A New Approach for Controlling Active Magnetic Levitation Systems

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Topic 10.2

Abstract - Control of active magnetic bearings and EMS systems is typically approached using PID algorithms. The equation of motion governing EMS systems has no analytic solution, but a close solution for short time intervals is possible. This equation can be used direct the motion of the suspended system.

Keywords : Magnetic bearing, control, analytic solution

Background

Control of active levitation systems has been approached using various classical control strategies [1][2],[3]. Self sensing levitation appears to be the prominent direction of many such systems in the future [4]. Knowing an analytic approximation to the equation of motion governing these systems would add yet another option to the type of control employed, relaxing to some extent the response time of the processors affecting current. The goal of this research is to present an analytic solution to the equations of motion for electromagnetic levitation systems (EMS) and active magnetic bearings (AMB). The solution will have validity fo only a limited time, but will allow one to dictate the current required to specify the movement of the levitated member with time. Thus, not only would it allow a targeted gap to be maintained, but dictate how the suspended member would reach that gap, and required to move the suspended member to a predetermined gap, or to any other gap.

Setup

Consider a C channel EMS Maglev guideway as depicted in Figure 2 with the electromagnet displaced a distance x.. The inductance [5] is a function of both x and z as

$$L = \frac{\mu_0 N^2 D}{2}\left\{\frac{w-x}{z} + \frac{4}{\pi}\ln\left(1+\frac{\pi x}{4z}\right)\right\} + C_{leakage}. \tag{1}$$

Boundary element analyses show this expression to be very accurate. Shown in Figure 1 is a comparison of the analytical expression to the numerical prediction. The constant $C_{leakage}$ is a constant added to the analytic expression to account for leakage at the ends of the winding, and it will vary depending on the magnet design.

The coenergy of the system is

$$W' = \frac{\mu_0 N^2 I^2 D}{4}\left\{\frac{w-x}{z} + \frac{4}{\pi}\ln\left(1+\frac{\pi x}{4z}\right)\right\}. \tag{2}$$

The lift and guidance forces F_z and F_x are

$$F_z = \frac{\mu_0 N^2 I^2 D}{4}\left\{-\frac{w-x}{z^2} - \frac{4x}{4z^2+\pi zx}\right\}. \tag{3}$$

$$F_x = \frac{\mu_0 N^2 I^2 D}{4}\left\{-\frac{1}{z} + \frac{4}{4z+\pi x}\right\}. \tag{4}$$

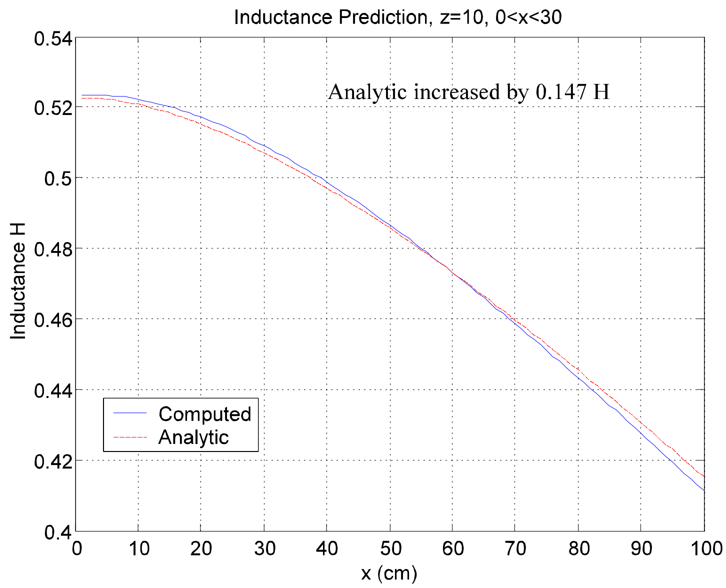

Figure 1 The analytic expression for inductance is quite good when leakage inductance is included.

Analytic Solution Approach

Consider only one dimension and one magnet for the moment. If the magnet and bogie have a mass m, then the equation of motion is simply

$$m\frac{d^2z}{dt^2} = F_z . \tag{5}$$

One can perform a Taylor expansion on z about time t=0, and approximate z at any time t>0 as

$$z = z_0 + v_z*t + a_2*t^2 + a_3*t^3 + a_4*t^4 + a_5*t^5 + a_6*t^6 + \ldots . \tag{6}$$

The magnet is assumed to have a velocity v_z at t=0. The unknown constants a_2, ..., a_6 can be determined by substituting (6) into (5), and collecting like powers in t. Consider first only movement in the z direction.

The governing equation is

Depth D

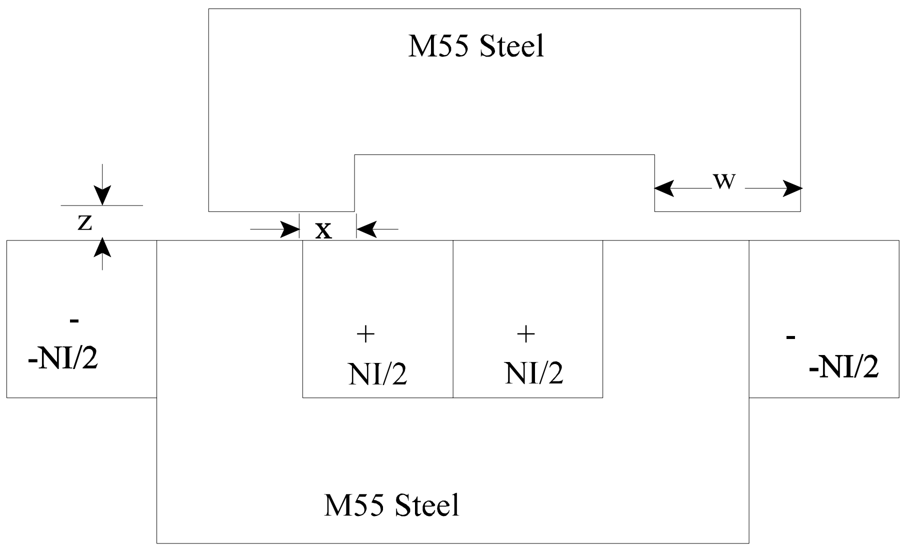

Figure 2 Displacement of the magnet yields natural restoring forces.

$$m\,\frac{d^2z}{dt^2} = mg - F_z,$$
$$\textit{where} \qquad (7)$$
$$\frac{d^2z}{dt^2} = 2\,a_2 + 6a_3t + 12\,a_4t^2 + 20a_5t^3 + 30\,a_6t^4 + \ldots$$

Define the term

$$\beta = \frac{\mu_0 N^2 I^2 D}{4m}\ . \qquad (8)$$

Inserting (3) into (7) yields the result

$$z^2\,(2a_2 + 6a_3t + 12a_4t^2 + \ldots - g)\,(4z^2 + \pi xz) + \beta\,((4z^2 + \pi\,xz)\,(w - x) + 4xz^2) = 0. \qquad (9)$$

An approximation for z, valid for short periods of time follows by setting like powers of t=0, along with the initial conditions for z and dz/dt at t=0.

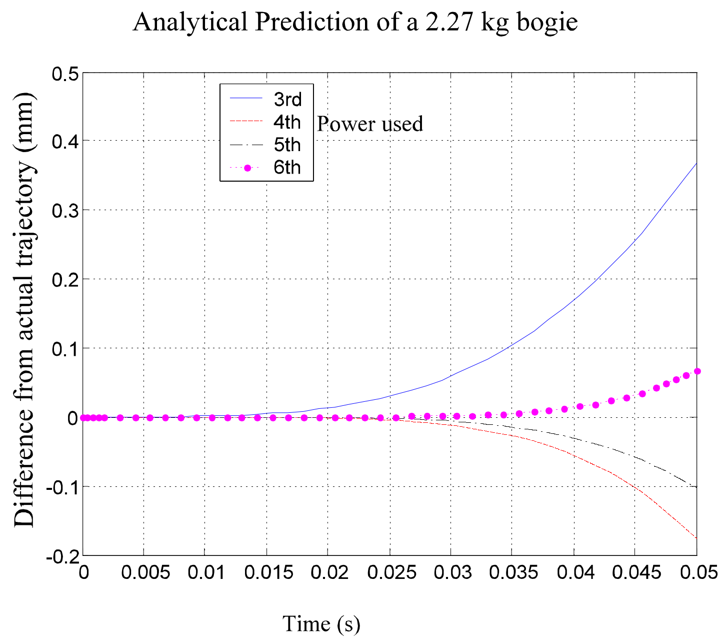

Figure 3 Difference of bogie position and various analytical approximations.

A 2.27 kg magnet (bogie) is placed at an arbitrary position in an 18 mm gap with a random initial velocity. The position z of the magnet is simulated using a Runge Kutta routine. The position is also computed using (9) with various powers of t. The error in mm between the position at various times between the actual magnet and (9) is plotted in Figure 3. As might be expected, representing z with more terms generally results in a better solution for longer times. The solutions suggest that a 4^{th} order polynomial representation in z is a good balance between accuracy obtained and computational time expended. These results suggest that the magnet's position can be predicted with confidence for 20 ms. Conversely, a current can be chosen at time t=0 that will guarantee that at time t=20 ms, the vehicle will be at a desired gap position $z_{desired}$. For a 4^{th} order polynomial,

$$a_2 = \frac{1}{2}\frac{(4z_0^3 g + z_0^2 g \pi x - 4\beta z_0 w - \beta \pi x w + \beta \pi x^2)}{z_0^2 (4z_0 + \pi xs)} . \tag{10}$$

$$a_3 = \frac{1}{3} v_z \beta \frac{(16 z_0^2 w + 8\pi x z_0 w - 6\pi x^2 z_0 + \pi^2 x^2 w - \pi^2 x^3)}{z_0^3 (4z_0 + \pi x)^2} . \tag{11}$$

$$a_4 = -\frac{1}{12}\beta(\pi^3 x^5 \beta + 64\beta z_0^3 w^2 - 32 v_z^2 z_0^2 \pi^2 x^3 + 48\pi x \beta z_0^2 w^2 + 10\pi^2 x^3 z_0^3 g + 144 v_z^2 z_0^3 \pi x w - 96 v_z^2 z_0^3 \pi x^2 + 36 v_z^2 z_0^2 \pi^2 x^2 w + 192 v_z^2 z_0^4 w + 3 v_z^2 \pi^3 x^3 z_0 w - 3 v_z^2 \pi^3 x^4 z_0 + 6\beta\pi^2 x^4 z_0 + 24\pi x^2 z_0^4 g - 40\beta z_0^2 w \pi x^2 - 12\pi^2 x^2 z_0^3 g w - 48\pi x z_0^4 g w - 2\pi^3 x^4 \beta w + \pi^3 x^4 z_0^2 g - \pi^3 x^3 z_0^2 g w + 12\pi^2 x^2 \beta z_0 w^2 - 18\pi^2 x^3 \beta z_0 w - 64 z_0^5 w g + \pi^3 x^3 \beta w^2)/[z_0^5 (4z_0 + \pi x)^3]. \quad (12)$$

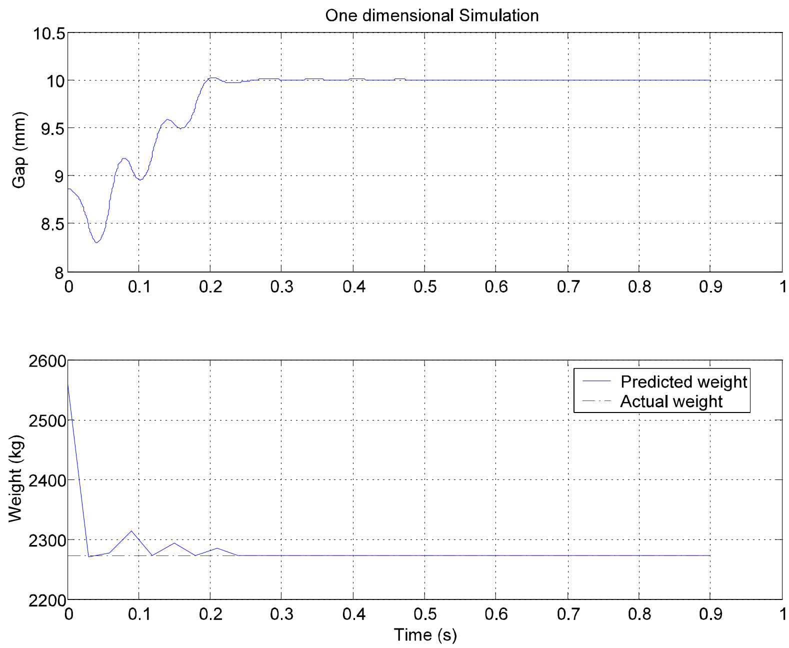

Figure 4 Gap position and weight estimate for a 30 ms update cycle.

Suppose the weight is unknown. The difference between the body's actual position and the desired position can be used to "weigh" the body. Usually it is sufficient to make this adjustment only after the first time increment in which the mass is in question. Since the adjustments on position are made a priori, ride quality can to some extent be predetermined. Suppose the desired displacement from the present position to the desired position is to be linear. Suppose also that the gap is randomly selected so that 2<gap<18 mm with zero initial velocity. If the target gap is 10 mm, then the largest deviation is 8 mm. Requiring that the controller place the bogie at the target in no less than 1 s, and using a 30 ms update cycle, implies a minimum displacement per cycle of 0.24 mm. Shown in Figure 4 is the actual gap position and weight estimate with time.

Requiring that the vehicle move 0.24 mm every 30 ms is an arbitrary assignment. If that requirement is relaxed to 0.12 mm every 30 ms, the simulation changes to that shown in Figure 5. Note that the oscillation period is dictated by the chosen update period. The amplitude of that oscillation is reduced by requiring that the vehicle move a smaller distance every period. The oscillations also diminish as the bogie nears the target.

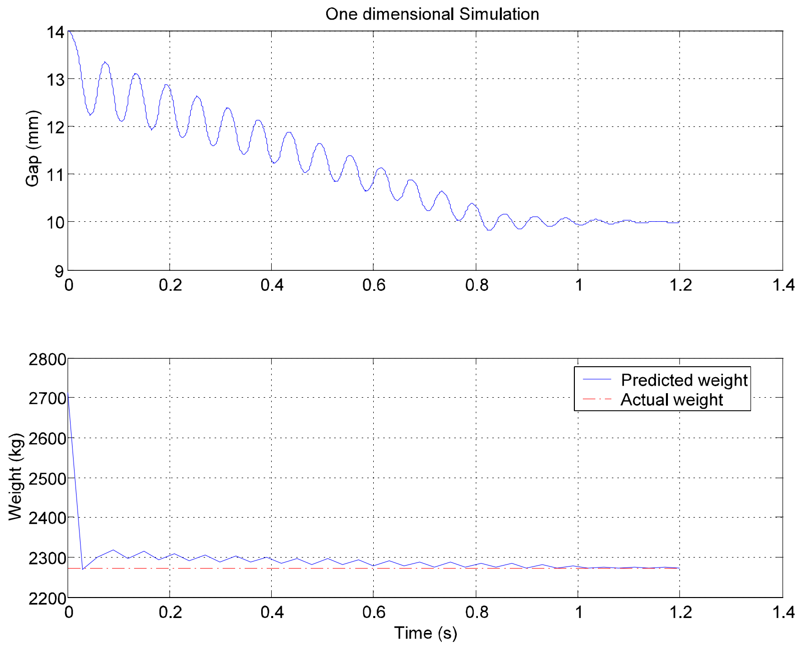

Figure 5 One dimensional simulation when the vehicle is commanded to move 0.12 mm every 30 ms.

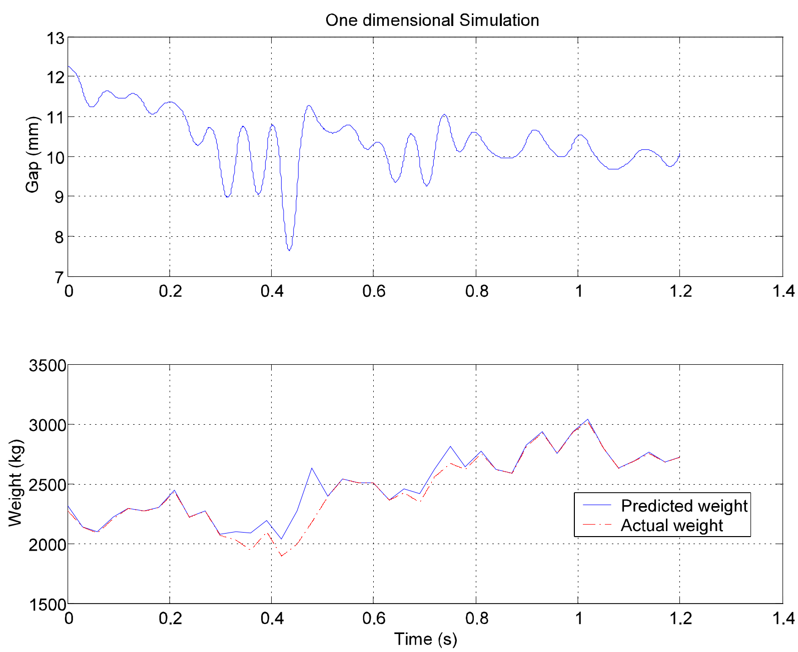

Figure 6 Response of the controller when the weight is randomly altered every 30 ms.

One way of checking the robustness is to randomly change the bogie weight every update period (30 ms) by as much as 10%. Shown in Figure 6 is this prediction. The algorithm is able to perform reasonably well even under these rather extreme conditions.

Adding Lateral Fluctuations

Since both (3) and (4) depend on both x and z, incorporating lateral motion forces a simultaneous solution of both (7) and

$$m\frac{d^2x}{dt^2} = F_x(x,z). \tag{13}$$

As with (6), a general solution for x is assumed in the form

$$x = x_0 + v_x*t + b_2*t^2 + b_3*t^3 + b_4*t^4 + b_5*t^5 + b_6*t^6 + \ldots . \tag{14}$$

The unknowns a2,a3,..,b2,b3,... must be determined simultaneously, and they each become functions of the excitation current, the bogie mass, as well as z_0,v_z,x_0,v_x.

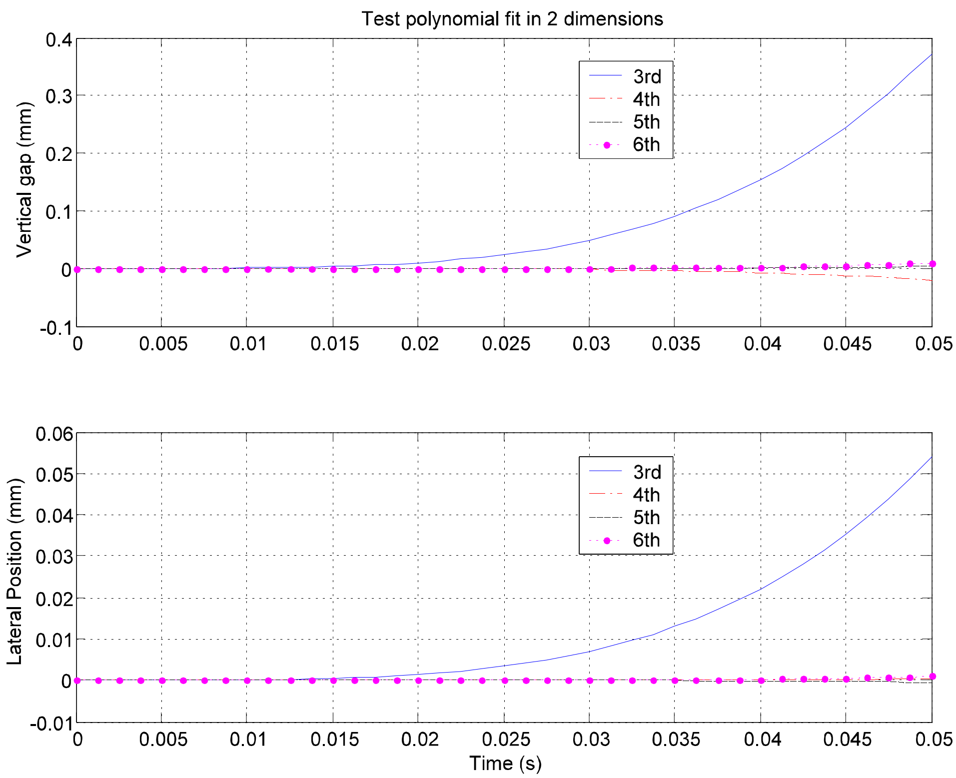

Figure 7 Comparison of bogie movement and polynomial prediction fo 50 ms.

The accuracy of this approach in the same way as with Figure 3. Figure 7 shows the accuracy of the various polynomial power estimates. The fit in both directions is quite accurate even for 50 ms.

Consider the criteria of Figure 5, with 30 ms time steps, with a requirement that the gap change by 0.12 mm in the direction of the target gap, 10 mm. Consider using only a 4th order polynomial estimate for the prediction of bogie movement. The performance for this condition is shown in Figure 8. Note that the lateral movement causes small errors in the weight prediction, but that effect is small on the control for the vehicle. As expected with only one magnet, suppression of lateral oscillation is not possible. Among the techniques for suppressing lateral oscillation is the technique of offsetting the magnets laterally about the midline. The two magnets swap a small percentage of current depending on the velocity. The magnet with the largest lateral displacement x will have an increase in current as the bogie is moving to increase the displacement.

Conclusions

Analytic expressions for the forces on U shaped magnets and shape are quite accurate, but usually require a constant to more closely approximate the actual forces. The equations of motion in both x and z can then be solved as a summation of various powers of time t. These solutions are very accurate for 30 ms. Since they are analytic solutions, they can be inverted in terms of current. Then the current can be computed to place the bogie at a specified position 30 ms into the future.

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