = y = 0$ and $n_x = n_y = 0$ in a point $z = z_0$.

$$z_0 = \frac{1}{2}(2x_{03})^{2/3}\left(\frac{m_0 m}{\Psi_3 - \Phi_3}\right)^{1/3}, \tag{22}$$

where z_0 is determined from the condition of a vanishing of interaction force between a free magnetic dipole and the system of superconductive coils. Obviously, that by a choice of system parameters it is possible always to obtain justice of dipole approach.

To define the sufficient conditions of the potential energy minimum in the point $x = y = 0$, $n_x = n_y = 0$ and $z = z_0$, it is enough to have 2-th order expansion in series of the potential energy in the given point.

The analysis of coefficients of an obtained quadratic form gives following requirements of the positive definiteness:

$$\frac{9}{32}\frac{m_0^2 x_{01}^4 m^2}{z_0^8 L_{11}} > 0 \quad and \quad \frac{81}{1024}\frac{m_0^4 x_{01}^4 m^2 x_{02}^4}{z_0^{16} L_{11} L_{22}} > 0,$$
$$\frac{81}{2048}\frac{m_0^4 x_{01}^4 m^5 x_{02}^4 B_0}{z_0^{16} L_{11} L_{22}} > 0 \quad and \quad \frac{81}{4096}\frac{m_0^4 x_{01}^4 m^6 x_{02}^4 B_0^2}{z_0^{16} L_{11} L_{22}} > 0 \tag{23}$$

and, 1, 2 and 4 conditions are executed automatically, and 3 is reduced to the fulfillment of an inequality $mB_0 > 0$ (sufficient condition of stability). Similar condition for the first time was obtained in the work [15] for the circuits of the ring shape.

Thus it is shown, that, when necessary and sufficient conditions are fulfilled, there is a minimum of potential energy of the system or the MPW.

Magnetic Levitation Based on the MPW

Let us return to the task of magnetic levitation. *Does it mean that, if the MPW system provides equilibrium without gravity, that it won't provide it by the gravity presence?*

Let us consider the MPW behavior in a field of the gravity force. Gravity force independently from the magnetic force acts on the «free body», displacing it in a direction of ground, and the magnetic force tries to return it to the MPW point. If near the MPW-point the magnetic force surpasses the gravity force in magnitude, then the body will be counterbalanced on some distance from the MPW-point, where the appropriate components of the forces will be equal by quantity and opposite by a sign.

Physically it means that, if the MPW «depth» is more than a change of potential energy of the gravity force, then the gravity force plays a role of disturbance and dose not destroy stability of equilibrium. Using of superconductive coils, allows receiving the high values of the magnetic force and its quantity variation in a wide range. It is easy to show that the mentioned above conditions can be easily executed for the considered configuration.

So the MPW systems are the new approach for the development of the ground systems magnetic levitation.

Resume

For the strict consideration of dynamics of observed above systems and investigation of the equilibrium condition in them we offered the adapted variant of Lagrange formalism, which allows to describe electromechanical interaction in the systems, composed of the superconductive coils and the permanent magnets.
Such an approach gives an opportunity, being based on the classical equations for the quasi-stationary chains, to obtain strictly mathematically the expressions for the magnetic potential energy and to investigate minimum of the function.
The developed mathematical apparatus and software, based on it, can be applied for the investigation and designing of systems of that specified type.
The demonstration model used dipole approximation, special geometry and external homogeneous magnetic field. All this, certainly, simplifies the task and allows receiving the decision in the analytical form. However it does not mean that the MPW exists only in such specific systems. On contrary, the carried research have revealed a very wide class of the systems, in which the MPW exists and no of the mentioned above simplifications is needed, but in those cases one fails to obtain the proof only by analytical calculations.
Stability in the system, based on the MPW, is reached on the physical level, that probably more preferable then using of systems, based on the automatic control of the magnetic force.
To all appearance, now there are no reasonable alternatives to the using of the MPW for the matter of providing contact free equilibrium in the space systems.
Using of the MPW for the magnetic levitation in ground transport systems in future can become one of the alternatives to the traditional systems.

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References

1. Edward M. Purcell. Electricity and Magnetism. Berkeley Physics Course. Vol. 2, Nauka Publisher. 1971.
2. M.D. Simon, L.O. Heflinger, A.K. Geim. Diamagnetically stabilized magnet levitation. October 30, 2000. Typescript.
3. Braunbeck W., Zeitschr. Fur Phys., 112, 7/8, 753, 1939.
4. Arkadjev V.K. Selected Transactions. Moscow- Leningrad, Press of the Ac. Of Sc. Of USSR, 331p, 1961.
5. Kozoriz V.V. et al, Proceed. Of the Ac. Of Sc. Of Ukrainian SSR, A, 3, 248, 1976.
6. Kozoriz V.V., Tcheborin O.G. Proceed. Of the Ac. Of Sc. Of Ukrainian SSR, A, 1, 80, 1977.
7. Zub S.S., Demutskiy V.P., V.M. Rushkovan. Raus' Method and Hertz' Principle for Electromechanical Systems Consisting of Permanent Magnets and Superconductive Coils and their applying for Magnetic Levitation' Researches // Vistnyk Kharkiv University, N421, -pp.95-100, 1998.
8. Michalevich V.S., Kozoriz V.V., et al. "Magnetic Potential Well" as Stabilizing Effect for Superconductive Dynamic Systems. Kyiv, Scientific Thought Press, 336p, 1991.
9. Maxwell J. C. A Treatise on Electricity and Magnetism. Oxford: Clarendon press, 1873, v.1-2.
10. Levich V.G. Theoretical Physics Course. Vol. 1, Moscow, Nauka Printing Office, 910p, 1969.
11. Landau L.D., Lifshits E.M. Electrodynamics of Continuous Media. Moscow, State Phys. Math. Press, 532p, 1959.

12. Gantmakher F.R. Lecture on Analytical Mechanics. Moscow. Nauka Printing Office. 1966.

13. White D.C., Wudson H.H. Electromechanical Energy Transformation. Moscow- Leningrad, Energy Press, 528p, 1964.

14. Matveev A.N. Electricity and Magnetism. Moscow, Higher School Printing Office, 463p, 1983.

15. Zub S.S., Demutskiy V.P., V.M. Rushkovan. Mathematical Simulating of the Magnet Potential Well in the System of Superconductive Coils and Permanent Magnets // Vistnyk Kharkiv University, N438, -pp.43-45, 1999.

16. Zub S.S., Demutskiy V.P., V.M. Rushkovan. Analysis of Static Stable Equilibrium of Free Superconductive Coil in the System of Three Mechanically Coupled Superconductive Coils // Vistnyk Kharkiv University, N443, -pp.34-39, 1999.

17. Zub S.S. Spatial Magnetic Potential Well and MagLev in the System of Magnetic Dipole and Superconductive Sphere // Vistnyk Kharkiv University, N453, -pp.48-53, 1999.

18. Stas Zub, Vasyl Rashkovan, Irina Ponomaryova. Basic Magnetic Properties of Arbitrarily Disposed Current Rings. // Proceed. Of 7th Conference of Electrical Engineer, Mexico, 5-7 September, -pp.8-11, 2001.